Scientific report

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Purpose of the visit

The purpose of the visit was:

- to develop methods permitting to compute the cohomology group of a homogeneous Q-manifold;

- to compare Lu's result about the K-invariant Poisson cohomology on $P = K/T$, where K is a compact semisimple Lie group and $T \subset K$ is a maximal torus, with the classical Borel-Weil-Bott Theorem.

- to discuss our ideas with Prof. Th. Voronov, Prof. H. Khudaverdian and Prof. K. Mackenzie;

Description of the work carried out during the visit

In the University of Manchester we started to develop methods permitting to compute the cohomology group of a homogeneous Q-manifold. First of all we studied results of the papers [Dr, Lu] and [LuWe].

Further, we considered Poisson-Lie groups and Poisson homogeneous spaces as spaces with additional super-symmetries. For instance, the Poisson bivector π on a Poisson homogeneoius manifold P and the mapping $X \mapsto L_X(\pi)$, where X is a fundamental vector field on P and L_X is the Lie derivative, may be regarded as an odd vector field on the supermanifold $(P, \bigwedge TP)$.

More precisely, we proved that any Poisson-Lie group (G, π) is a homogeneous space of a Lie supergroup $\mathcal{G}$ such that the mapping $X \mapsto L_X(\pi)$ is an element of the Lie superalgebra $\mathfrak{g} = \operatorname{Lie} \mathcal{G}$. In other words, it was shown that (G, π) is a homogeneous Q-manifold. We described also a class of Lie superalgebras and their subsuperalgebras such that the quotient space is a (local) Poisson-Lie group. In the case $\mathfrak{su}(2)$ we computed the Lie superalgebra and the Lie subsuperalgebra explicitly. The well-known theorem of Drinfeld

The category of local Poisson-Lie groups is equivalent to the category of bialgebras.

may be deduced from our result.

We also studied the case $P = G$, i.e. P is a Poisson homogeneous G-manifold with the Poisson structure compatible with the action of G on P by left translations. For instance, we described deformations of the vector field Q, which lead to Poisson structures compatible with this action of G on P. Our result is equivalent to the following result of Lu, Dazord and Sondaz:

The Poisson structures on P compatible with the action of G on P by left translations are in one-to-one correspondence with Lagrangian subalgebras $L \subset D(g)$ transversal to $\mathfrak{g}$.

We also started to study the general case $P = G/H$.

We discussed our ideas with Prof. Th. Voronov in the context of the possible applications. For instance, I learnt about the latest achievements in integration theory in the super-context. In Manchester we communicated also with Prof. H. Khudaverdian, the PhD student Adam Biggs, and other members of the School of Mathematics. We visited one of leading specialists in the theory of Lie algebroids Prof. K. Mackenzie in the University of Sheffield. We gave a talk at Manchester Geometry Seminar.

Description of the main results obtained

- Description of a class of Lie superalgebras and their subsuperalgebras such that the quotient space is a local Poisson Lie group. It was shown that we obtained all local Poisson-Lie groups.

- Computation of this Lie superalgebra and the Lie subsuperalgebra in the case $\mathfrak{su}(2)$ explicitly.

- Description of deformations of the vector field Q, which lead to Poisson structures compatible with the action of G on P by left translations.

Future collaboration with the host institution

We expect further collaborations with the members of the School of Mathematics, the University of Manchester, concerning the subject, i.e. visits, co-working.

Projected publications

We are going to study the general case $X = G/P$ in the super-context and to develop methods permitting to compute the cohomology group of a homogeneous Q-manifold for this case. Afterwards, we are going to publish our results in peer-reviewed journals.

References

[Dr] *Drinfeld, V.G.*, On Poisson homogeneous spaces of Poisson-Lie groups. (English, Russian summary) Teoret. Mat. Fiz. 95 (1993), no. 2, 226-227; translation in Theoret. and Math. Phys. 95 (1993), no. 2, 524-525.

[Lu] *Lu, Jiang-Hua*, Poisson homogeneous spaces and Lie algebroids associated to Poisson actions. Duke Math. J. 86 (1997), no. 2, 261304.

[LuWe] *Lu, Jiang-Hua, Weinstein, A.* Poisson Lie groups, dressing transformations, and Bruhat decompositions. J. Differential Geom. 31 (1990), no. 2, 501-526.