Research Networking Programmes

Science Meeting – Scientific Report

The scientific report (WORD or PDF file - maximum of seven A4 pages) should be submitted online within two months of the event. It will be published on the ESF website.

***Proposal Title**:* Special Session "Applied Algebraic Topology"

Application Reference N°: 5830

1. Summary

A 4-days international joint meeting of the American, European and Portuguese Mathematical Societies took place in Porto, Portugal, June 10-13 2015 (http://aep-math2015.spm.pt). It gathered around 1100 mathematicians from all over the world. The scientific programme included 8 plenary lectures, 53 special sessions and 9 contributed talks sessions.

The special session Applied Algebraic Topology (special session # 6) was given three blocks scheduled on Thursday June 11, 13:00-15:30, Friday June 12, 9:00-11:30 and Saturday June 13, 9:00-11:30. The session consisted of 9 talks (six 45-minute talks and three 20-minute talks) covering the main themes of ACAT, namely:

- Topological Data Analysis and Persistent Homology
- Directed Algebraic Topology and Concurrency Theory
- Computational Algebraic Topology
- Topological Robotics
- Stochastic Topology

In addition to these invited talks, a 10-minute contributed talk on Statistical Topology has been delivered within the Geometry and Topology contributed talks session (Wednesday June 10).

The session has attracted over 30 mathematicians from 8 European countries, Israel and the USA. Among the invited speakers, two speakers came from the USA while the remaining speakers came from European universities.

The funding of 10.000 euros from the European Science Foundation enabled the organisers to cover the main expenses of the 9 invited speakers as well as to provide partial funding (travel and/or accommodation and/or registration) for a further 5 European participants.

2. Description of the scientific content of the event

The following talks have been given (see appended schedule):

Massimo Ferri: *Biomedical applications of persistent Homology*

Algebraic Topology succeeds in formalizing qualitative aspects of spaces. Persistent Homology, in particular, focusses on pairs (X;f) of spaces endowed with a function; this adds the possibility of probing together topological and geometrical features – through the filtration given by sublevel sets – and the possibility of taking the observer's concept of shape into account.

This approach turns out to be particularly convenient when dealing with data of natural origin, in particular in the biomedical domain. This talk will survey some applications in protein docking, analysis of echocardiographic data, classification of leukocytes and of hepatic cells, diagnosis and retrieval of melanocytic lesions.

Matthew Kahle: *The length of the longest bar in random persistent homology*

In recent years, a number of papers have studied topological features of "random geometric complexes", particularly various facts about their expected homology. One of the motivations for this is establishing a probabilistic null hypothesis for topological data analysis. In practice, however, one usually computes persistent homology over a range of parameter, rather than homology alone. Detailed results for persistent homology of random geometric complexes have been harder to come by.

I will present new work which quantifies the length of the longest bar in persistent homology, up to a constant factor. This is an important step toward quantifying the statistical significance of topological signals in data. This is joint work with Omer Bobrowski and Primoz Skraba.

Armindo Costa: *Topology of random simplicial complexes*

Random objects often have desirable properties for which explicit examples are hard to construct; eg often random graphs are good expanders and have strong Ramsey properties. The most well-studied model of random graphs is the Erdos- Renyi model $G(n, p)$. In the $G(n, p)$ model one generates a random graph with n vertices by adding each possible edge with independent probability p. Properties of random graphs are often studied asymptotically, ie by having n tend to infinity and the probability parameter p depend on n. A pioneering result of Erdos and Renyi on $G(n, p)$ is topological; it studies in detail the phase transition of connectivity. In this talk we will look at models of random simplicial complexes. Unlike the graph setup, one can study several interesting topological properties of random complexes. Additionally, by considering the fundamental group of a random complex one also has a model of random groups.

Martin Raussen: *Combinatorial and topological models for spaces of schedules*

Higher Dimensional Automata are topological models for concurrent computation in the form of cubical complexes. A schedule gives rise to a directed path (d-path), and d-homotopies (preserving the directions) of such d-paths leave the results of computations invariant.

I shall describe and discuss several models for the homotopy type of the space of traces (schedules up to reparametriza- tion) for particularly simple HDAs: as a prodsimplicial complex – with products of simplices as building blocks – and as a configuration space living in a product of simplices. In favourable cases, these models allow calculations of homology groups and other topological invariants of the trace spaces.

Joint work with Krzysztof Ziemiański (Warsaw) and Roy Meshulam (Technion Haifa).

Ana Romero: *Zigzag persistence: application to processing stacks of neuronal images*

The theory of zigzag persistence provides an extension of persistent homology to diagrams of topological spaces which do not define a filtration and has been used in different contexts such as topological bootstrapping, parameter thresholding and real-valued functions.

In this work, a new application of this theory is presented: we use zigzag persistence to determine the objects of interest in stacks of neuronal images, obtained by a microscope as a set of slices corresponding to different levels of the Z-axis. This method allows us, in particular, to recognize some 3D properties of the objects, identifying different dendrites which could appear mixed in the maximal projection image.

The algorithm has been implemented as a plugin in the Fiji/ImageJ framework, a Java image processing package which is used for research in life sciences and biomedicine. This plugin is an auxiliary tool in a more ambitious project, aimed at locating neurons in a low-scale picture of a fragment of the brain. Some results from actual neuronal images will be presented in the talk.

José Manuel Garcia-Calcines: *On some approaches of topological complexity*

The topological complexity of a space X; $TC(X)$, is the sectional category (or Schwarz genus) of the end-points evaluation fibration $\pi_X : X^I \to X \times X$; $\pi_X(\alpha) = (\alpha(0), \alpha(1))$. This homotopical invariant was defined by M. Farber in order to give a topological approach to the motion planning problem in robotics. If one regards the topological spaceX as the configuration space of a mechanical system, the motion planning problem consists of constructing a program which takes pairs of configurations $(A; B) \in X \times X$ as an input and produces as an output a continuous path in X which starts at A and ends at B. Broadly speaking, $TC(X)$ measures the discontinuity of any motion planner in the space. In this talk I will explain the use of certain approaches of the topological complexity of a space X and see how they are related. Among such approaches we can mention $wTC(X)$ the weak topological complexity, $cat(C_\Delta)$ the Lusternik- Schnirelmann category of the homotopy cofibre of the diagonal map $\Delta : X \to X \times X$ or $TC^M(X)$ the monoidal topological complexity of X.

Graham Ellis: *Computing with homotopy 2-types*

The fundamental notions of (i) CW-complex, (ii) simple homotopy type and (iii) crossed module were all introduced by Henry Whitehead in his work on combinatorial homotopy during the 1930s and 1940s. The first two notions are the basis for much recent research in computational topology whereas the third notion seems to be less well known. In this talk I will advertize the notion of crossed module and its suitability for computer calculations.

José Gabriel Carrasquel-Vera: *On rational topological complexity and related invariants*

We will present new advances in the study of Michael Farber's topological complexity of rational spaces. We also study some related invariants such as Rudyak's higher topological complexity and Iwase-Sakai's monoidal topological complexity. This is done by giving a new characterisation of rational sectional category in the spirit of the Jessup-Murillo-Parent conjecture.

Sanjeevi Krishnan: *Dynamic sensor networks*

A class of pursuit-evasion games, where an evader tries to avoid detection by a time-evolving sensed space, is considered. Earlier results give homological criteria for evasion that are necessary or sufficient, but not both. We give a necessary and sufficient ordered cohomological criterion for evasion in the general case. The main idea is to refine the (n-1)-cohomology of a coverage region with a positive cone encoding orientation, refine the 1-homology of the coverage gaps with a positive cone encoding time, and prove a positive Alexander Duality in homological degree 1. Positive cohomology, the limit of a sheaf of local positive cohomology semigroups on the real number line, can be computed as a linear programming problem. We demonstrate such a calculation for a prototypical case of evasion that eludes ordinary homological criteria.

3. Assessment of the results and impact of the event

As a special session in a very large international meeting of mathematics, this event was a great opportunity to catch the attention of pure or applied mathematicians from other areas to the themes of the emergent area of Applied Algebraic Topology. The program has been designed in order to cover most of the themes discussed within the ESF RNP ACAT. Besides the speakers and some other participants who have an already confirmed interest for the ACAT topics, the session has effectively attracted many mathematicians working in other fields, mainly in Geometry and Topology related areas but also in more applied areas such as Applied Dynamical Systems or Statistics. Some of them asked very interesting questions showing that they followed the talks with interest.

The event also contributed to strengthen and develop connections between European and American researchers in the area of applied topology. Questions asked during the session gave rise to interesting discussions in the breaks between the talks.

At a local level, the conference provided a new opportunity, after the ACAT Lisbon meeting (XXI Oporto Meeting, 4-7 February 2015), for mathematicians from Portuguese institutions and interested in ACAT themes to meet. It also permitted to bring the attention of more Portuguese mathematicians to this

area.

4. Programme of the session and List of participants

The programme is given on the next page.

At least the following people participated to the session:

- Irene Brito, University of Minho, Portugal
- José Gabriel Carrasquel Vera, University of Louvain - UCL, Belgium
- Michele Cirafici, University of Lisbon, Portugal
- Armindo Costa, Queen Mary, University of London, UK
- Eugénia César de Sá, University of Porto, Portugal
- Jonathan Dawes, University of Bath, UK
- Barbara di Fabio, Arces, University of Bologna, Italy
- Graham Ellis, National University of Ireland, Galway, Ireland
- Lucía Fernández Suárez, ISEL, Portugal
- Ana Cristina Ferreira, University of Minho, Portugal
- Massimo Ferri, University of Bologna, Italy
- Rita Fioresi, University of Bologna, Italy
- José Manuel GarcÌa Calcines, University of La Laguna, Spain
- Thomas Kahl, Univeristy of Minho, Portugal
- Matthew Kahle, Ohio State University, Columbus, USA
- Sanjeevi Krishnan, University of Pennsylvania, USA
- Luc Lemaire, ULB, Belgium
- Nelson Martins-Ferreira, IP Leiria, Portugal
- João Miguel Nogueira, University of Coimbra, Portugal
- Ana Pereira do Vale, University of Minho, Portugal
- Roger Picken, University of Lisbon, Portugal
- Martin Raussen, Aalborg University, Denmark
- Antonio Rieser, Technion, Haifa, Israel
- Ana Romero, University of La Rioja, Spain
- Lucile Vandembroucq, University of Minho, Portugal
- Friedrich Wagemann, University of Nantes, France

2015 INTERNATIONAL MEETING

10 - 13 June, **Porto** - Portugal

AMS / EMS / SPM

american mathematical society | european mathematical society | sociedade portuguesa de matemática

Special Session 6
Applied Algebraic Topology

Thursday June 11, 2015 – Room F-116

13:00-13:50	Massimo Ferri *Biomedical applications of persistent Homology.*	abstract # 1111-55-221
14:00-14:50	Matthew Kahle *The length of the longest bar in random persistent homology.*	abstract # 1111-55-248
15:00-15:20	Armindo Costa *Topology of random simplicial complexes.*	abstract # 1111-55-531

Friday June 12, 2015 – Room F-116

9:00-9:50	Martin Raussen *Combinatorial and topological models for spaces of schedules.*	abstract # 1111-55-48
10:00-10:20	Ana Romero *Zigzag persistence: application to processing stacks of neuronal images.*	abstract # 1111-55-335
10:20-11:00	**Coffee/Tea Break**	
11:00-11:50	Jose Manuel Garcia-Calcines *On some approaches of topological complexity.*	abstract # 1111-55-614

Saturday June 13, 2015 – Room F-116

9:00-9:50	Graham Ellis *Computing with homotopy 2-types.*	abstract # 1111-55-338
10:00-10:20	Jose Gabriel Carrasquel-Vera *On rational topological complexity and related invariants.*	abstract # 1111-55-550
10:20-11:00	**Coffee/Tea Break**	
11:00-11:50	Sanjeevi Krishnan *Dynamic sensor networks.*	abstract # 1111-55-606

Applied & Computational Algebraic Topology
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