ALTA
Institute for Algebra, Geometry, Topology and their applications

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Scientific report of the conference

Applied and Computational Algebraic Topology

July 15 – 19, 2013
Universität Bremen

1. Summary.

The conference took place July 15 - July 19, 2013, on the campus of the University of Bremen. The intensive program (attached to this report) was attended by approximately 50 participants.

The scientific program was comprised of a number of 1-hours talks as well as several 30-minute presentations. In addition there was a board meeting of the steering committee of the ESF Network Program "Applied and Computational Algebraic Topology" under whose auspices this conference was organized.

Both the presentations themselves, as well as lively discussions which followed the talks and often lasted long into the coffee breaks und into the evenings, provided an essential forum for the exchange of cutting-edge research in the field.

2. Description of the scientific content.

Several important directions within the general subject of Applied and Computational Algebraic Topology were pursued. We would like to single out the following ones:

- Computational Algebraic Topology;
- Stochastic Topology;
- Applications to Concurrency and Distributed Computing.

2.1. Computational Algebraic Topology.

A central theme has been the notion of persistent homology, introduced by Edelsbrunner, which is used to understand high-dimensional data sets, and is of vital importance to many applications, including, but not restricted to the analysis of shapes and images. The conference opened with a presentation by Herbert Edelsbrunner about the recent developments connecting persistent homology to dynamical systems. Given a self-map $f : X \to X$, the dimension of an eigenspace of the map induced on the homology can be computed using persistent homology of a multi-scale representation of a finite sampling of the self-map. The talk also dealt with the algorithmic side of the computation.

There were many talks which followed the theme of persistence after the Edelbrunner presentation. Paul Bendich was talking about how to adopt Frechet means to connect statistics and persistence; a theme which also provides a link to the stochastic topology below. Facundo Memoli talked about curvature and persistence. Dmitriy Morozov talked about merge trees and 0-persistence. Claudia Landi talked about a bridge between continuous and discrete multidimensional persistent homologies.

There were several other talks on the topic and it was fascinating to see how many different seemingly unrelated fields of mathematics could all come to fruitful interaction with persistence and be used for the analysis of large data sets.

2.2. Stochastic Topology.

The general framework of stochastic topology is as follows: given a probability space P and an algebraic functor on P, we want to understand the induced probability distribution on the algebraic invariants. This is a relatively new area of research, yet there have been an astonishing amount of progress.

The first talk on this topic was by Michael Farber who discussed geometric and topological properties of random 2-complexes. One of the central questions is whether one can generate randomly aspherical 2-complexes (i.e. such that $\pi_2(Y) = 0$) and whether random aspherical 2-complexes satisfy the Whitehead Conjecture. This conjecture was proposed in 1941 by J.H.C. Whitehead; it states that any subcomplex of an aspherical 2-complex is also aspherical. A result presented in the talk stated that (under certain assumptions) any aspherical subcomplex $Y' \subseteq Y$ of a random 2-complex Y satisies the Whitehead conjecture, with probability tending to 1.

Roy Meshulam talked on topology and combinatorics of Ramanujan complexes. Ramanujan complexes are a certain family of finite quotients of affine buildings associated with the linear group over a local field. Ramanujan complexes are on one hand highly structured, e.g. all their vertex links are isomorphic to the order complex of subspaces of a finite vector space. On the other hand they exhibit random like properties which makes them (potentially) useful in various extremal problems in topological combinatorics. Meshulam discussed some applications of Ramanujan complexes, including e.g. their essential optimality with respect to the higher dimensional Moore bound for the diameter of a top dimensional cycle in a simplicial complex.

2.3. Applications to Concurrency and Distributed Computing.

There are many applications of algebraic topology to concurrency. One such connection is provided by the field of directed algebraic topology, where the ambient space has some sort of direction, and homotopy is required to respect that direction. The resulting theory of homotopy invariants is quite intricate and rather different from the classical one, yet very useful in certain concurrency models phrased in terms of higher-dimensional automata.

Martin Raussen discussed particular classical examples of directed spaces, a class of Higher Dimensional Automata (HDA). For such a space, he described a method that determines the homotopy type of the space of traces (executions) as a prodsimplicial complex. A description of that complex opens up for machine calculations of homology groups and other topological invariants of the trace space. The determination of path components is particularly important for applications.

Lisbeth Fajstrup followed up on this theme by studying cut-off theorems in Dijkstra's PV-models. In such a model a program is given by its use of shared resources. A thread is a list of requests for access and release of resources. She considered the special case, where a thread T is run in parallel with itself n times, given the joint execution. A cut-off theorem is a result that a property holds for all n, if and only if it holds up to a fixed n. Two theorems of that kind were presented in the talk.

3. Assessment of the results and impact of the event on the future directions of the field.

The conference was a very important platform for the exchange of ideas. Central concepts of the fields (such as persistence homology) were brought in connection with very many different topics which was very informative and provided an excellent environment for the cross-pollinating of the ideas. Many minor issues which arose in the talks could be resolved immediately in the discussions which followed. Furthermore, teams of researchers coming from different corners of the fields were build spontaneously during the meeting to approach more sophisticated questions in the long run.

In addition, both the informal discussions as well as the board meeting on Tuesday led to consolidation of future plans. As one immediate outcome, it was decided to pursue an application for a trimester on the topic at the Hausdorff Center of Mathematics in Bonn. This application is now in the final stage of preparation. Other plans for the continuation of the current ESF network were discussed.

4. Annexes.

Programme of the meeting and full list of speakers and participants are attached to this report.

The scientific program

Monday, July 15, 2013

08:50–09:00 Opening of the conference

09:00–09:50 **Herbert Edelsbrunner**
Sampled dynamical systems

10:00–10:50 **Paul Bendich**
Towards statistics on vineyards with fuzzy Frechet means

coffee break

11:20–12:10 **Rick Jardine**
Homotopy theories of dynamical systems

lunch break

14:00–14:50 **Dmitriy Morozov**
Back to basics: merge trees

coffee break

15:20–16:10 **Eric Goubault**
Determination of tree spaces, and the geometric nature of synchronisation

16:20–16:50 **Daniel Müllner**
Stability of levelset zigzag persistence and discretized Reeb graphs

16:50–17:20 **Justin Curry**
Persistent homology via cellular cosheaves

Tuesday, July 16, 2013

09:00–09:50 **Michael Farber**
Geometry and topology of random 2-complexes

10:00–10:50 **Roy Meshulam**
Topology and combinatorics of Ramanujan complexes

coffee break

11:20–11:50 **Kevin Knudson**
Syzygies and multi-dimensional persistence

11:50–12:20 **Vitaliy Kurlin**
A persistence-based reconstruction of homotopy types of graphs from noisy samples in the plane

lunch break

14:00–14:50 **Martin Raussen**
Spaces of directed paths as simplicial complexes

coffee break

15:20–15:50 **Claudia Landi**
A bridge between continuous and discrete multidimensional persistent homologies

15:50–16:20 **Lisbeth Fajstrup**
Cut-off theorems in PV-models, a geometric approach

16:30–18:00 **ACAT Board Meeting**

19:30 **Conference dinner**

Wednesday, July 17, 2013

09:00–09:50 **Graham Ellis**
Applied computational group theory?

10:00–10:50 **Vin de Silva**
Persistent cohomology and the topological analysis of recurrent signals

coffee break

11:20–12:10 **Neza Mramor Kosta**
Birth and death in discrete Morse theory

lunch

Excursion:
Auswandererhaus / German Emigration Center

Thursday, July 18, 2013

09:00–09:50 **Peter Bubenik**
Metrics on diagrams and persistent homology

10:00–10:50 **Frederic Chazal**
Optimal rates of convergence for persistence diagrams in topological data analysis

coffee break

11:20–11:50 **Sanjeevi Krishnan**
Higher dimensional flow-cut dualities

11:50–12:20 **Thomas Kahl**
On topological abstraction of higher dimensional automata

lunch break

14:00–14:50 **Dominique Attali**
Collapsing Rips complexes for shape reconstruction in high dimensions

coffee break

15:20–16:10 **Matthew Kahle**
Topology of random flag complexes

16:20–16:50 **Sefi Ladkani**
Derived categories arising from combinatorial data

16:50–17:20 **Hubert Wagner**
Persistent homology in text mining

Friday, July 19, 2013

09:00–09:50 **Facundo Memoli**
Curvature sets over persistence diagrams

10:00–10:50 **Patrizio Frosini**
Adapting persistent homology to invariance groups

coffee break

11:20–12:10 **Lucile Vandembroucq**
On topological complexity and related invariants

lunch

Participants/speakers

Michal Adamaszek	University of Bremen
Dominique Attali*	CNRS Grenoble, France
Ulrich Bauer	IST Austria, Austria
Paul Bendich*	Duke University, USA
Christoph Bey	University of Bremen
Magnus Botnan	NTNU Trondheim, Norway
Roman Bruckner	University of Bremen
Peter Bubenik*	Cleveland State University, USA
Frederic Chazal*	INRIA, France
Justin Curry*	University of Pennsylvania, USA
Emanuele Delucchi	University of Bremen
Martin Dlugosch	University of Bremen
Herbert Edelsbrunner*	IST Austria, Austria
Graham Ellis*	National University of Ireland, Ireland
Lisbeth Fajstrup*	Aalborg University, Denmark
Michael Farber*	University of Warwick, UK
Eva-Maria Feichtner	University of Bremen
Dmitry Feichtner-Kozlov	University of Bremen
Massimo Ferri	University of Bologna, Italy
Patrizio Frosini*	University of Bologna, Italy
Eric Goubault*	CAE Saclay, France
Gerrit Grenzebach	University of Bremen
Tim Haga	University of Bremen
Zur Izhakian	University of Bremen
Rick Jardine*	University of Western Ontario, Canada
Thomas Kahl*	University of Minho, Portugal
Matthew Kahle*	Ohio State University, USA
Kevin Knudson*	University of Florida, USA
Marek Krcal	Charles University, Czech Republic
Sanjeevi Krishnan*	University of Pennsylvania, USA
Vitaliy Kurlin*	Durham University, UK
Catherine Labruère	University of Bourgogne, France
Sefi Ladkani*	University of Bonn, Germany
Claudia Landi*	University of Modena, Italy
Facundo Memoli*	University of Adelaide, Australia
Roy Meshulam*	Technion, Israel
Dmitriy Morozov*	LBNL, USA
Neza Mramor Kosta*	University of Ljubljana, Slowenia
Daniel Müllner*	Stanford University, USA
Martin Raussen*	Aalborg University, Denmark
Sonja Riedel	University of Bremen
Jan Senge	University of Bremen
Vin de Silva*	Pomona College, USA
Lucile Vandembroucq*	University of Minho, Portugal
Hubert Wagner*	Jagiellonian University, Poland

Note: * denotes speakers.