ESF Research Networking Programme NEWFOCUS
Final Scientific Report

Project title: **Analysis and design of curved metasurface lenses**

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HOST
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TIME SCHEDULE
- Project duration: 6 months
- Start date: August 29, 2014
- End date: February 15, 2015

Purpose of the visit

Within this project the idea was to develop a model and numerical method for solving scattering from curved multilayered dielectric or metalic objects. Challenges in solving Maxwell's equations for considered structures are not in the equation themselves but in data describing the electromagnetic problem (large electric size, geometric coplexity, inhomogeneous discontinuous materials...). In order to successfully design curved lens structure one needs to develop specialized algorithms since the considered structures are very large in terms of wavelengths and they contain a lot of small details within each of the layers (by definition, the unit cell of the structure pattern is smaller than a wavelength). The need for a specialized program lies in the fact that these large finite structures with numerous small cells cannot be efficiently designed using general electromagnetic solvers since memory requirements would be extremely large and the successful optimization would be too slow. In order to solve described project some kind of reduction of dimension is derived which allows us to reduce 3D integral equations to 2D integral equations over thin structures. With this reduction it is possible to reduce computational time which means that optimization procedures becames faster and more efficient.

Description of the work carried out and main results obtained

In this report we propose an approximate method for solving the scattering problem from curved thin dielectric layer which allows us to model and produce curved multilayered lenses. We assume that permittivity ε is scaled as *1/h* where h describes thickness of the scatterer. The method starts with Helmholtz equation which is transformed to a Lippmann-Schwinger type of equation using standard Greens function method. In order to use information about small thickness of the structure perturbation method in terms of small parameter which describes thickness is applied. Solution to a full 3D problem is described as a series in dimensionless parameter h. Using that procedure starting integral equation over 3D structure is reduced to an integral equation over 2D structure. Transition from Helmholtz to Lippmann-Schwinger equation greatly reduces computational time but still we have to solve 3D integral equation. After perturbation analysis it is enough to solve only 2D integral equation.

Scattering from dielectric thin structure is described by scalar Helmholtz equation with permittivity ε:

$$\Delta u(\vec{x}) + k_0^2 \varepsilon(\vec{x}) u(\vec{x}) = 0$$

Solution of the problem is field u which can be decomposed as $u=u_i+u_s$ where u_i is incident field (excitation) and u_s is scattered field (solution of the scattering problem). Thin object will be marked as S_h where h is thickness of the object. We will assume that the object has a constant thickness.
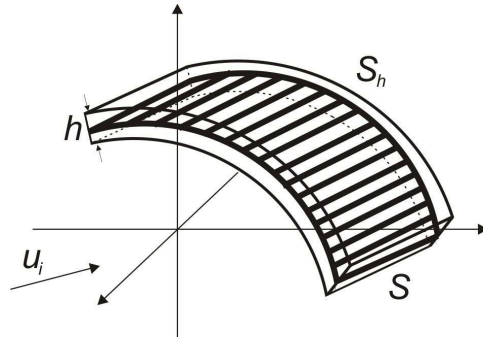

Figure 1. Scattering from curved object with thickness h

Inside object we have permittivity:

$$\varepsilon(\vec{x}) = \begin{cases} \dfrac{\varepsilon_0(\vec{x})}{h} & , \vec{x} \in S_h \\ 1 & , \vec{x} \notin S_h \end{cases}$$

We will assume that ε_0 is bounded inside object S_h. If we decompose solution u into incident and scattered field we can treat right side of the previous equation as an excitation for scattered field. Using free space Green's function G last equation could be transformed into integral equation for scattered field:

$$u_s(\vec{x}) = k_0^2 \int_{\mathbb{R}^3} \left(1 - \varepsilon(\vec{y})\right) G(\vec{x}, \vec{y}) u(\vec{y}) d^3 y$$

We assume that outside our object is free space which mean that $1-\varepsilon=0$ and integration is taken just over our object S_h. We can add incident field on both sides of last equation. With that we get integral equation of Lippmann-Schwinger type for total field u:

$$u(\vec{x}) = u_i(\vec{x}) + k_0^2 \int_{S_h} \left(1 - \varepsilon(\vec{y})\right) G(\vec{x}, \vec{y}) u(\vec{y}) d^3 y$$

Last equation is implicit equation for total field u. We have total field u on both sides of the equation. That is a god and a bad point. If we know total field inside object S_h we can use it to compute field in every point in space. That means that we reduced portion of space in which we will use numerical solution to solve the problem. Bad thing is that we need total field inside object S_h which is not explicitly given in the equation. Mathematical theory behind equations of similar type are given in the theory of compact operators and three Riesz theorems about operators of $I+A$ type where I is identity operator, and A is an compact operator. That equation is a starting point for perturbation procedure in which we will use information that our structure is very thin.

In order to use perturbation method we will have to write object S_h in terms of parametrization with set Ω_h. We will assume that our object could be parametrized with diffeomorphic parametrization and with only one chart. If object is more complicated and more chart are needed that computation will be more complicated but all ideas will be the same. After all derivation only what will be complicated in application of our results is finding the parametrization for different objects. We will assume that y is parametrization of our object S_h in terms of variable z from Ω_h.
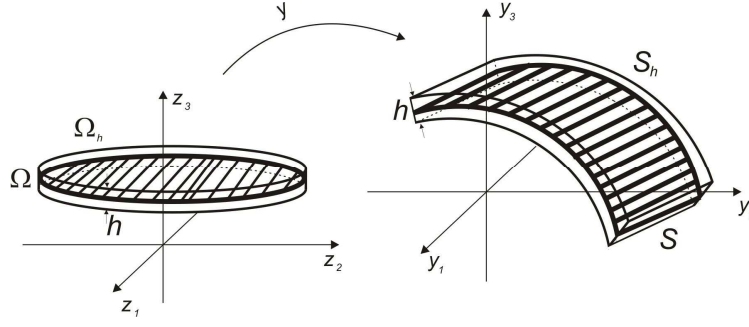
Figure 2. Parametrization of curved structure

In order to define integration over structure S_h we have to define metric tensor g of the structure and redefine our integral equation as:

$$u(\vec{x}) = u_i(\vec{x}) + k_0^2 \int_\Omega \int_{z_0-\frac{h}{2}}^{z_0+\frac{h}{2}} \left(1 - \frac{\varepsilon(\vec{y}(\vec{z}))}{h} \right) G(\vec{x}, \vec{y}(\vec{z})) u(\vec{y}(\vec{z})) \sqrt{|g(\vec{z})|} d^3 z$$

We have to scale variable in z_3 in order to get dimensionless parameter h and to remove h from integral boundaries. After transformation we get equation which is prepared for perturbation analysis:

$$u(\vec{x}) = u_i(\vec{x}) + k_0^2 \int_\Omega \int_{z_0-\frac{1}{2}}^{z_0+\frac{1}{2}} \left(h - \varepsilon(\vec{y}(\vec{z})) \right) G(\vec{x}, \vec{y}(\vec{z})) u(\vec{y}(\vec{z})) \sqrt{|g(\vec{z})|} d^3 z$$

We assume that our solution could be described as local perturbation series in terms of dimensionless parameter h which describe thickness of the structure. In the rest of the report we will use two notation. One represent point in space in terms of parametrization domain (x), and other represents point on the surface S (x_0):

$$\vec{x} = \vec{x}(z_1, z_2, z_0 + z_3)$$
$$\vec{x}_0 = \vec{x}(z_1, z_2, z_0)$$

Same is with point y. Incident field is a plane wave so it could be decompose in terms of it Taylor series. If we put both series into equation we will get:

$$u_0(\vec{x}) + h u_1(\vec{x}) + \ldots = u_i(\vec{x}_0) + h \frac{\partial u_1}{\partial z_3}(\vec{x}_0) + \ldots$$

$$+ k_0^2 \int_\Omega \int_{z_0-\frac{1}{2}}^{z_0+\frac{1}{2}} \left(h - \varepsilon(\vec{y}) \right) G(\vec{x}, \vec{y}) \left(u_0(\vec{y}) + h u_1(\vec{y}) + \ldots \right) \sqrt{|g|} d^3 z$$

We can collect terms without h into equation for first term approximation:

$$u_0(\vec{x}) = u_i(\vec{x}) - k_0^2 \int_\Omega \int_{z_0-\frac{1}{2}}^{z_0+\frac{1}{2}} \varepsilon(\vec{y}) G(\vec{x}, \vec{y}) u_0(\vec{y}) \sqrt{|g|} d^3 z$$

Idea is to take limit $h \to 0$ and get implicit equation of first order approximation u_0. In order to show convergence and error estimate we have to go deeper into the analysis of previous equation.

Everything connected with the analysis behind this equations and all numerical estimates will be presented in forthcoming paper. Here is just the final solution which is a good starting point for an analysis of the problem, numerical methods, eventual homogenization and optimizations:

$$u_0(\vec{x}) = u_i(\vec{x}) - k_0^2 \int\limits_{\Omega} \varepsilon(\vec{y}_0) G(\vec{x}_0, \vec{y}_0) u_0(\vec{y}) \sqrt{|g_0|}\, d^2z$$

At the end we have difference between approximate and real solution, where u_0 represents approximate solution, and v is equal to full solution:

$$\|v - u_0\|_\infty \leq Ch$$

where C is a constant which is independent on h but could be dependent on k_0. That means that the solution of an approximate 2D scattering problem converge to a solution of a full 3D scattering problem. With this procedure we decreased computational time because we reduced our 3D problem to a solution of a 2D problem. We reduced one dimension which will reduce matrix size in our numerical solution of the problem. We also know speed of this convergence which is $O(h)$.

In order to show previous asymptotic solution in action we will solve scattering in 2D space from a one part of the thin sphere of thickness h with $r=0.03$ m, $\phi_1=180^0$ and $\phi_2=0^0$. Our object will be homogeneous with permittivity $\varepsilon_0=10$. For scattering problems in 2D space we have to use different Greens function than in 3D space. In first problem incident field have a direction k=(1,1). There is no special reasons for picking up a 2D problem. Idea is just to show how powerfull is our approximation so we used simpler case because it is easier and faster to solve it.
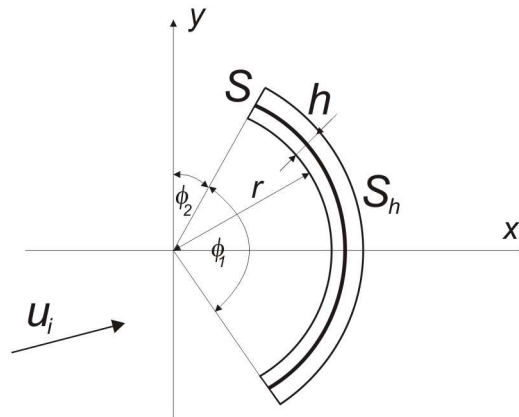

Figure 3. Test model for approximation derived in this report

We will look at total field in one line behind the object and on absolute error between full and approximate solution. All solutions will be obtained through numerical solution of integral equation. In order to get rid of numerical error we will use high density mesh for both numerical solutions. On the Figure 4. we can see the field values calculated for a full model and approximate solution. We look at the field value on a position x=0.15 m, and y=-0.3 m to y=0.3 m. We can see that the difference between approximate and a full solution is very small. On a Figure 5. we have comparison between 2 different full solution. One is for h=10^-3 m and second one is for h=10^-4 m. As we can see it is very hard to distinguish approximate solution and full solution for h=10^-4 m so I put a Figure 6. on which one small portion of previous Figure is zoomed. On a Figure 7. absolute error between approximate and a full solution is presented. As we predicted in a theoretical part of the report, error have an O(h) behaviour. From this theoretical work we see that for a thin dielectric we can use approximation which is derived through this visit to University of Michigan, and get a very accurate results. Generalization of this result to a multilayer structure is straightforward and it is not so hard to use it in some of those situations.
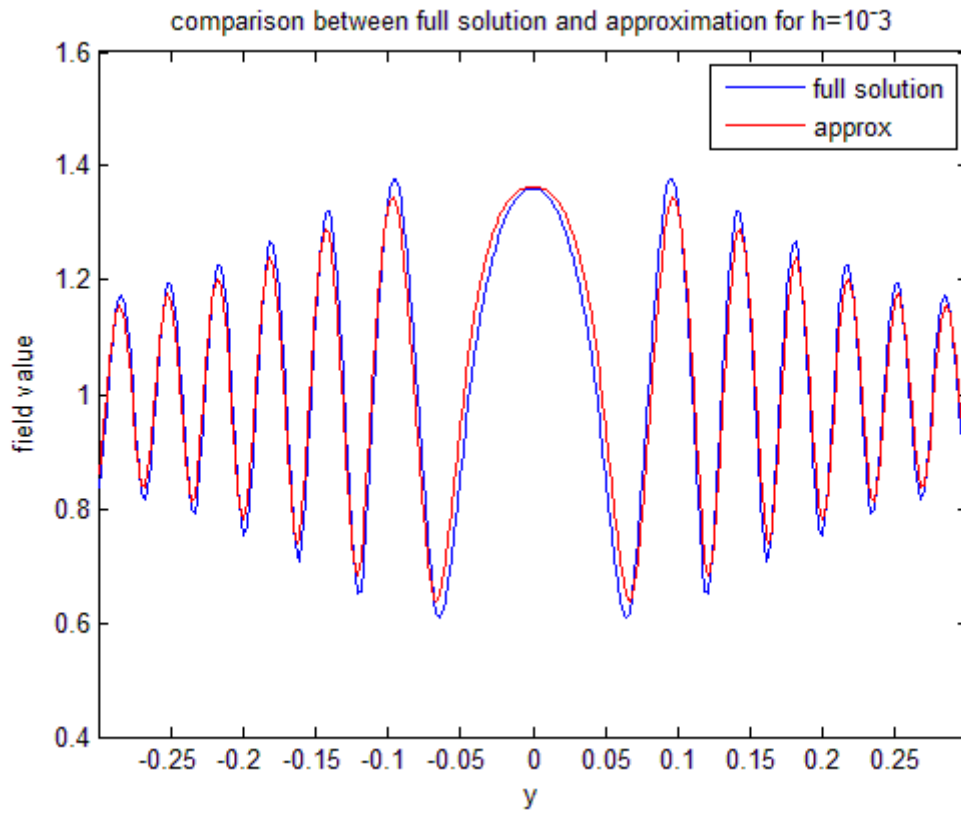
comparison between full solution and approximation for h=10⁻3

Figure 4. Comparison between solution of a full 3D model and
approximate 2D model for h=10^-3 m

comparison between full solution and approximation
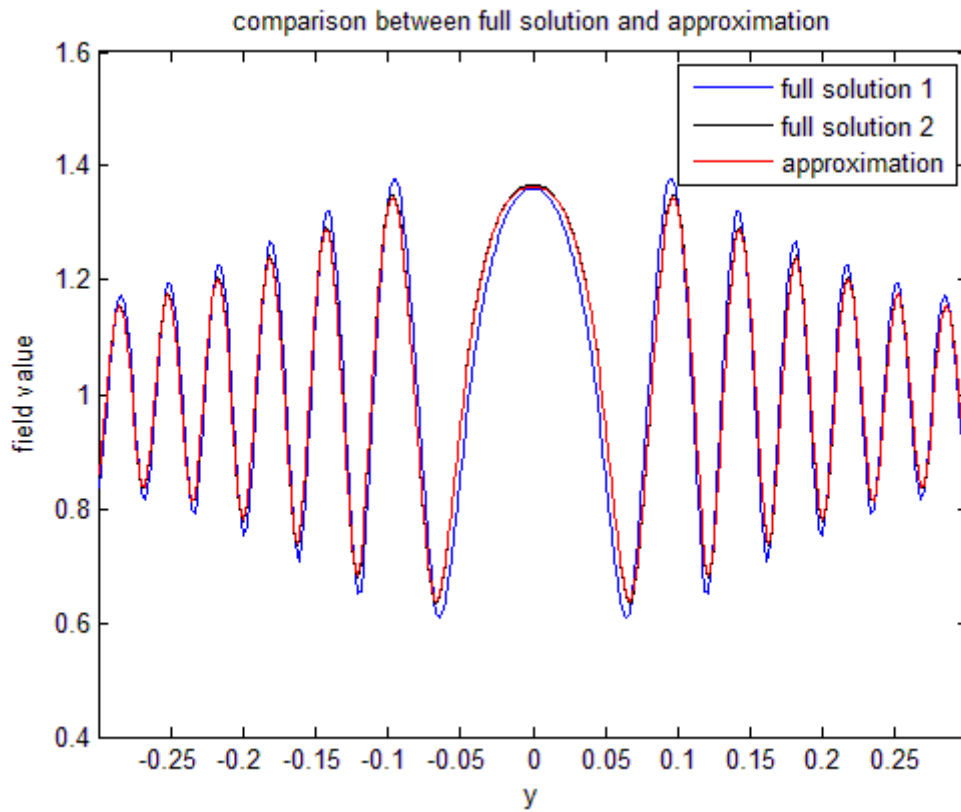

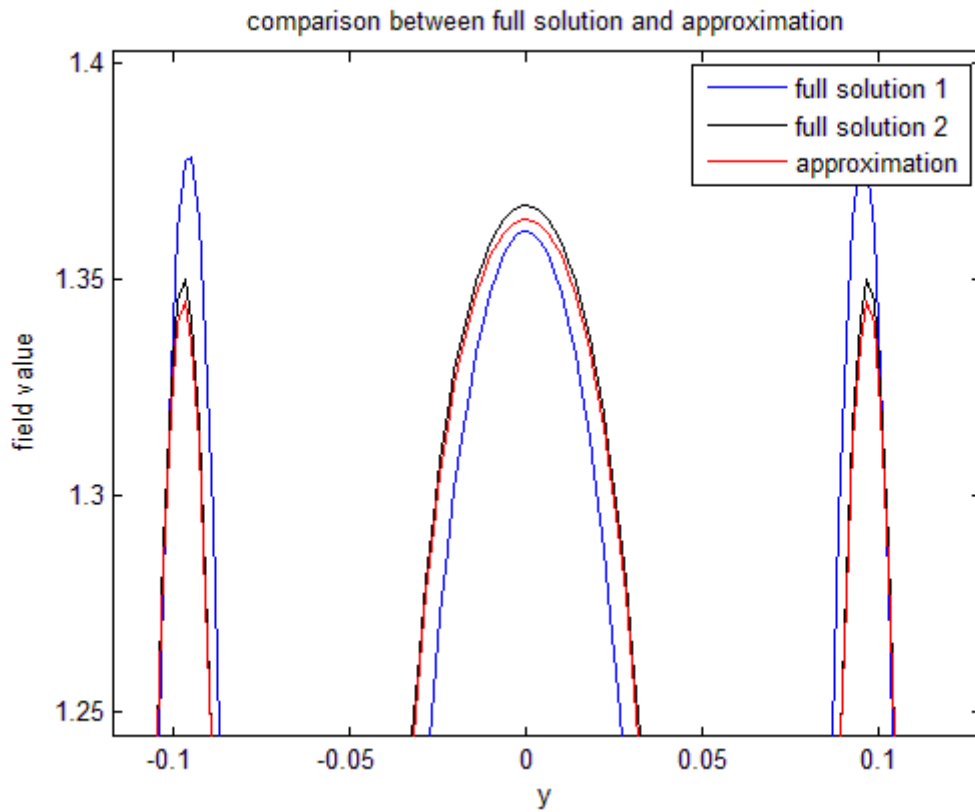Figure 5. Comparison between solution of a full 3D model and approximate 2D model

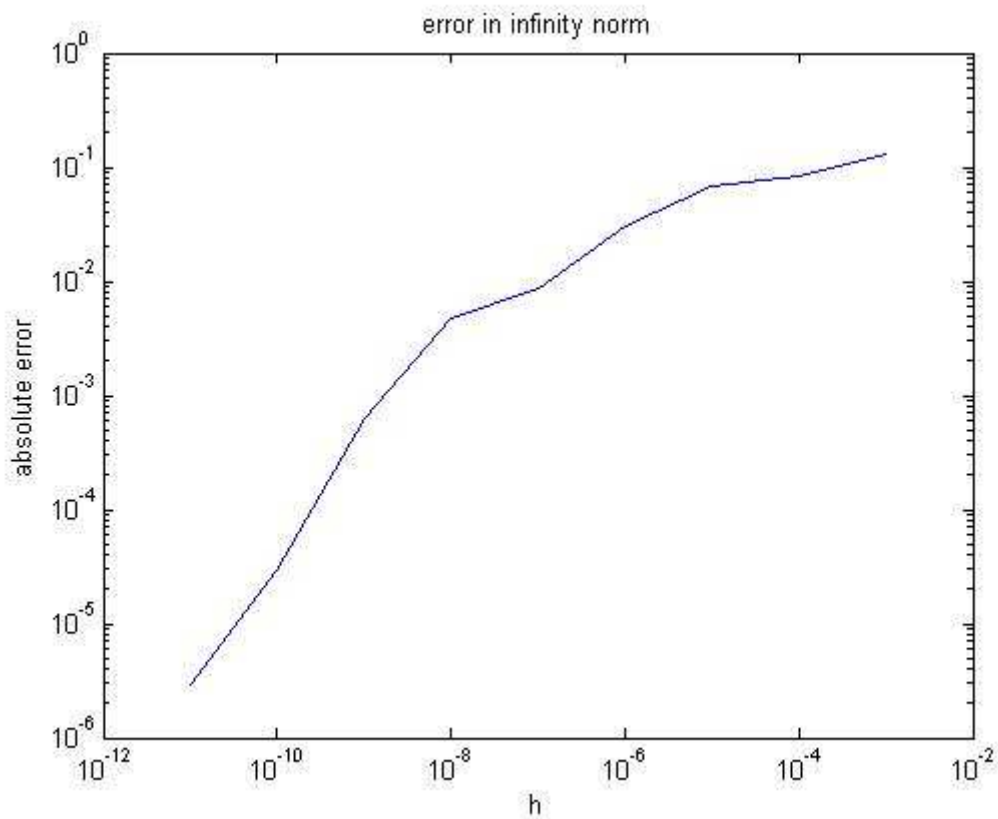Figure 6. Zoomed one small portion of a Figure 6 for h=10^-3 m and h=10^-4 m

Figure 7. Absolute error between approximate solution and a full solution for h=10^-3 m to h=10^-11 m

Future collaboration with host institution

Solved problem is just a first step in a scientific work in this area. This method allows us to work on models of multilayered curved lenses without any constraint on a permitivity inside or a shape of the lense. This method allows us to put some kind of periodic structure inside and then made standard two scale homogenization which is very usefull in lense design. Because this is just a first step in developing general procedure for designing curved multilayered structures we expect to collaborate with prof. Anthony Grbic's group a lot in the future. We expect to finish one design as soon as possible and produce it in University of Michigan Nanofabrication Laboratory and measured it in Radiation Laboratory at the University of Michigan.

Projected publications/articles resulting from the grant

During the exchange period one journal paper was prepared and it will be submitted as soon as possible to IEEE Transaction on Antennas and Propagation, and one conference paper is submitted and accepted on the EuCAP conference. We expect to write one more paper about our theoretical work and one paper about design.