Gaia science for the dynamics of NEOs

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1 Purpose of the visit

The aim of this study is first to prepare some simulation codes for the imminent launching of the Gaia satellite (expected on October 2013). It is important to assess the applications of the data that will be delivered to the astronomer community throughout the five years of mission. This visit also aimed to continue and to strengthen the collaboration already established with the DAG group in the Institut of Astronomy of University of Vienna.

2 Description of the work done and main results obtained

2.1 Comparison of existing numerical integrators

In this section, I describe the work done for the comparison of four numerical integrators: Runge Kutta order 5th, Bulirsch-Stoer, Gauss-Radau and Lie integrators (for a description of each integrator, see Eggl and Dvorak [2010], Everhart [1974], Stoer and Bulirsch [1980]). These integrators are the main one used for the dynamical study of objects for short or long term integrations. Existing optimised codes for Gauss-Radau and Bulirsch-Stöer were used. The algorithm for Lie was implemented following Bancelin et al. [2012] and the algorithm for Runge Kutta is from Press et al. [1992]. Our aim was to test the accuracy of one integrator when a close encouter of an asteroid with the Earth occurs. This test was done with a backwards and forwards integration; the elapsed time between the backwards and forwards time integration should contain a close encounter with the Earth. The tested Near-Earth Objects were choosen according to their relative deep close distance with the Earth. These objects are (99942) Apophis, 2012 DA14, 2007 UD6 and 2008 TC3. For each asteroid and each integrator, we computed the dispersion on the semi-major axis value $\Delta a = a_0 - a_0'$ where a_0 is the initial value of a and a_0' is the initial value of a after the backwards and forwards integration. Table 1 contains the results obtained with the four integrators mentionned above. We listed the mininum distance for each asteroid (Min. Dist.), the value Δa for each integrator and the nominal uncertainty on the semi-major axis σ_a. As seen on the table, the order of magnitude of Δa is related to the current 1σ value of the semi-major axis. As a matter of fact, for a well-known orbit – e.g. asteroid Apophis with $1\sigma_a \sim 10^{-12}$ – the divergence after the close encounter is quite small, with Δ a $\sim 10^{-12}$ - 10^{-11}, even for a deep close encounter. For comparable distance but with a higger $1\sigma_a$, the divergence is more significant as the $\Delta a \sim 10^{-10}$.

Table 1: Divergence Δa for integrator Gauss-Radau (GS), Runge-Kutta (RK5), Bulirsch-Stoer (BS) and Lie (LIE). σ_a is for the current semi-major axis uncertainty.

Asteroid	Min dist [AU]	Δa_{GS} [UA]	Δa_{RK5} [AU]	Δa_{BS} [AU]	Δa_{LIE} [AU]	σ_a [AU]
Apophis	2.56×10^{-4}	9.65×10^{-12}	2.33×10^{-11}	1.01×10^{-11}	9.73×10^{-12}	1.27×10^{-12}
2012 DA14	2.27×10^{-4}	3.35×10^{-10}	7.82×10^{-10}	4.21×10^{-10}	3.42×10^{-10}	6.11×10^{-8}
2007 UD6	6.92×10^{-4}	3.87×10^{-10}	9.41×10^{-10}	4.42×10^{-10}	3.90×10^{-10}	1.02×10^{-4}
2008TC3	3.91×10^{-5}	7.81×10^{-10}	1.73×10^{-9}	8.23×10^{-10}	7.83×10^{-10}	4.28×10^{-6}

Figure 1 shows the performance for each integrator for the value indicated on Tab.1. We can see that Gauss-Radau (numbered 1) and Lie (numbered 4) integrators show similar performances whereas Runge Kutta integrator (numbered 2) presents the weakest performances.

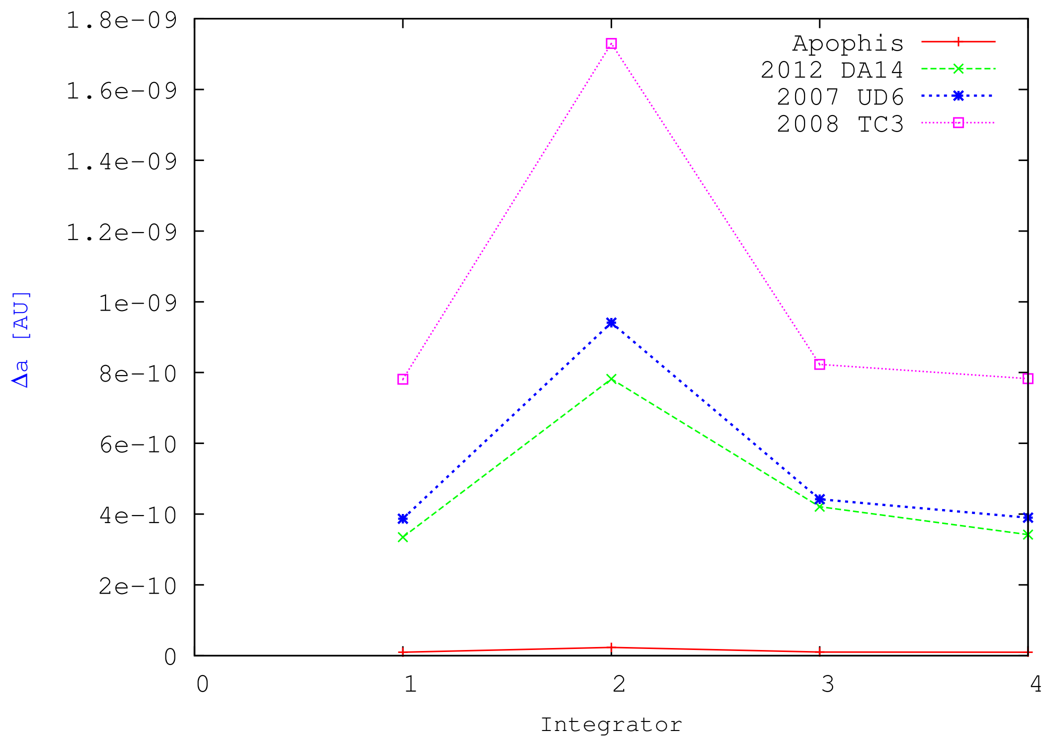

Figure 1: Divergence Δa for each integrator: 1 is for Gauss-Radau, 2 for Runge Kutta, 3 for Bulirsch-Stoer and 4 for Lie.

2.2 Orbital adjustment

The satellite Gaia will provide an unprecedented high astrometry for Solar System Objects. Data will be delivered throughout the five years of mission. A pipeline to identify objects entering the field of view of the satellite is implemented an rely on an auxiliary database of *astorb* [Bowell, 2011]. This database contains all the known asteroid and their relative orbital elements. For those which will be classified as known by the pipeline, a process of orbital adjustment will start. This process is fundamental for the mission because the database will continually be updated.
For this task, I implemented an algorithm for orbital adjusment using ground-based data (optical and range/range rate) and Gaia data. The orbital fit process is based of the minimisation of the residuals (Observed minus Computed data – hearafter O-C). The minimisation of this function is performed using the so famous Linear Least Square method (hearafter LLS). I used the algorithm of Press et al. [1992] for its implementation.
The computation of the O-C for optical data is trivial but not for radar and range rate data. For those data, I implemented the algorithm described Yeomans et al. [1992]. I used an existing code provided by a member of the Gaia-DPAC consortium[1] (*private communication*) to compute the barycentric position of the Gaia satellite during the 5-years mission. It mainly relies on an interpolation method using Tchebychev polynomial. The optical data are taken from the Minor Planet Center [2] and the radar data

[1] http://www.rssd.esa.int/index.php?project=GAIA&page=DPAC_Introduction
[2] http://minorplanetcenter.net/

are from the JPL website[3]. No Gaia data are available but they can be simulated. For this purpose, I used a Java simulator provided by the Gaia-DPAC to simulate observational rendez-vous between the asteroid and the satellite.
The object choosen for this study is the asteroid (99942) Apophis. Figure 2 shows the distribution of the Gaia observations obtained with the Java simulator. This shows that only half the orbit of Apophis would be covered by the satellite.

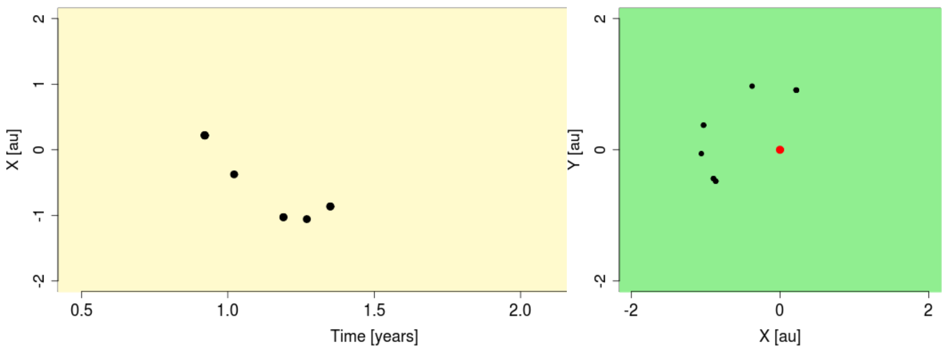

Figure 2: Left panel: Time distribution of Apophis observations. The abscissa are expressed as a function of the number of years after the beginning of the mission. Right panel: Spatial distribution of the Gaia observations of Apophis projected on the ecliptic plane and centered in the Sun (●).

This set of Gaia observations were used together with the optical and radar data for the orbital fit process described above and fully implemented. The input format for the Gaia data matches with those of the MPC[4]. Last but not least, the code observatory is required to assess the position of the observer. For an Earth-based observatory, it a combination of three characters (letters and/or numbers). For the Gaia satellite, this code is set to **GAI**. Considering a 5 mas astrometric accuracy for the Gaia data, I used the orbital fit process to assess the orbital improvement for Apophis, regarding the uncertainty on the keplerian elements. Table 2 shows the ratio of 1σ value for the keplerian elements considering Gaia data (σ_{O+G}) and without Gaia data (σ_O). This test shows that Gaia data will enable to gain a factor 1000 of accuracy on the semi-major axis.

The implementation of this pipeline for orbital fit is ready for running as soon as data coming from the satellite ($\sim$ 48h to send data to Earth) will be sent to the Earth.

[3] http://ssd.jpl.nasa.gov/?radar&fmt=html&grp=ast
[4] http://www.minorplanetcenter.net/iau/info/OpticalObs.html

Table 2: 1σ improvement of Apophis keplerian elements using Gaia data (O+G) or not (O).

	σ_O/σ_{O+G}
a [AU]	~ 1000
e	~ 10
i [°]	~ 10
Ω [°]	~ 10
ω [°]	~ 10
M [°]	~ 100

2.3 Variational equations for Lie series

The integration of the equations of motion in gravitational dynamical systems – either in our Solar System – or for extra-solar planetary systems – being non integrable in the global case, is usually performed by the means of numerical integration. Among the different numerical techniques available for solving ordinary differential equations, the numerical integration using Lie series has shown advantages. In its original form [Hanslmeier and Dvorak, 1984], it was limited to the N-body problem where only gravitational interactions are taken into account. In the paper of Bancelin et al. [2012], the relativistic perturbation and Yarkovsky effect were added so that this integrator can be used for the integration of near-Earth objects and impact risk computation. Nevertheless, it is impossible to assess the accuracy of an orbit or the stability of a system with this integrator because it is necessary to propagate the variational equations i.e. the partial derivatives of the orbital solution w.r.t. the components of the initial state vector. The purpose of this task is to develop a recurrence formula for the variational equations particular to the Lie series.
Considering a particule ν with barycentric coordinates $\mathbf{x}_\nu, \mathbf{v}_\nu$, we remind that, according to Bancelin et al. [2012], this coordinates, expressed as a Taylor expansion, are:

$$\mathbf{x}_\nu(\tau) = e^{\tau D}\mathbf{x}_\nu(0) = \left(\sum_{n=0}^{\infty} \frac{\tau^n D^n}{n!}\right)\mathbf{x}_\nu(0) \tag{1}$$

$$\mathbf{v}_\nu(\tau) = e^{\tau D}\mathbf{v}_\nu(0) = \left(\sum_{n=0}^{\infty} \frac{\tau^n D^n}{n!}\right)\mathbf{v}_\nu(0) \tag{2}$$

where D is the generalised Lie operator defined as:

$$D = \sum_{\mu=1}^{N}\left[\mathbf{v}_\mu \cdot \frac{\partial \cdot}{\partial \mathbf{x}_\mu} + \mathbf{H}_\mu \cdot \frac{\partial \cdot}{\partial \mathbf{v}_\mu}\right] \tag{3}$$

where $\mathbf{H}$ is the contribution of all the accelerations derived from the forces acting on a particle μ. Here, N denotes the number of bodies involved in the system. The variationnal equations are thus deduced from Eq. (1) and Eq. (2):

$$\left.\frac{\partial \mathbf{x}_\nu(\tau)}{\partial(\mathbf{x},\mathbf{v})}\right|_{\nu,0} = \sum_{n=0}^{\infty} \frac{\tau^n}{n!} \left.\frac{\partial(D^n \mathbf{x}_\nu(0))}{\partial(\mathbf{x},\mathbf{v})}\right|_{\nu,0} \tag{4}$$

$$\left.\frac{\partial \mathbf{v}_\nu(\tau)}{\partial(\mathbf{x},\mathbf{v})}\right|_{\nu,0} = \sum_{n=0}^{\infty} \frac{\tau^n}{n!} \left.\frac{\partial(D^n \mathbf{v}_\nu(0))}{\partial(\mathbf{x},\mathbf{v})}\right|_{\nu,0} \tag{5}$$

The initial conditions of our system, i.e. at time $\tau = 0$, of the partial derivatives of $\mathbf{x}_\nu$ and $\mathbf{v}_\nu$ are:

$$\begin{cases} \left.\dfrac{\partial \mathbf{x}_\nu(0)}{\partial(\mathbf{x},\mathbf{v})}\right|_{\nu,0} = (\mathbb{I} \quad \mathbb{O}) \\ \left.\dfrac{\partial \mathbf{v}_\nu(0)}{\partial(\mathbf{x},\mathbf{v})}\right|_{\nu,0} = (\mathbb{O} \quad \mathbb{I}) \end{cases}$$

For n≥2, the partial derivatives are non trivial. As a matter of fact, for n≥2:

$$\begin{cases} \left.\dfrac{\partial (D^n \mathbf{x}_\nu)}{\partial(\mathbf{x},\mathbf{v})}\right|_{\nu,0} = \left.\dfrac{\partial (D^{n-2} \mathbf{H}_\nu)}{\partial(\mathbf{x},\mathbf{v})}\right|_{\nu,0} \\ \left.\dfrac{\partial (D^n \mathbf{v}_\nu)}{\partial(\mathbf{x},\mathbf{v})}\right|_{\nu,0} = \left.\dfrac{\partial (D^{n+1} \mathbf{x}_\nu)}{\partial(\mathbf{x},\mathbf{v})}\right|_{\nu,0} \end{cases} \tag{6}$$

According to Eq. (6), the next step consists on developping a formula for the partial derivatives of $D^n\mathbf{H}$. In Bancelin et al. [2012], three accelerations are considered. In this work, we mainly focused on the gravitational and relativistic acceleration .

2.3.1 Partial derivatives for the gravitational acceleration

The gravitational acceleration acting on particule ν is written like:

$$\boldsymbol{\gamma}_{G/\nu} = GM_\star \sum_{\mu=1,\mu\neq\nu}^{N} m_\mu \Phi_{\nu\mu,3} \mathbf{x}_{\nu\mu}$$

and the reccurence formula for the Lie terms $D^n\boldsymbol{\gamma}_{G/\nu}$ given in Bancelin et al. [2012] is:

$$D^n \boldsymbol{\gamma}_{G/\nu} = GM_\star \sum_{\mu=1,\mu\neq\nu}^{N} m_\mu \sum_{k=0}^{n} \binom{n}{k} D^k \Phi_{\nu\mu,3} D^{n-k} \mathbf{x}_{\nu\mu} \tag{7}$$

where $\Phi_{\nu\mu,p} = \rho_{\nu\mu}^{-p}$ with $\rho_{\nu\mu}$ the relative distance between the bodies ν and μ. The work done in this study lead to the formula:

$$\left.\frac{\partial (D^n \boldsymbol{\gamma}_{G/\nu})}{\partial(\mathbf{x},\mathbf{v})}\right|_{\nu,0} = GM_\star \sum_{\mu=1,\mu\neq\nu}^{N} m_\mu \sum_{k=0}^{n} \binom{n}{k} \left[\left.\frac{\partial (D^k \Phi_{\nu\mu,3})}{\partial(\mathbf{x},\mathbf{v})}\right|_{\nu,0} D^{n-k}\mathbf{x}_{\nu\mu} + D^k \Phi_{\nu\mu,3} \left.\frac{\partial (D^k \mathbf{x}_{\nu\mu})}{\partial(\mathbf{x},\mathbf{v})}\right|_{\nu,0} \right] \tag{8}$$

The partial derivatives of $D^n\Phi_{\nu\mu,3}$ is required but unfortunalty, a reccurence formula has not already be found for the evolution of $\dfrac{\partial (D^n \Phi_{\nu\mu,3})}{\partial(\mathbf{x},\mathbf{v})}$. It is still in progress with the collaboration of the DAG group.

However, the other recurrence formulas have been successfully determined. $D^n \Lambda_{\nu\mu}$ is defined as

$$D^n \Lambda_{\nu\mu} = \sum_{k=0}^{n} \binom{n}{k} D^{n-k} \mathbf{x}_{\nu\mu} D^k \mathbf{v}_{\nu\mu} \tag{9}$$

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Therefore, the work done in this study leads to:

$$\left.\frac{\partial (D^n \Lambda_{\nu\mu})}{\partial(\mathbf{x},\mathbf{v})}\right|_{\nu} = \sum_{k=0}^{n} \binom{n}{k} \left[D^k \mathbf{x}_{\nu\mu} \left.\frac{\partial (D^{n-k} \mathbf{v}_{\nu\mu})}{\partial(\mathbf{x},\mathbf{v})}\right|_{\nu} + D^k \mathbf{v}_{\nu\mu} \left.\frac{\partial (D^{n-k} \mathbf{x}_{\nu\mu})}{\partial(\mathbf{x},\mathbf{v})}\right|_{\nu} \right] \tag{10}$$

2.3.2 Partial derivatives for the relativistic acceleration

The relativistic acceleration is defined as:

$$\boldsymbol{\gamma}_{R/\nu} = \frac{GM_\star}{c^2}\,\Phi_{\mu\nu,3}\,(\boldsymbol{\gamma}_1+\boldsymbol{\gamma}_2) \tag{11}$$

where

$$\begin{cases} \boldsymbol{\gamma}_1 & = \mathbf{x}_{\mu\nu}\left(4\,GM_\star\,\Phi_{\mu\nu,1} - \mathbf{v}^2_{\mu\nu}\right) \\ \boldsymbol{\gamma}_2 & = 4\,\mathbf{v}_{\mu\nu}\,\Lambda_{\mu\nu} \end{cases} \tag{12}$$

The evolution of the Lie series applied to $\boldsymbol{\gamma}_{R/\nu}$ is given in Bancelin et al. [2012] as:

$$D^n\,\boldsymbol{\gamma}_{R/\nu} = \frac{GM_\star}{c^2}\sum_{k=0}^{n}\binom{n}{k}D^k\,\Phi_{\mu\nu,3}\,D^{n-k}\,(\boldsymbol{\gamma}_1+\boldsymbol{\gamma}_2) \tag{13}$$

and

$$\begin{cases} D^n\,\boldsymbol{\gamma}_1 = \sum_{k=0}^{n}\binom{n}{k}D^{n-k}\,\mathbf{x}_{\mu\nu}\left(4\,GM_\star\,D^k\,\Phi_{\mu\nu,1} - \sum_{k'=0}^{k}\binom{k}{k'}D^{k'}\,\mathbf{v}_{\mu\nu}\,D^{k-k'}\,\mathbf{v}_{\mu\nu}\right) \\ D^n\,\boldsymbol{\gamma}_2 = 4\sum_{k=0}^{n}\binom{n}{k}D^k\mathbf{v}_{\mu\nu}\,D^{n-k}\Lambda_{\mu\nu} \end{cases} \tag{14}$$

Thus, the partial derivatives applied to Eq. (13) and (14) is done in this work and is expressed as:

$$\left.\frac{\partial(D^n\,\boldsymbol{\gamma}_{R/\nu})}{\partial(\mathbf{x},\mathbf{v})}\right|_\nu = \frac{GM_\star}{c^2}\sum_{k=0}^{n}\binom{n}{k}\left[D^k\Phi_{\nu\mu,3}\left.\left(\frac{\partial\boldsymbol{\gamma}_1}{\partial(\mathbf{x},\mathbf{v})}+\frac{\partial\boldsymbol{\gamma}_2}{\partial(\mathbf{x},\mathbf{v})}\right)\right|_\nu + \left.\frac{\partial(D^k\Phi_{\nu\mu,3})}{\partial(\mathbf{x},\mathbf{v})}\right|_\nu D^{n-k}(\boldsymbol{\gamma}_1+\boldsymbol{\gamma}_2)\right] \tag{15}$$

with

$$\frac{\partial(D^n\boldsymbol{\gamma}_1)}{\partial(\mathbf{x},\mathbf{v})} = \sum_{k=0}^{n}\binom{n}{k}\left[A\cdot\left.\frac{\partial(D^{n-k}\mathbf{x}_{\nu\mu})}{\partial(\mathbf{x},\mathbf{v})}\right|_\nu + B\cdot D^{n-k}\mathbf{x}_{\nu\mu}\right] \tag{16}$$

where

$$\begin{cases} A = 4\,GM_\star\,D^n\Phi_{\nu\mu,1} - \sum_{k'=0}^{k}\binom{k}{k'}D^{k'}\mathbf{v}_{\nu\mu}\,D^{k-k'}\mathbf{v}_{\nu\mu} \\ B = 4\,GM_\star\left.\frac{\partial(D^k\Phi_{\nu\mu,1})}{\mathbf{x},\mathbf{v}}\right|_\nu - \sum_{k'=0}^{k}\binom{k}{k'}\left(D^{k-k'}\mathbf{v}_{\nu\mu}\left.\frac{\partial(D^{k'}\mathbf{v}_{\nu\mu})}{\partial(\mathbf{x},\mathbf{v})}\right|_\nu + D^{k'}\mathbf{v}_{\nu\mu}\left.\frac{\partial(D^{k-k'}\mathbf{v}_{\nu\mu})}{\partial(\mathbf{x},\mathbf{v})}\right|_\nu\right) \end{cases}$$

and

$$\left.\frac{\partial(D^n\boldsymbol{\gamma}_2)}{\partial(\mathbf{x},\mathbf{v})}\right|_\nu = 4\sum_{k=0}^{n}\binom{n}{k}\left[D^k\mathbf{v}_{\nu\mu}\left.\frac{\partial(D^{n-k}\Lambda_{\mu\nu})}{\partial(\mathbf{x},\mathbf{v})}\right|_\nu + \left.\frac{\partial(D^n\mathbf{v}_{\nu\mu})}{\partial(\mathbf{x},\mathbf{v})}\right|_\nu D^{n-k}\Lambda_{\mu\nu}\right] \tag{17}$$

3 Future collaboration

During my visit, the collaboration with the DAG was mostly for the numerical integration and for the Lie series. Now, as a full member of the DAG – as a postdoc – the collaboration continues mainly for the implementation and test of the recurrence formula developped for the variational equations using Lie series. This study would certainly lead to a publication into an international revue (mainly Celestial Mechanics and Dynamical Astronomy).

References

D. Bancelin, D. Hestroffer, and W. Thuillot. Numerical integration of dynamical systems with Lie series. Relativistic acceleration and non-gravitational forces. *Celestial Mechanics and Dynamical Astronomy*, 112:221–234, February 2012.

E. Bowell. The asteroid orbital elements database, 2011. ftp://ftp:lowell.edu/pub/elgb/astorb.html.

S. Eggl and R. Dvorak. An Introduction to Common Numerical Integration Codes Used in Dynamical Astronomy. In J. Souchay & R. Dvorak, editor, *Lecture Notes in Physics, Berlin Springer Verlag*, volume 790 of *Lecture Notes in Physics, Berlin Springer Verlag*, pages 431–480, March 2010.

E. Everhart. Implicit Single-Sequence Methods for Integrating Orbits. *Celestial Mechanics*, 10:35–55, August 1974.

A. Hanslmeier and R. Dvorak. Numerical Integration with Lie Series. *A&A*, 132:203–+, March 1984.

W. H. Press, S. A. Teukolsky, W. T. Vettering, and B. P. Flannery. *Numerical recipes in Fortran 77*. Cambridge University Press, 1992.

J. Stoer and R. Bulirsch. Introduction to Numerical Analysis. *Introduction to Numerical Analysis, Second Edition, New York*, 1980.

D. K. Yeomans, P. W. Chodas, M. S. Keesey, S. J. Ostro, J. F. Chandler, and I. I. Shapiro. Asteroid and comet orbits using radar data. *AJ*, 103:303–317, January 1992. doi: 10.1086/116062.