Exchange Visit Grant: Final Scientific Report

(Holographic Methods for Strongly Coupled Systems)

The Trivalent–Tree Phase of Strongly Coupled Random Matrix Models

1. General Information

Exchange Grant Number: HoloGrav 4076

Project Title: The Trivalent–Tree Phase of Strongly Coupled Random Matrix Models

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2. Purpose of the Visit

The main goal of the present exchange visit was to develop recent techniques for the analysis of random matrix models, namely, to develop the resurgent analysis of random matrix integrals and its applications in the study of the strongly coupled phase of these systems.

Let us briefly recall the main scientific context of this project, following the initial proposal submitted to the ESF. At large N, a random one–matrix integral Z may be well described in saddle–point approximation via a spectral curve, whose cuts essentially describe the clustering of eigenvalues in the complex plane. While this is a good large N approximation to computing the free energy $F = \log Z$, it is still just an approximation: it gets perturbatively corrected in $1/N^2$ (the usual 't Hooft expansion) and nonperturbatively corrected in $\exp(-N)$ (the instanton corrections describing eigenvalue tunneling between different cuts). All possible corrections to the saddle–point approximation may be assembled together in what is known as a resurgent transseries solution, and this was previously studied in detail by the applicant.

One fundamental point in these resurgent transseries solutions is that their validity may be thoroughly tested by matching large–order analysis via resurgence, which makes us very confident of their full content: some numerical tests have errors smaller than $10^{-60}\%$. But one other fundamental point of these solutions is that, besides the aforementioned corrections in $1/N^2$ and in $\exp(-N)$, they also have corrections of the form $\exp(+N)$ and these do not have a first principle *physical* interpretation. If one regards the resurgent transseries solution as the construction of the grand–canonical partition function of the random matrix model, then the natural question to address is: to what phase do these new terms correspond to?

The purpose of the visit was to investigate the physical content of these transseries solutions, in particular their phase content. In fact, note that by studying the full physical information encoded in a transseries solution one is also uncovering all possible phases of a given gauge theory and, in particular, understanding which phases admit large N closed string duals. While this seems to be always the case in the common 't Hooft Stokes phase, and never the case in the (oscillatory) anti–Stokes phase, there are other phases which are more mysterious at this point, such as the trivalent–tree phase (where, as the name gives it away, eigenvalues cluster into trees in the complex plane). Fully understanding what possible phases are encoded in random matrix models (zero–dimensional gauge theory) and which phases admit holographic duals was thus the main point associated to the present project.

3. Description of the Work Carried Out

Due to the limited amount of information concerning the trivalent–tree phase, and due to the fundamental question of whether this phase admits (or not) a closed string holographic dual description, one possible approach to its study is to start investigating what happens *directly* in the closed string side (as opposed to the gauge theory side, which has mostly prevailed in earlier studies). If one may approach resurgent transseries solutions directly in closed string theory, and if these solutions do not include a trivalent–tree phase, we will immediately learn that this (gauge theoretic) phase will *not* admit an 't Hooft expansion and we will further learn that, generically, gauge theoretic phases with closed string duals are just a small corner of the gauge theory phase space. If, on the other hand, closed string resurgent transseries solutions do include the trivalent–tree phase, it might well be the case that the closed string side will prove simpler to analyze than the gauge theoretic side. Given this strategy, our work thus focused on constructing resurgent transseries solutions for closed strings.

To understand how this may be done we must first recall the Dijkgraaf–Vafa large N duality between matrix models and B–model topological strings. In fact, one may wonder why is it the case that some random matrix integral may be solved, at large N, by a geometrical construction (the spectral curve). This may be understood if one first sees the spectral curve solution as actually being part of a Calabi–Yau geometry[1]. In this case, one may understand the geometric nature of the solution to the random matrix integral as the geometric transition implementing the large N duality that brings us from open topological B–strings on a resolved geometry (whose string field theory localizes into a matrix model) to the closed topological B–model on the corresponding deformed geometry—which is precisely the Calabi–Yau constructed out of the spectral curve. In this context, one may either solve B–model topological strings on the deformed geometry, or the random matrix model associated to the resolved geometry—via large N duality both approaches should be equivalent. But now one has an extra ingredient when trying to solve the closed string side: B–model topological strings on a specific background are solved via the famous holomorphic anomaly equations. These equations provide a setting for applying resurgent transseries techniques and, thus, try to uncover what closed string theory allows in its phase diagram, as described above. This was the main focus of the work carried out within this project.

4. Description of the Main Results

In this context, we may now specify our main results. Indeed, as hinted above, one may solve the holomorphic anomaly equations for closed B–model topological strings on a local Calabi–Yau geometry using resurgent transseries. This generalizes perturbative approaches to solving these equations—such as the direct integration method—which have been very successful in the past. We have found that the holomorphic anomaly equations always require the instanton action to be holomorphic and, thus, they do not fix the instanton action. In this way, it is still given by either an A or B cycle in the dual geometry, but one requires specific knowledge of the

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[1] This would be a local Calabi–Yau threefold constructed as a specific fibration over the spectral curve.

gauge theoretic dual in order to properly fix the instanton action. Once this is done, one may solve for the generalized multi–instanton sectors, and these may be understood generically as having a polynomial structure which follows in a similar fashion to the perturbative solution. Furthermore, the holomorphic anomaly equations allow for some rather general statements on the nature of the resurgent transseries solutions, in particular on what regards which types of sectors will have closed string expansions (*i.e.*, expansions in g_s^2) and which types of sectors will have D–brane expansions (*i.e.*, expansions in g_s). The holomorphic ambiguity in all nonperturbative sectors may be fixed by comparing to the conifold points, where previous work by the applicant has already fixed the "local" nonperturbative structure.

In summary, many new generalized multi–instanton sectors may appear, opening room to having new phases within the closed string side—much like it happened in the gauge theoretic side. However, actually understanding what these phases are requires working out examples. Furthermore, while we have obtained hints on how to develop it, the complete understanding of the trivalent–tree phase, from either gauge theoretic or closed string side, is still an open problem which will require further work and, eventually, further research visits.

5. Future Collaboration with the Host

The present visit and its subsequent research activities have made it possible to move forward in answering questions first formulated in the research proposal. As described above, many goals were achieved but, at the same time, many others still have to be fully understood and many new research questions were also put forward. In this way many new scientific projects between host and visitor can be envisaged for the near future. In particular, in order to proceed with the present line of research, more visits of scientific exchange will be required in order to fully pursue all scientific research lines which started with the present collaboration.

6. Projected Publications

One publication is presently being written and, as soon as it becomes available, a copy will be forwarded to the ESF. In this publication we report on the results which were described above as our main results, concerning the existence of resurgent transseries solutions to the holomorphic anomaly equations and exploring their general structure.