Homotopy in Concurrency and Rewriting
scientific report

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1 Summary

This conference organized at École Polytechnique from June 9 to 11 on directed algebraic topology, concurrency and rewriting aimed at bringing together researchers interested in applying methods originating in topology and in higher-dimensional rewriting. It was motivated by the discovery, over the recent years, of many connections between the study of concurrent processes from a topological point of view and rewriting methods and structures. It was interested in applications of those fields to problems commonly encountered in computer science, in the study of concurrent and distributed processes, in semantics of programming languages, but also to more theoretical questions arising from the study of structures found in algebraic topology such as operads and higher-dimensional categories.

On the one hand, there has been an increasing interest in the potential applications of algebraic topology to study concurrency over the past decades. This new field at the intersection of topology and computer science was started by a series of works which introduced semantics of concurrent programs based on topological spaces, whose points model the states of programs and paths model executions of the program. Since executions of programs always go forward in time, these topological spaces should be equipped with a notion of direction for paths, which requires adapting the usual tools in algebraic topology to this new setting. Two paths which are homotopic (in a directed sense) correspond to two schedulings of a concurrent program which are equivalent in the sense that they give rise to the same results. Starting from this observation, the study of the space of paths up to homotopy in a directed space provides compact representations of the possible schedulings of concurrent programs, in the same way that homotopy (or homology) groups describe in a concise way the essential characteristics of classical topological spaces. This point of view has provided new tools in order to efficiently verify concurrent programs by providing compressed representations of their state space, and open new research fields such as the investigations of variants of homology adapted to the directed setting.

On the other hand, directed methods appear also in homological algebra throughout rewriting. Since the eighties, several results theory to compute homological invariants (resolutions, homology groups, Hilbert and Poincaré series). Moreover, convergent presentations, and Gröbner bases for linear structures, allow the computation of normal forms and thus, one can address decision problems through computational methods. Recently, these rewriting methods have been generalized from strings to richer algebraic structures such as operads, monoidal categories, and more generally higher-dimensional categories. These extensions of rewriting fit in the general scope of higher-dimensional rewriting theory, which has emerged as a unifying algebraic framework. This approach allows one to perform homotopical and homological analysis of rewriting systems. It also provides new computational methods in combinatorial algebra, in homotopical and homological algebra.

The project was supported by the CATHRE ANR projet, FOCAL IDEX project and LIX laboratory.

2 Scientific content and discussions

The conference lasted for three days and featured four talks each day, which were quite interactive. The schedule was designed so that it could leave room for discussions between and after the talks among participants, so that people could exchange more privately about technical aspects of the presentations. From impressions gathered from participants, this idea was quite successful since there were lots of fruitful scientific exchange. On the day after the conference, took place a meeting from the CATHRE French ANR project on closely related themes (which was sponsored by the ANR project), to which many participants of the conference also assisted.

Presentations on a broad range of topics were given, both from young and senior researchers, with homotopical methods as a central theme. The objective was for the various communities to present their tools around this notion. They can roughly be grouped according to major themes as follows:

- homotopical and homological methods in computer science through the use of topological models (Krishnan, Dubut, Porter),
- categorical approaches to the formalization and the study of algebraic structures occurring in mathematics and computer science (Fiore, Dotsenko, Burgunder, Burroni, Obradović),
- homotopical methods for higher-dimensional categories (Porter, Steiner, Ara, Lafont).

In conclusion, this meeting was very stimulating for the various communities involved, enjoying the interplay between pure mathematics and computer science, with a crossed enrichment.

3 Results and impact

The goal of the conference was to create a meeting point for young and confirmed researchers working in fields at the interaction between mathematics and computer science, with homotopy as central theme. Around 50 participants were present, which is quite a high figure for such an event.

We are particularly proud of the

- diffusion with high quality lectures and an important participation of PhD students and young researchers,
- numerous discussions that popped up at the interplay of the different fields.

Let us mention a few examples of topics which have been discussed from different viewpoints during the conference and, we hope, which will have a real impact in the development of the research in these areas:

- Porter's talk about various models of $(\infty, 1)$-categories was particularly pedagogic and illuminating, it turns out that some of the ideas that he presented entered in resonance with Dubut's talk and might provide, on the long term, a good framework for a definition of homology for directed topological spaces.
- Steiner has discovered that his work on ∞-categories and chain complexes was extensively used by Ara (and his co-workers), which might lead to collaborations about the subject.
- Many works used polygraphs, a notion which was introduced by Burroni, and the new developments on the subject that he presented at the conference might lead to many progress.
- More generally, rewriting theory appeared to be a central underlying technical tool in many of the presented works and there were lot of knowledge exchange around this subject, which hopefully lead to new scientific advances.

A Programme

A.1 Schedule

A.1.1 Tuesday 9 June

9:00	*Arrival*	
9:15	*Opening*	
9:30	Sanjeevi Krishnan	Positive Alexander Duality
10:30	*Break*	
11:00	Steve Oudot	Reflections in quiver and persistence theories
12:00	*Lunch*	
14:00	Carlos Simpson	A two-dimensional generalization of Stallings core graphs
15:00	*Break*	
15:30	Marcelo Fiore	Algebraic Theories for Type and Homotopy Theories

A.1.2 Wednesday 10 June

9:30	Vladimir Dotsenko	Associahedra and noncommutative topological field theory
10:30	*Break*	
11:00	Timothy Porter	Steps towards a “directed homotopy hypothesis”
12:00	*Lunch*	
14:00	Emily Burgunder	Free algebraic structures on the permutehedra
15:00	*Break*	
15:30	Richard Steiner	Chain complexes and higher categories

A.1.3 Thursday 11 June

9:30	Jérémy Dubut	Natural Homology
10:30	*Break*	
11:00	Dimitri Ara	A Quillen’s Theorem A for strict ∞-categories
12:00	*Lunch*	
14:00	Viktoriya Ozornova	Factorability structure
15:00	*Break*	
15:30	Albert Burroni	Machines et grammaires polygraphiques

A.2 Abstracts

A.2.1 Dimitri Ara – A Quillen's Theorem A for strict ∞-categories

Quillen's Theorem A is one of the main tool in the study of homotopy types of categories. It gives a sufficient condition on a functor for its nerve to be a weak homotopy equivalence of simplicial sets. In this talk, I will explain how to generalize Quillen's Theorem A to strict ∞-categories using Street's nerve. The proof involves various tools interesting in their own right such as a join construction, generalized slices and a connection between lax transformations and simplicial homotopies. An essential tool to obtain these results is Steiner's theory of augmented directed complexes. This is joint work with G. Maltsiniotis.

A.2.2 Albert Burroni – Machines et grammaires polygraphiques

A l'instar des systèmes de réécriture de mots qui sont des présentations de monoïdes, les polygraphes sont des présentations de catégories en dimensions supérieures (autrement dit, ce sont des systèmes de réécriture en dimensions supérieures). Ces polygraphes, complétés en logographes (i.e. munis de systèmes d'entrées et de sorties) deviennent des automates qui permettent la reconnaissance de langages en dimensions supérieures. Nous illustrerons cela avec des langages d'images qui sont formés de mots de dimension 2.

A.2.3 Emily Burgunder – Free algebraic structures on the permutehedra

Tridendriform algebras are a type of associative algebras, introduced independently by Chapoton and by Loday-Ronco, in order to describe operads related to the Stasheff polytopes. The vector space ST spanned by the surjections from $\{1, \ldots, n\}$ to $\{1, \ldots, r\}$, which describes the faces of permutohedra, has a natural structure of tridendriform bialgebra. We prove that it is free as a tridendriform algebra and exhibit a basis.

This is joint work with Pierre-Louis Curien and Maria Ronco.

A.2.4 Vladimir Dotsenko – Associahedra and noncommutative topological field theory

I shall explain how toric varieties of (Loday's realisations of) associahedra give rise to a new remarkable algebraic structure that is in many ways a noncommutative counterpart of the notion of genus 0 cohomological field theory. This is a joint work with Sergey Shadrin and Bruno Vallette.

A.2.5 Jérémy Dubut – Natural Homology

We propose a notion of homology for directed algebraic topology, based on so-called natural systems of abelian groups, and which we call natural homology.

Contrary to previous proposals, and as we show, natural homology has many desirable properties:

- it is invariant by dihomeomorphism
- it satisfies a Hurewcz-like theorem
- it is non-trivial on non dihomotopically trivial spaces
- it lives in a homological category
- it is equivalent to a finite computable natural system in the case of cubical complexes
- it is invariant by refinement (subdivision)

A.2.6 Marcelo Fiore – Algebraic Theories for Type and Homotopy Theories

I will first present foundations for (untyped and simply typed) algebraic theories with variable-binding operators, addressing the mathematical theory from the viewpoints of universal algebra, equational logic, and categorical algebra. In an exploratory vein, I will subsequently pursue the perspective provided by this model theory in the context of homotopy theory, introducing an abstract categorical framework for uniform Kan-like filler algebraic structure and discussing its intended applications.

A.2.7 Sanjeevi Krishnan – Positive Alexander Duality

The real homology groups of a directed space naturally come equipped with cones, as constructed by Grandis, encoding some of the directionality of the space. Alexander Duality is a duality between the homology of a space and the cohomology of its complement in a sphere. We accordingly construct positive cones on the real cohomology of certain directed spaces, sufficiently smooth spaces parametrized over the real number line, and give a positive Alexander Duality in homological degrees 0,1. Positive cohomology, the limit of a sheaf of local positive cohomology semigroups on the real number line, can be computed as a linear programming problem. A natural application of this Positive Alexander Duality is to characterize when an evader can avoid capture in certain pursuit-evasion games. After describing this application, we briefly outline possible other applications (like concurrency) that should fall out of suitable generalizations from the case presented here. This is joint work with Rob Ghrist.

A.2.8 Steve Oudot – Reflections in quiver and persistence theories

Reflections are among the classical operations used in quiver theory to transform a given quiver Q into another quiver Q'. The nice feature of such a reflection is to be associated with a functor mapping the representations of Q into representations of Q'. This is interesting in the context of the classification of quiver representations, where decompositions of representations of Q can be transferred to decompositions of representations of Q'. This principle has been applied with success by Bernstein, Gelfand and Ponomarev to prove Gabriel's theorem and some of its extensions in the 70's and 80's. In this talk I will draw a connection between reflection functors and the Diamond Principle from persistence theory, then I will present an application to the computation of persistence for so-called zigzags.

A.2.9 Viktoriya Ozornova – Factorability structure

This is a joint work with A. Heß. A factorability structure on a group or a monoid is a choice of geodesic normal forms with respect to a fixed generating system with certain compatibility properties. Factorability structure yields a smaller chain complex than the bar complex for computing the homology of the given monoid or group. Moreover, under some additional assumptions, factorability yields a complete string rewriting system on the monoid. Examples of factorability structures can be found on symmetric groups, braid groups (and more general, Garside groups), and on arbitrary Artin monoids.

A.2.10 Tim Porter – Steps towards a "directed homotopy hypothesis": $(\infty, 1)$-categories, directed spaces and rewriting: An overview of some techniques and some questions

It is now almost "classical" that ∞-groupoids correspond to "spaces" and encode certain aspects of "rewriting" for (low dimensional) categories. We will review some of the models for this. It has more recently been suggested that $(\infty, 1)$-categories might correspond in a similar way to directed spaces. We will examine various aspects hopefully looking at the same time for any means of applying insights from this study to rewriting.

A.2.11 Carlos Simpson – A two-dimensional generalization of Stallings core graphs

Two-dimensional constructions, glued from polyhedra modeled on the affine Weyl chambers for SL_3, are the structures forming pieces of two dimensional euclidean Bruhat-Tits buildings. In joint work with Katzarkov, Noll and Pandit, we propose a combinatorial reduction process starting from such a construction but with positively curved points, going towards a negatively curved "pre-building". An important role is played by a subcomplex homeomorphic to a Riemann surface, which appears to be analogous to the Stallings core graph as studied by Parzanchevski, Puder and others in similar one-dimensional reduction processes.

A.2.12 Richard Steiner – Chain complexes and higher categories

It is well known that there is an equivalence from the category of chain complexes to the category of ω-category objects in the category of abelian groups, where an ω-category is a particular type of higher category. I will define a category of augmented directed chain complexes in which the objects are chain complexes with additional structure and the morphisms are structure-preserving chain maps. By modifying the equivalence one obtains a functor from augmented directed complexes to ordinary ω-categories (that is, ω-category objects in the category of sets). In turn, this functor restricts to an equivalence between two important full subcategories, yielding an algebraic description of certain important ω-categories. These ω-categories include the simple ω-categories (associated to globes and finite discs) and Street's orientals (associated to simplexes). They also include the ω-categories associated to cubes and (with some adjustments) the ω-categories associated to opetopes. One can obtain alternative descriptions of the class of ω-categories: they are equivalent to simplicial sets with additional structure (complicial sets) and to cubical sets with additional structure (cubical sets with compositions and connections).

B Attendees

B.1 Speakers

- Emily Burgunder (Université Paul Sabatier Toulouse)
- Albert Burroni
- Vladimir Dotsenko (Trinity College Dublin)
- Jérémy Dubut (ENS Cachan)
- Marcelo Fiore (University of Cambridge)
- Sanjeevi Krishnan (University of Pennsylvania)
- Steve Oudot (INRIA Saclay)
- Viktoriya Ozornova (University of Bremen)
- Timothy Porter (University of Wales)
- Carlos Simpson (Université de Nice)
- Richard Steiner (University of Glasgow)

B.2 Participants

- Matteo Acclavio (I2M)
- Clément Alleaume (Institut Camille Jordan)
- Dimitri Ara (Institut de mathématiques de Marseille)
- Christian Ausoni (Université Paris 13)
- Emily Burgunder (Université Paul Sabatier Toulouse)
- Albert Burroni
- Andrea Cesaro (Université Lille 1)
- Cyrille Chenavier (INRIA, laboratoire PPS, université Paris Diderot)
- Pierre-Louis Curien (CNRS)
- Jacques Darné (Ecole Normale Supérieure)
- Patrick Dehornoy (Universite de Caen)
- Vladimir Dotsenko (Trinity College Dublin)
- Jérémy Dubut (ENS Cachan)

- Eric Finster (École Polytechnique)
- Marcelo Fiore (University of Cambridge)
- Soichiro Fujii (The University of Tokyo)
- Stéphane Gaussent (Université Paul Sabatier Toulouse)
- Marc Giusti (CNRS)
- Éric Goubault (École Polytechnique)
- Adrien Guatto (ENS – INRIA)
- Yves Guiraud (INRIA, Université Paris 7)
- Emmanuel Haucourt (École Polytechnique)
- Eric Hoffbeck (Université Paris 13)
- Sanjeevi Krishnan (University of Pennsylvania)
- Yves Lafont (I2M-AMU)
- Zachery Lindsey (Indiana University)
- Maxime Lucas (INRIA / Univ. Paris 7)
- Kenji Maillard (ENS Ulm)
- Philippe Malbos (Institut Camille Jordan, Université Claude Bernard Lyon 1)
- Georges Maltsiniotis (CNRS-Paris 7)
- Paul-André Melliès (CNRS, Paris Diderot)
- François Métayer (Laboratoire PPS)
- Joan Millès (Université de Toulouse)
- Samuel Mimram (École Polytechnique)
- Nicolas Ninin (École Polytechnique)
- Hage Nohra (Institut Camille Jordan)
- Jeremy Nusa (Université de Montpellier)
- Jovana Obradovic (PPS Laboratoire)
- Steve Oudot (INRIA Saclay)
- Viktoriya Ozornova (University of Bremen)

- Loïc Paulevé (CNRS/LRI)
- Cagne Pierre (PPS – Université Paris Diderot)
- Timothy Porter (University of Wales)
- Philippe Saade (ICJ – Université Claude Bernard Lyon 1)
- Carlos Simpson (Université de Nice)
- Richard Steiner (University of Glasgow)
- Lutz Strassburger (INRIA)
- Noam Zeilberger (MSR-Inria)