Final report of the event

"COMPUTER ALGEBRA IN ALGEBRAIC TOPOLOGY AND ITS APPLICATIONS"

Summary

The workshop "Computer Algebra in Algebraic Topology and its Applications", as originally projected, was a special session within the international conference "ACA2013 Applications of Computer Algebra" hold in Málaga, from July 2nd to July 6th. of the present year. The workshop itself took place from July 3rd to July 4th. It contained eight talks covering different computational aspects of applied algebraic topology including, in particular, data analysis, digital images, algorithmic of algebraic topology, and persistence.

Description of the Scientific Content and assessment of results

As originally planned, the event "Computer Algebra in Algebraic Topology and its Applications" was mainly devoted to the computational aspects of applied algebraic topology. It is well known how, during last two decades, algebraic topology has become the core discipline from which several areas of application-oriented research highly benefit. From the topological methods of intrinsic nature used by these applications, to their final implementations, there is a whole range of computational procedures, involving topological results, interesting on its own rite, to be developed and/or refined.

The main aspects covered by the workshop were:

Computational and topological analysis of digital images
Equivariant applied algebraic topology
Computational homology and discrete Morse theory
Computational and topological aspects of data analysis
Persistence

The event consisted in eight scheduled talks and discussion between them. Also, bein a special session of an international conference, the participant of our session benefited from other sessions which were scientifically closed: for instance, general session on computer algebra, computer algebra in coding theory, and non standard applications of computer algebra.

As a summary of the scientific content of the workshop and the results covered, we include the titles and abstracts of the eight scheduled talks.

"Spectral sequences for computing persistence homology of digital images" (ana Romero)

Persistent homology is an algebraic method for measuring topological features of shapes and functions, which can be applied to study digital images. More concretely, this technique consists in identifying homological features that persist within the different stages of a filtration. On the other hand, spectral sequences are a tool for computing homology groups by taking successive approximations. Both concepts are deeply related. In a previous work, we showed that a slight modification of our previous programs for computing spectral sequences [4] is enough to compute also persistent homology. By inheritance from our spectral sequence program, we obtained for free

persistent homology programs applicable to spaces not of finite type (provided they are spaces with effective homology) and with Z coefficients (significantly generalizing the usual presentation of persistent homology over a field). In this work, we will use our programs in order to compute persistent homology of digital images, which will allow us to determine relevant features, that will be long-lived on contrast with the noise which will be short-lived. As a test case, our programs could be applied on a fingerprint database.

“A chain contraction approach to the computation of cubical homology and cohomology” (Pawel Pilarczyk)

Algorithms for the computation of homology, cohomology, and related operations on cubical cell complexes are introduced, using the construction of a chain contraction from the original chain complex to a reduced one that represents its homology. As opposed to the traditional" approach in which the Smith Normal Form of boundary matrices is computed, the additional structure provides considerably more comprehensive homological information. With this technique, one can instantly determine the homology class of any cycle, which allows computing (co)homological operations (like the cup product in cohomology) much more easily than in the approach in which the SNF alone is computed. This work is based on previous results for simplicial complexes obtained by Pedro Real, Roxio Gonzalez-Diaz, and their collaborators, and uses Serre's diagonalization for cubical cells.

“Discrete Morse theory and computational homology” (Pawel Dlotko)

During the last 15 years some classical concepts from pure mathematics have become computationally tractable. Also a few other concepts have been discovered until that time. The classical concepts which are now routinely computable are homology and cohomology groups and their generators. The new concepts, which were introduced along with algorithms to compute them, are standard and zigzag persistence. At the same time, somehow independently discrete Morse theory and its computational methods has been developed. They have been used to simplify functions on surfaces, denoising, and in some situation to obtain Betti numbers. Still, both computational homology and discrete Morse theory provide information about evolution of level sets of some function de ned on a cell complex. In this talk we will show how, using discrete Morse theory, one can obtain information about field homology, persistence and zigzag persistence. Consequently, we will show, that discrete Morse theory is a main branch from which all the described computational methods can be derived. This talk is based on joint work with Vidit Nanda and Hubert Wagner.

“Some advances in G-invariant topology and homology” (Patrizio Frosini)

It is well known that classical persistent homology is invariant under the action of the group of all self-homeomorphisms of a topological space X. As a consequence, this theory is not able to distinguish certain filtering functions. The following question naturally arises: How can we adapt the concept of persistence in order to get invariance just under the action of a proper subgroup of Homeo(X) rather than under the action of the whole group Homeo(X)? In this talk we will illustrate how this problem

can be managed by means of G-invariant persistent homology and other recently developed techniques.

“Computations homological algebra for advanced topological analysis of 4D digital images” (Pedro Real)

We deal with here the complex problems involved in adapting and efficiently applying algebraic topology-based methods for the analysis of digital images up to four dimensions. To achieve this, an underlying mathematical and computational framework to exploit homology-based tools (related to the notion of n-dimensional holes) in diverse 4D discrete settings is generated. The computational nature of homology information and close connection to applications is highlighted using new homological algebra notions such as chain-integral complexes and equivalences. This algebraic machinery works with two nilpotent algebraic operators acting on the same graded module [2]. It is is more than a mere extension of the classical homotopy category of chain complexes, allowing us to establish a graph-based topological representation of a subdivided object and a strong interplay between Discrete Morse Theory and Algebraic-Topological Models. Restricted to real coefficients, it also help us in reinterpreting and exploiting homology information and in finding general harmonic representative classes. From a theoretical point of view, the research will be focused on the following topics: Homological Modeling, Homological Analysis and Homological Acuity for 4D digital images. These three issues are developed using chain-integral tools.

Concerning Homological modelling, the idea is to construct a continuous analogous of 4D digital objects (based on square hypercubes) and to develop homology and geometry computation algorithms based on the chain-integral homology (CHI) framework. 402 non-isometric hyper-polyhedra are the elementary bricks of this local-to-global topological approach [5] which aims to establish results harmoniously combining geometry and topology.

For advancing in 4D-knowledge, it is compulsory to clarify the nature and role of topology in the digital imagery setting, and to try and positively answer the related problems of robustness with respect to noise and dimensionality reduction. We develop a topological processing framework of 4D digital images, which is consistent, robust, flexible and reusable [4, 2, 3, 1]. We use global combinatorial stuff (mainly, graphs and trees) in order to do advanced topological analysis at two levels: Cocyclic calculus (topological skeletons, Reeb graphs, classification of cycles, contractibility and transformability of cycles, cocyclic operations, ...) and Homological Calculus (homology and cohomology classes, homology A(∞)-coalgebra, cohomology ring, cohomology operations, homotopy operations,...).

“Boundary and acyclicity operators of primal and dual elementary cell complexes” (Ana María Pacheco)

In this paper, we compute the boundary and acyclicity algebraic operators of each of the 23 elementary cell complexes in the context of the discrete combinatorial geometry developed by Kenmochi and Imiya. Moreover, we compute the boundary and acyclicity operators of the barycentric dual cell complex of each of these elementary cell complexes. Finally, we present some conclusions about the relationship between the boundary and acyclicity operators of an elementary cell complex and its corresponding dual cell complex.

“On Higher dimensional cocyclic Hadamard matries” (Victor Pacheco)

Little is known about the existence of improper higher dimensional Hadamard matrices. Since the cocyclic framework has showed to be a promising technique for handling with planar Hadamard matrices, we wonder if higher dimensional cocyclic matrices might be suitable as well for looking for higher dimensional improper Hadamard matrices. In this paper we first give a method for computing a basis for n-cocycles over a finite group G, from which some different techniques for looking for higher dimensional cocyclic Hadamard matrices over G are derived. Some examples are given for illustrating these procedures.

“A-infinity persistence” (Francisco Belchí)

Classical persistence, in very general terms, gives the observer a good presentation of data related with the homology of a (time or other parameters dependent) complex which usually arises from some applied setting (digital images, sensor networks, data analysis,...). However, any (co)homology theory can be endowed with extra algebraic structures which may also reveal some special behaviour of the considered situation. Thus, inspired by computational approaches to A(∞)-structures by Pedro Real et al. through Discrete Morse Theory and the Homotopy Perturbation Lemma, we develop a theory of persistence for A(∞)-structures on (co)homology, with the hope of filtering in a finer way the noise arising in 3D digital images.

Program of the meeting and full list of speakers and participants

July 3 10:00 - 10:30	Computer algebra in algebraic topology and its applications
	Spectral Sequences for computing persistent homology of digital images A. Romero, G. Mata, J. Rubio, J. Heras and F. Sergeraert
11:00- 11:30	A chain contraction approach to the computation of cubical homology and cohomology P. Pilarczyk and P. Real
12:00 - 12:30	Discrete Morse Theory and Computational Homology P. Dlotko

July 4 11:00 - 11:30	Some Advanced in G-invariant topology and homology P. Frosini
12:00 - 12:30	Computational Homological Algebra for Advanced Topological Analysis of 4D Digital Images P. Real
13:00 - 13:30	A-infinity persistence Kiko Belchí, Aniceto Murillo Mas
15:00 - 15:30	On Higher Dimensional Cocyclic Hadamard Matrices V. Álvarez Solano, J. A. Armario, M. D. Frau and P. Real
16:00 - 16:30	Boundary and Acyclicity Operators of Primal and Dual Elementary Cell Complexes A. M. Pacheco and P. Real

List of Speakers

Victor Álvarez, University of Sevilla,
Kiko Belchí, University of Málaga
Pawel Dlotko, University of Pennsylvania
Patrizio Frosini, University of Bolonia
Ana María Pacheco, University of Sevilla,
Pawel Pilarczyk, University of Minho
Pedro Real, University of Sevilla,
Ana Romero, University of La Rioja,

List of Participants

Being this event a special sesión of the international conference “ACA2013 Applications of Computer Algebra”, we refer to the list of participants of this conference which appear in the following link

http://www.aca2013.uma.es/participants.html