

# Report for the Exchange Visit Grant of the ESF

Application Reference Number: 4473

**Purpose of the visit** The purpose of my visit was to establish a collaboration with Prof. Koszmider with the aim of conducting a preliminary exploration of the combinatorial aspects of operators on  $\ell_\infty/c_0$  and to investigate the question of embeddability of Banach spaces into  $\ell_\infty/c_0$ .

**Description of the work carried out during the visit** During my stay at Warsaw, Prof. Koszmider and I had several meetings per week to discuss the progress of the work. Each week I participated in the “Working group in applications of set theory” and I attended the Functional Analysis seminar at the Institute of Mathematics of the Polish Academy of Sciences (IM PAN). In the former seminar, I gave a talk on October 3rd entitled *The Open Coloring Axiom and embeddings into  $\wp(\mathbb{N})/Fin$* .

**Description of the main results obtained** In our investigation of the combinatorial aspects of operators on  $\ell_\infty/c_0$ , we were able to find a stronger version of the Drewnowski–Roberts lemma (Lemma of [1]) for a certain class of operators from  $\ell_\infty/c_0$  into  $\ell_\infty/c_0$ .

Regarding the question of embeddability of Banach spaces into  $\ell_\infty/c_0$ , we were interested in determining whether under OCA the space  $\ell_\infty(\ell_\infty/c_0)$  is isomorphically embeddable into  $\ell_\infty/c_0$ . We obtained a partial result in this direction having found a natural class of operators which cannot be isomorphic embeddings of  $\ell_\infty(\ell_\infty/c_0)$  into  $\ell_\infty/c_0$  under OCA.

More specifically, consider an ideal  $\mathcal{I}$  over the integers and denote by  $K_{\mathcal{I}}$  the Stone space of the Boolean algebra  $\wp(\omega)/\mathcal{I}$ . Our main result states that for a certain class of ideals, every isomorphic embedding of  $C(K_{\mathcal{I}})$  into  $C(\omega^*)$  of a certain type induces a Boolean embedding of  $\wp(\omega)/\mathcal{I}$  into  $\wp(\omega)/Fin$ . In other words, we have reduced the problem of isomorphic embeddability of  $C(K_{\mathcal{I}})$  into  $C(\omega^*)$  to the problem of embeddability of  $\wp(\omega)/\mathcal{I}$  into  $\wp(\omega)/Fin$ , for a certain type of operators. The next step is then

to generalize this and enlarge the class of operators and the class of ideals for which this is true. What we have achieved so far has opened a path for further exploration and gives indications that this type of reduction may be possible for any operator.

**Future collaboration with host institution** The results we obtained have been very encouraging. We are now interested in enlarging as much as possible the class of operators which satisfies the properties of the results we have proved. What we have achieved so far has opened a path for further exploration and gives strong indications that our results may hold for any operator.

We are now looking for ways of continuing our collaboration in order to explore these ideas. In particular, I will be returning to Warsaw for three weeks starting at the end of November. We are also planning future visits of myself to Warsaw and of Prof. Koszmider to Paris next year.

#### **Projected publications / articles resulting or to result from the grant**

1. P. Koszmider, C. Rodriguez Porras. *Combinatorial aspects of operators on  $\ell_\infty/c_0$* . Initial version in preparation.
2. C. Rodriguez Porras. *Linear embeddings into  $\ell_\infty/c_0$  under OCA*. Initial version in preparation.

## **Références**

- [1] Lech Drewnowski and James W. Roberts, *On the primariness of the Banach space  $\ell_\infty/c_0$* , Proceedings of the American Mathematical Society **112** (1991), no. 4, 949 – 957.