Research Networking Programmes

Science Meeting – Scientific Report

The scientific report (WORD or PDF file - maximum of seven A4 pages) should be submitted online within two months of the event. It will be published on the ESF website.

Proposal Title: Seventh de Brun Workshop on Homological Perturbation Theory

Application Reference N°: 5656

1 Summary

A 5-day workshop on homological perturbation theory was held at the National University of Ireland, Galway from late afternoon on Monday 1st December to mid-morning on Friday 5th December 2014. This was the seventh in a series of de Brún Workshops which began in 2008.

The ESF Research Network on Applied and Computational Algebraic Topology is centred around four broad themes:

1. The topological and statistical analysis of shapes, images, and large multidimensional data sets.

2. Algorithms for motion planning and the study of configuration spaces of mechanical systems.

3. Statistical topology and the study of large growing systems.

4. The theory of concurrent computation and computer networks..

Computational homological algebra underlies each of these four research themes. Techniques from standard computational linear algebra suffice for certain basic applications of homological algebra. However, when dealing with large data sets the techniques of linear algebra soon become inadequate and need to be enhanced with methods from the newly emerging area of computational homotopy theory. Techniques from homotopy theory are also essential for the more mathematically sophisticated applications, involving notions such as cohomology operations and fundamental group, that are aimed at analysing finer features of data sets.

The members of the workshop organizing committee are each involved in the engineering of software for computational homological algebra and its applications. Their software projects are, to a large extent, independent of each other. However, each project has ended up with the need to engineer implementations of various aspects of homological and homotopical perturbation. The committee felt that the time was right for holding this intensive international research workshop devoted to theoretical and computational aspects of homological perturbation theory. Over 50 scientists participated, including many young researchers in the area as well as more well-established contributors.

2 Description of the scientific content of the event

Vladimir Dotsenko (Mon 17.00-17.45)

Pre-Lie symmetries and homotopy transfer

I shall explain a new proof of homotopy transfer theorem for P-infinity algebra structures (where P is any Koszul operad), which in fact proves more than its classical statement does, and provides one with better understanding of combinatorics of transfer formulas. The key idea is to (develop and) use formulas for gauge symmetries in pre-Lie algebras. This is a joint work with Sergey Shadrin and Bruno Vallette.

Jim Stasheff (Mon 17.45-18.30)

HPT: The Once and Future Theory
In memory of Victor Gugenheim

A view of HPT from it's beginnings from one who was there through to recent applications in mathematical physics.

Thomas Hüttemann (Tue 9.00-9.45)

Totalisation of homotopy commutative cubes and multicomplexes

A chain complex of R-modules is called R-finitely dominated if it is homotopy equivalent to a bounded complex of finitely generated projective R-modules. (This notion is relevant, for example, in topology of manifolds, group theory, and - in the guise of "perfect complexes" - in algebraic geometry.) I will discuss a homological criterion for finite domination, obtained in collaboration with David Quinn. The criterion involves homology with coefficients in certain rings of formal Laurent series. I will explain how homotopy commutative cubical diagrams occur quite naturally in this context, and how the formalism of multicomplexes enters the picture.

Domenico Fiorenza (Tue 9.45-10.30)

The string Lie 2-algebra from homotopy transfer

A compact simple and simply connected Lie group G carries a canonical closed G-invariant 3-form, which can be realized as the curvature 3-form of a U(1)-bundle gerbe over G. The infinitesimal automorphisms of this gerbe form a differential graded Lie algebra which, by homotopy tranfer, can be equivalently realized as a Lie 2-algebra, the so called Lie 2-algebra of quantomorphisms of the gerbe. The string Lie 2-algebra of G sits inside the quantomorphisms Lie 2-algebra, exactly in the same way as the Heisenberg algebra sits inside the Poisson algebra of smooth functions on a symplectic vector space. The talk will be completely introductory and no prior knowledge of gerbe theory will be assumed. Based on joint work with Chris Rogers and Urs Schreiber (HHA 16(2), 2014,107-142 ; arXiv:1304.6292).

Ronnie Brown (Tue 11.00-11.45)

Compositions of cubes in a homotopical approach to algebraic topology

Cubical compositions allow the expression in all dimensions of "algebraic inverses to subdivision" and so apply to nonabelian local-to-global problems, such as the Seifert-van Kampen Theorem, now groupoidal in dimension 1. The extension to higher dimensions enables replacing the usual singular chains approach to basic homology. In this theory, one needs two types of algebraic model: "broad", using cubes, for geometry, conjectures, and proof; and "narrow", more like chain complexes, for relating to classical ideas and for calculation. Also the methods apply not to spaces but to filtered spaces.

Incorporating HPT into these ideas is an open problem!

Alexander Berglund (Tue 11.45-12.30)

Homological perturbation theory for algebras over operads

Pawel Pilarczyk (Tue 14.30-15.15)

A chain contraction approach to the algorithmic computation of homology, cohomology, and homological operations on cubical complexes

The differential approach to the computation of (co)homology of a finite cell complex aims at reducing the topological information to a minimal system that describes the degree of its connectivity. On the contrary, the integral approach aims at representing the chain contraction of the entire chain complex to a complex that represents its homology (an Algebraic-Topological Model or an Algebraic Minimal Model). In this talk, algorithms for the computation of this kind of effective homology and cohomology on cubical cell complexes will be introduced. Constructing an AT Model or an AM Model of a chain complex, which includes the chain contraction, opens new possibilities, e.g., for efficient computation of a variety of (co)homological operations, which is not possible in the differential approach. This work is based on previous results for simplicial complexes, and uses Serre's diagonalization for cubical cells. An implementation of the algorithms in C++ using the technique of generic programming (templates) will also be introduced. This is joint work with Pedro Real.

Kiko Belchi (Tue 15.15-15.45)

The game of guessing links

Let's suppose a man named Graham samples points from a circumference and sends only this set of points to another man, Jim. Then Jim can use Persistent Homology to tell that the space Graham started with is very likely to have Betti numbers 1, 1, 0, 0, 0...

Let's step up the game: If Graham starts with an arbitrary n-component link, how can Jim try to guess, in an automated manner, the Milnor n-fold invariant of that link?

In this talk, I will show a way to approach this game of guessing and how it can be particularly useful in Topological Data Analysis.

Ana Romero (Wed 9.00-9.45)

Basic Perturbation Lemma and effective homology: application to the computation of homology of 2-types

The effective homology method, developed by Francis Sergeraert, has allowed the computation of homology and homotopy groups of complicated spaces of infinite type which where not known before. This technique is implemented in the system Kenzo, a Common Lisp Program where the Basic Perturbation Lemma plays an essential role. In the talk we present some applications of the Basic Perturbation Lemma and the effective homology method, focusing our attention on the computation of homology of 2-types.

Francis Sergeraert (Wed 9.45-10.30)

The Homological Hexagonal lemma

The Homological Hexagonal lemma is a direct consequence of the elementary Gauss linear reduction of linear systems. This lemma gives a limpid simultaneous understanding of the Homological perturbation theorem and Forman's theorems for the discrete vector fields. Makes also obvious extensions of the perturbation theorem to topological situations.

Jónathan Heras (Wed 11.00-11.45)

A formal proof of the Basic Perturbation Theorem

In the last decade, proof assistants have been successfully used in the formalisation of non-trivial mathematical results — e.g. the Four Colour Theorem, the Odd-Order theorem, or the Kepler conjecture. In this talk, we will provide an overview of the use of interactive theorem provers in the context of the homological perturbation theory. In addition, we will explain a recent formalisation of the Basic Perturbation Theorem using the Coq proof assistant.

Mark Walsh (Wed 11.45-12.30)

Loop Spaces, Operads and Positive Scalar Curvature

In this talk we discuss an application of the theory of operads in differential geometry. In particular, we consider the problem of understanding the space of metrics of positive scalar curvature on a sphere. Recently much progress has occurred in this topic. In the case when $n \geq 3$, this space is a homotopy commutative, homotopy associative H-space. In particular, we show that it admits an action of the little n-disks operad, which, via theorems of Stasheff, Boardman, Vogt and May, allow us to demonstrate that this space is weakly homotopy equivalent to an n-fold loop space.

Marek Golasinski, Daciberg Lima Gonçalves (Wed 14.30-15.15)

Periodic groups and space forms

A finite group G is called *periodic* if there is an integer $q > 0$ and $\alpha \in H^q(G, \mathbb{Z})$ such that the cup product map $\alpha \cup - : H^i(G, M) \longrightarrow H^{i+q}(G, M)$ is an isomorphism for every G-module M and

$i > 0$. A complete classification finite periodic groups has been given by Suzuki-Zassenhaus:

Family	Definition	Conditions
I	$\mathbb{Z}/a \rtimes_\alpha \mathbb{Z}/b$	$(a,b) = 1$
II	$\mathbb{Z}/a \rtimes_\beta (\mathbb{Z}/b \times \mathcal{Q}_{2^i})$	$(a,b) = (ab, 2) = 1$
III	$\mathbb{Z}/a \rtimes_\gamma (\mathbb{Z}/b \times T_i)$	$(a,b) = (ab, 6) = 1$
IV	$\mathbb{Z}/a \rtimes_\tau (\mathbb{Z}/b \times O_i^*)$	$(a,b) = (ab, 6) = 1$
V	$(\mathbb{Z}/a \rtimes_\alpha \mathbb{Z}/b) \times SL_2(\mathbb{F}_p)$	$(a,b) = (a, p(p^2-1)) = 1$
VI	$\mathbb{Z}/a \rtimes_\mu (\mathbb{Z}/b \times TL_2(\mathbb{F}_p))$	$(a,b) = (ab, p(p^2-1)) = 1$

R. Swan showed that every periodic finite group acts freely on a $2n+1$-homotopy sphere $\Sigma(2n+1)$ (a finite dimension CW-complex with the homotopy type of the $2n+1$-sphere $\mathbb{S}^{2n+1}$). But, in view of the discovery by J. Milnor, some periodic groups could not act freely on any sphere.

Given a periodic group G, write $\mathcal{K}_G^{2n+1}$ for the set of all homeomorphic classes of orbit spaces $\Sigma(2n+1)/G$ (called *space forms*) and $\mathcal{K}_G^{2n+1}/\simeq$ for the associated set of homotopy classes. The aim of this talk is to show:

Theorem. *There is a bijection*

$$\mathcal{K}_G^{2n+1}/\simeq \xrightarrow{\cong} \mathrm{Aut}(\mathbb{Z}/|G|)/\{\mp\varphi^*; \ \varphi \in \mathrm{Aut}(G)\}.$$

Then, computations of $\mathcal{K}_G^{2n+1}/\simeq$ and classifications of homotopy types of spaces forms for all families of groups from the table above follow.

An extension of those results on other classes of discrete periodic (not necessary finite) groups might be approached as well.

Sebastian Gutsche and Sebastian Posur (Wed 15.15-15.45)

Cap - Categories, algorithms, and programming

Many calculations in homological algebra boil down to a small set of basic algorithms, given by the existential quantifiers of ABELian categories, e.g. compositions, kernels, direct sums. Categorical programming is the strategy of reducing complex algorithms, e.g. spectral sequences, to those basic algorithms. So implementations become more abstract and are applicable in many contexts. In the talk we first present the idea and motivation for categorical programming. After that we show the power and convenience of categorical programming in CAP, a categorical programming language implemented in GAP.

Tornike Kadeishvili (Wed 17.00-17.45)

Minimality Theorem and HPT

Minimality theorem states the existence of minimal A-infty algebra structure in homology of a dg algebra. Particularly such a structure appears on cohomology of a topological space and is more

informative than the standard cohomology algebra structure. For example it determines cohomology of loop space, furthermore, its commutative version determines rational homotopy type. We intend to compare minimal A-infty algebra structures obtained using various constructions such as basic obstruction argument, homology perturbation theory, Hodge decomposition.

Andy Tonks (Thu 9.00-9.45)

Towards a twisted, and crossed, tensor product

I will present joint work in progress with my student Pedro Santos, and my former student Jo Gill, on generalising classical homological perturbation theory of Shih et al. to ∞-groupoids (that is, to crossed complexes). The twisted crossed tensor of the title should generalise the crossed complex tensor product on one hand and twisted tensor product of chain complexes on the other.

Alban Quadrat (Thu 9.45-10.30)

Robust control theory as a basic application of the homological perturbation lemma

Control theory is a branch of mathematical systems theory which aims at studying stabilization problems of interconnected dynamical systems defined by ordinary or partial differential systems, differential time-delay systems, ... Robust control theory deals with the uncertainties about the system coming from the errors of mathematical models, the external perturbations, ... Robust control theory bas been a major success of control theory and is nowadays applied in industry. The purpose of this talk is to introduce the audience to robust control theory and to show how central results of robust control theory can be obtained in a unified way as a basic application of the homological perturbation lemma.

Martin Markl (Thu 11.00-11.45)

Natural operations and Koszul hierarchy

The Koszul hierarchy (aka higher brackets or Koszul brackets) is an explicit construction that, for any commutative associative algebra A with a differential Δ (which is, very crucially, not necessary a derivation), produces a sequence of multilinear maps

$$\Phi_n \colon A \times ... \times A \longrightarrow A \quad \text{(n copies of A)}$$

such that

(1) the operations Φ_n form a strongly homotopy Lie algebra, and

(2) $\Phi_n = 0$ implies $\Phi_{n+1} = 0$ (heredity property).

Koszul brackets are used for instance to define higher-order derivations: Δ is a degree n derivation if $\Phi_{n+1}(\Delta) = 0$. Higher order derivations play an important role e.g. in BRST approach to closed string field theory.

Recently, a similar construction appeared also for associative (non-commutative) algebras. I was able to show that both brackets are given by the twisting by a specific unique automorphism and that they are essentially unique. Consequently, the notion of higher-order derivations is God given, not human invention.

The proof is based on careful analysis of a space of natural operations. Here I employed the technique developed in my work with M. Batanin on the Deligne conjecture. As a matter of fact, the core of my theory is a vanilla version of Deligne's conjecture. I plan to focus my talk on this side of the story.

Lukáš Vokřínek (Thu 11.45-12.30)

Universal properties of certain resolutions and homotopy R-modules

I will show how certain canonical resolutions are in fact free in a particular sense (implying easily that they are indeed free resolutions). Further, I will use such free resolutions to define strongly homotopy modules over differential graded algebras and describe an explicit transfer of structure for them.

Martin Doubek (Thu 14.30-15.15)

Perturbations in operad theory

Various homotopy structures, such as A_∞ algebras, are algebras over a differential graded operad R resolving a certain operad P. If P is perturbed to $\tilde{P}$, in a broad sense, it is sometimes possible to perturb, in an explicit way, the resolution R to $\tilde{R}$ resolving $\tilde{P}$. I will review some examples of these perturbations including homotopy derivation and differential of arbitrary homotopy algebra, $A_{\infty\infty}$ algebra and diagrams (of morphisms) of homotopy algebras.

Sebastian Gutsche and Sebastian Posur (Thu 15.15-15.45)

Cap - Categories, algorithms, and programming

Continued.

Marian Mrozek (Thu 17.00-17.45)

Morse-Forman-Conley theory for combinatorial multivector fields.

In late 90' R. Forman defined a combinatorial vector field on a CW complex and presented a version of Morse theory for acyclic combinatorial vector fields. He also studied combinatorial vector fields without acyclicity assumption, studied its chain recurrent set and proved Morse inequalities in this setting.

In this talk we consider a generalized concept of combinatorial multivector field and present an extension of the Morse-Forman theory towards the Conley index theory.

This is research in progress.

Alexander Rahm (Fri 9.00-9,45)

From Homological Perturbation Theory to the Quillen-Wendt conjecture

Homological Perturbation Theory can be used to carry out cohomological computations on Bianchi groups that give insights into the cohomology of higher rank arithmetic groups. This is currently worked on by Tuan Anh Bui at NUI Galway, with view on a project of Gael Collinet. Moreover, cohomological computations on the Bianchi groups have allowed the speaker to develop a new technique (called Torsion Subcomplex Reduction) for computing the Farrell-Tate cohomology of discrete groups acting on suitable cell complexes. Some methods of the technique were already in use beforehand as a set of ad-hoc tricks, e.g. in calculations by Christophe Soulé. However, giving a comprehensive formulation of this technique has not only already yielded general formulae for the cohomology of the tetrahedral Coxeter groups as well as, above the virtual cohomological dimension, of the Bianchi groups (and at odd torsion, more generally of SL_2 groups over arbitrary number fields), it also very recently has allowed Matthias Wendt to reach a new perspective on the Quillen conjecture; gaining structural insights and finding a variant that can take account of all known types of counterexamples to the Quillen conjecture. If no counterexample of completely new type surprisingly shows up, then this refined conjecture must be valid.

Pedro Real (Fri 9.45-10,30)

Compressing and simplifying homology flows without using filtrations

In this talk, a homological perturbation technique for computing integer homology information of a finite simplicial complex K embedded in $\mathbb{R}^n$ is developed. This technique is based on spanning trees over the connectivity graph of K (graph having as nodes the cells of K and as edges the pairs of incident cells of dimension p and $p \pm 1$). The algorithm determines the potential integer homology torsion area in terms of a set of cells involved, specifies a series of simplicial collapses for killing this area and generates in this way a localization space $L_F(K)$ for K, being F a field (in the sense of Bousfield or Sullivan). The rationalization, realification and complexification of K are all the same space. Possible applications of this F-ization of the space are the fast computation of homotopy groups with coefficients in F or fast FEM simulation computations.

3   Assessment of the results and impact of the event

The workshop attracted over 50 participants from 12 european countries, Georgia, India and the USA. It brought together scientists with a purely theoretical interest in homological perturbation (theory of operads, mathematical physics, ...) as well as those with a more practical interest (software engineering, computational topology, control theory, ...). The diverse backgrounds of participants, all with a common interest in HPT, gave rise to a varied programme of lectures and synergetic coffee-time discussions. Some of these will no doubt lead to future research projects and publications. The workshop helped to maintain and enlarge the active community of European researchers in the area of applied topology. At a local level it provided an opportunity for topologists from Irish universities to meet; in that way it was a support to the small but growing Irish topology community.

A permanent public record of the slides of most lecture talks has been stored at
`http://hamilton.nuigalway.ie/HPT/HPTprogramme.html` .

4 Annexe 4a): Programme of the meeting

Lectures from Monday to Thursday will be held in AMBG065 (Arts Millenium Building)

Lectures on Friday will be held in THBG010 (James Hardiman Building)

Morning coffee/tea from Monday to Thursday will be served near AMBG065 (Arts Millenium Building

Afternoon tea/coffee and Friday's morning coffee will be self-served in ADBG001 (Aras de Brun)

	Monday	Tuesday	Wednesday	Thursday	Friday
9.00-9.45		Thomas Hüttemann	Ana Romero	Andy Tonks	Alexander Rahm
9.45-10.30		Domenico Fiorenza	Francis Sergeraert	Alban Quadrat	Pedro Real
10.30-11.00		Coffee	Coffee	Coffee	Coffee
11.00-11.45		Ronnie Brown	Jónathan Heras	Martin Markl	
11.45-12.30		Alexander Berglund	Mark Walsh	Lukáš Vokřínek	
12.30-14.30		Lunch	Lunch	Lunch	
14.30-15.15		Paweł Pilarczyk	Marek Golasinski	Martin Doubek	
15.15-15.45		Kiko Belchi	Sebastian Gutsche	Sebastian Posur	
15.45-16.30		Discussion	Discussion	Discussion	
16.30-17.00	Coffee	Poster &	Coffee	Coffee	
17.00-17.45	Vladimir Dotsenko	Cheese &	Tornike Kadeishvili	Marian Mrozek	
17.45-18.30	Jim Stasheff	Wine session			
20.00-			Dinner in The House Hotel		

4   Annexe 4b) Full list of speakers and participants

4.1   Speakers

- Alexander Berglund (Stockholm)
- Ronnie Brown (Bangor)
- Kiko Belchi (Malaga)
- Vladimir Dotsenko (Dublin)
- Martin Doubek (Prague)
- Domenico Fiorenza (Rome)
- Marek Golasinski (Olsztyn)
- Sebastian Gutsche (Kaiserslautern)
- Jonathan Heras (La Rioja)
- Thomas Httemann (Belfast)
- Tornike Kadeshvili (Tbilisi)
- Martin Markl (Prague)
- Marian Mrozek (Krakow)
- Pawel Pilarczyk (IST Austria)
- Sebastian Posur (Aachen)
- Alban Quadrat (Inria Saclay)
- Alexander Rahm (Galway)
- Pedro Real (Seville)
- Ana Romero (La Rioja)
- Francis Sergeraert (Grenoble)
- Jim Stasheff (North Carolina) [via video lecture]
- Andy Tonks (London Met)
- Luk Voknek (Brno)
- Mark Walsh (Wichita)

4.2   Organizers

- Mohamed Barakat (Kaiserslautern)
- Graham Ellis (Galway)
- Pedro Real (Seville)
- Julio Rubio (Logrono)
- Emil Skldberg (Galway)

4.3 Other visiting participants

- Ethan Berkove (Easton, USA)
- Martin Bies (Heidelberg)
- Soutrik Roy Chowdhury (Dublin)
- Ciaran Corvan (Belfast)
- Alexander Engstrm (Helsinki) [Cancelled]
- Marek Filakovsky (Brno)
- Ashis Mandal (Kanpur)
- Malte Milatze (ETH Zrich)
- Aniceto Murillo (Malaga) [Cancelled]
- David Quinn (Aberdeen)
- Luke Steers (Belfast)

4.4 Other local participants

- Patrick Browne (Galway)
- Bui Anh Tuan (Galway)
- Isaac Burke (Galway)
- John Burns (Galway)
- Ted Hurley (Galway)
- Niall Madden (Galway)
- Brendan Masterson (Galway)
- Aisling McCluskey (Galway)
- John McDermott (Galway)
- Sejong Park (Galway)
- Götz Pfeiffer (Galway)
- Rachel Quinlan (Galway)
- Michael Tuite (Galway)
- James Ward (Galway)