Report on the workshop:

GESTA 2013 Workshop

Organizers:	Jean-François Barraud, Eva Miranda, Klaus Niederkrüger, and Fran Presas
Reference Number:	4688
Unit(s):	PESC
Activity Title:	Contact and Symplectic Topology
Activity Acronym:	CAST
Convenor Name:	Mr. Klaus Niederkrüger, Toulouse Cedex 9, France

The workshop *GESTA 2013* was held at the Mathematical Institute of the Université de Toulouse III in the week from June 3 to June 7, 2013. It was mainly sponsored by the ESF network CAST, but it also received minor funding from the university itself and from the ANR project *Topologie de contact en grandes dimensions*.

The workshop consisted of four mini-courses (of 5×45 minutes each) covering different aspects of symplectic geometry in a wide sense. The aim was to give a gentle introduction into several recent topics.

The audience consisted of roughly 50 people, most of whom were graduate students and young postdocs coming from different places. The atmosphere was friendly and it seemed that participants liked the courses.

Below we give an abstract of the four courses:

Joel Fine (Université Libre de Bruxelles). *Moment maps in differential and Kähler geometry.*

Moment maps enable one to take quotients of group actions in symplectic geometry. When the initial space is also a complex algebraic manifold and the group action lifts to a polarizing ("pre-quantum") line bundle, one can also take a quotient using algebraic geometry (via geometric invariant theory). The Kempf–Ness theorem shows that these two quotients are one and the same.

In a seminal article, Atiyah and Bott showed how these ideas could also be applied to the space of connections in a vector bundle, an infinite dimensional symplectic manifold. This work has since been extended in many directions. The infinite dimensional manifold typically parametrizes geometric objects (e.g., connections, Kähler metrics, maps into a symplectic manifold, . . .) Zeros of the moment map typically satisfy a special curvature condition amounting to solving a non-linear PDE (that the connection be Yang–Mills, the Kähler metric has constant scalar curvature, etc). From Kempf–Ness one believes that the PDE should have a solution (the orbit is represented in the symplectic quotient) if and only if a certain algebro-geometric "stability" condition is satisfied (so that the orbit is represented in the complex quotient). This motivates deep conjectures in Kähler geometry, some of which are still open.

Viktor Ginzburg (University of California, Santa Cruz). *From Hamiltonian dynamics to periodic orbits of Reeb flows*

How few periodic orbits can the Reeb flow have, when the contact form gives rise to the standard contact structure on a sphere? Can it have just one closed orbit? In this series of talks we discuss how methods from Hamiltonian dynamics (e.g., the proof of the Conley conjecture) translate to the realm of Reeb flows to answer, or at least to shed some light on these kind of questions.

In particular, drawing from a joint work of the speaker with Hein, Hryniewicz and Macarini, we prove that the existence of one simple closed Reeb orbit of a particular type (a symplectically degenerate maximum) forces the Reeb flow to have infinitely many periodic orbits. We use this result to give a different proof of a recent theorem of Cristofaro–Gardiner and Hutchings asserting that every Reeb flow on the standard contact three-sphere has at least two periodic orbits. We also show that this approach together with several other ingredients leads to a (more or less) purely symplectic proof of the existence of infinitely many geodesics on the two-sphere.

Emmy Murphy (Massachusetts Institute of Technology). *Flexibility in high dimensional contact and symplectic topology.*

Recently there have been a number of developments in high dimensional contact and symplectic flexibility. Here flexibility means that the only obstruction to the existence of objects are those arising from algebraic topology, which is to say that geometric obstructions totally vanish. There are two main results we will discuss: the classification of loose Legendrian embeddings, and the existence of Lagrangian cobordisms with a loose negative end.

We sketch the proof classifying loose Legendrians, which will lead us through "the philosophy of the h-principle" in contexts other than symplectic/contact geometry. As an application we define flexible Weinstein manifolds and deduce their properties, in particular we show that a flexible Weinstein h-cobordism is equivalent to the symplectization of a contact manifold.

We then turn to the topic of exact Lagrangian cobordisms between Legendrian manifolds. After an overview of the relevant concepts, we show that if the Legendrian link at the negative end has a loose component, then exact Lagrangian cobordisms exist as long as they are unobstructed homotopically. As a corollary we show that flexible Weinstein manifolds universally admit embeddings into symplectic manifolds, in particular we note that the only interesting geometry of a Weinstein domain occurs in a topological collar of the boundary. We also apply the theorem, for example we show that any three-manifold admits an exact Lagrangian immersion in $\mathbb{C}^3$ with a single transverse self intersection.

As time allows we will discuss further applications, as well as open questions. Possible topics include: interactions between loose Legendrians, plastikstufe, and the connect sum monoid of contact spheres; Hamiltonian isotopies of Lagrangian cobordisms; deformation invariance of Weinstein flexibility; and contact pseudo-isotopies.

Nguyen Tien Zung (Université de Toulouse III). *Integrable Hamiltonian systems.*

This mini course is about local and global topological and symplectic invariants of integrable Hamiltonian systems on symplectic manifolds. Topics to be covered include:

- Normal forms of integrable Hamiltonian systems
- Topological decomposition theorem for nondegenerate singularities
- Partial action-angle variables near singular fibers
- Symplectic invariants of singularities
- Global topological and symplectic invariants
- Quantization and quantum normal forms

Appendix A. Full list of participants.

(1) Hannah Alpert (MIT)
(2) Jean-François Barraud (IMT)
(3) Gabriele Benedetti (University of Cambridge)
(4) Léo Bénard (Paris VI)
(5) Patrick CABAU (Lycée Pierre Fermat)
(6) Roger Casals (CSIC)
(7) Robert Castellano (Columbia University)
(8) Guillem Cazassus (IMT)
(9) Baptiste Chantraine (Université de Nantes)
(10) Octav Cornea (Université de Montréal)
(11) Sylvain Courte (UMPA ENS Lyon)
(12) Mihai Damian (Université de Strasbourg)
(13) Álvaro del Pino (Universidad Autónoma de Madrid)
(14) Max Dörner (Universität zu Köln)
(15) Sinan Eden (Instituto Superior Tecnico, Lisboa)
(16) Joel Fine (Université Libre de Bruxelles)
(17) Viktor Ginzburg (UCSC)
(18) Vincent Guedj (IMT)
(19) Jean Gutt (Université Libre de Bruxelles)
(20) Kai JIANG (Universite Paris 7)
(21) Michael Khanevsky (University of Chicago)
(22) Anna Kiesenhofer (UPC Barcelona)
(23) Pradip Kumar (Harish Chandra Research Institute)
(24) Oleg Lazarev (Stanford University)
(25) Eveline Legendre (IMT)
(26) Guogang Liu (Nantes)

(27) Alejo Lopéz Ávila (Universidad Autonoma de Madrid)
(28) Mark Lowell (Massachusetts Amherst)
(29) Marco Mazzucchelli (ENS Lyon)
(30) Eva Miranda (Universitat Politècnica de Catalunya)
(31) Philippe Monnier (Université Paul Sabatier)
(32) Emmy Murphy (MIT)
(33) Keddari Nassima (Université de Strasbourg)
(34) Klaus Niederkrüger (Toulouse)
(35) Jean-Pierre Otal (IMT)
(36) Milena Pabiniak (CAMGSD IST Lisboa, Portugal)
(37) Francisco Presas Mata (ICMAT, CSIC)
(38) Marcelo Ribeiro de Resende Alves (Universite Libre de Bruxelles)
(39) Antonio Rieser (Department of Mathematics, Technion - Israel Institute of Technology)
(40) Guillaume Roux (ENS/Paris 7)
(41) Hironori Sakai (WWU Münster)
(42) Simon Schatz (IRMA)
(43) Kyler Siegel (Stanford University)
(44) Peter Smillie (Harvard)
(45) Romero Solha (Universitat Politecnica de Catalunya)
(46) Oldrich Spacil (University of Aberdeen)
(47) Dmitry Tonkonog (University of Cambridge)
(48) Bulent Tosun (CIRGET/UQAM)
(49) Jennifer Vaughan (University of Toronto)
(50) Ramón Vera (Durham University)
(51) Jagna Wiśniewska (Vrije Universiteit, Amsterdam)
(52) Marco Zambon (Universidad Autonoma Madrid, ICMAT)
(53) Alexandr Zamorzaev (Stanford University)