INFTY Exchange Visit - Scientific Report

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Project. *Implementing ordinals in set theory.*

Purpose of the visit. I visited Prof. Albert Visser to the Department of Philosophy of the University of Utrecht in order to share my own research on philosophical, mathematical and historical aspects of ordinal infinity with Visser's competences on this topic and on related areas. In particular, I wanted to profit from Visser's expertise on Interpretability theory and on Truth theories.

Description of the work carried out during the visit. Last October, I gave a talk about the Revision theory of truth (at the *Numbers and Truth* conference held in Gothenburg) which raised some commments by Visser about the alleged proof of my main result. Therefore, in the first part of my visit in Utrecht, we mainly focused on a series of informal seminars devoted to explain and refine my own proof of Gothenburg result.

In the second part of my visit, Visser gave me an overlook of his own research on *Interpretations as a notion of sameness* and I tried to apply his framework to my research on the mutual interpretation (over Zermelo set theory) between the theories of the ordinal numbers in Von Neumann and in Scott-Tarski style.

Besides this joint work on topics directly related with my research project, I have taken the opportunity of entering the academic life of the Department of Philosophy by informal interaction with other researcher and by attending invited talks on Carnap's semantics.

Description of the main result obtained. The main result obtained during my visit concerns a semantic theory of truth known as the *Revision theory of truth*. This theory was introduced in 1982 by Anil Gupta and Nuel Belnap and, independently, by Hans Herzberger, as an account for self-referential truth alternative to Kripke's. The question I addressed was

the following: is the role played by the transfinite ordinal numbers really essential for doing Revision? The theorem proved in Utrecht provides a negative answer for a wide range of revision theories, included Gupta's and Herzberger's original ones.

In the standard presentation, the Revision theory of truth aims to classify the sentences of a first-order language involving its own truth-predicate in *paradoxical* and *non-paradoxical* sentences. This task is accomplished by looking at the trend of the truth-values taken by a sentence along suitable transfinite sequences of models, called *revision sequences*. Any revision sequence is an ordinal-length sequence built-up by transfinite recursion: in Zermelo-Fraenkel set theory, this amounts to an essential involvement both of *proper classes* and of the axiom of Replacement.

My result shows that, for revision sequences like Gupta's or Herzberger's, two kinds of reduction are possible: first, it is possible to extract from an initial segment of the sequence the information carried by the full sequence, so that the classification of truth provided by the revision theory becomes formalisable in Zermelo set theory, avoiding both proper classes and Replacement. Secondly, we can dispense at all with ordinals and transfinite recursion by recasting the result in purely order-theoretic terms.

The result opens further research into two directions: on the set-theoretic side we have to investigate the possibility of extending these reductions to other kinds of revision sequences; on the truth-theoretic side we have to think about the philosophical significance of the class of revision sequences which admit these kinds of reductions.

Projected publication resulting from the grant. The result obtained will be recorded in the Logic Group Preprint Series of the University of Utrecht and will be submitted for publication, under the title *Revision without ordinals*, to an international academic journal operating in the Philosophical logic area.