Summer School in Cologne

Hamiltonian Systems and Celestial Mechanics

University of Cologne, July 29th - August 2nd, 2013

Scientific Report for CAST

1) Summary

Hamiltonian dynamical systems are one of the classical inspirations for symplectic and contact geometry. The recent work of Albers, Frauenfelder, Hofer, van Koert and others on applications of contact topology to the restricted 3-body problem has generated renewed interest in the interaction between contact and symplectic geometry on the one hand, and celestial mechanics as a special source of dynamical systems on the other.

One of the highlights of celestial mechanics in the last decade was the discovery of new periodic solutions of the 3-body problem (in the case of equal masses) bey Chenciner and Montgomery. This has led to further solutions of the *n*-body problem for other values of *n*, with all bodies moving on the same fixed curve – so-called *n*-body choreographies. These generalize the classical Lagrange solutions to the 3-body problem, where in the case of equal masses the three bodies form an equilateral triangle and move along a single circle.

In order to foster the interaction between symplectic geometry and celestial mechanics, we had short courses by three speakers who are working in these areas.

2) Description of the scientific content of and discussion at the event

The academic programme consisted of a series of morning lectures and working groups in the afternoon.

We had three series of lectures introducing the main subjects of the week. The goal of the lectures were to communicate the fundamental motivating questions in each field, the tools used to address them, and the important results.

Action minimizing periodic orbits in the N-body problem. Series of four lectures, by Prof. Jacques Féjoz

1. Central configurations
2. The Lagrangian action and its minima
3. Marchal's theorem
4. The example of the P_{12} family

Twist maps with non-periodic angles. Series of four lectures, by Prof. Rafael Ortega

Consider an annulus A with coordinates (q, r), $q + 2p \equiv q$, $r \in [a, b]$. An area-preserving map $(q, r) \longrightarrow (q_1, r_1)$ is twist if it satisfies $\partial q_1 / \partial r > 0$. Twist maps have been extensively studied and they are useful to understand the dynamics of autonomous or periodic Hamiltonian systems of low dimension. In this course we studied twist maps without assuming periodicity on q. In other words, the annulus A is replaced by a strip $S = R \times [a, b]$. This new class of twist maps can be applied to the study of ping-pong models when the motion of the racket is not periodic.

Closed orbits of classical Hamiltonian systems in cotangent bundles. Series of four lectures, by Prof. Felix Schlenk

The search for closed orbits in celestial mechanics was a driving force for the development of Hamiltonian systems and symplectic geometry. The most natural phase spaces are cotangent bundles. We looked at classical dynamical systems on theses phase spaces (over a compact configuration space) and described two methods for proving the existence of a closed orbit on a given energy level: The classical minimax method, and the action selector method. For the latter method, we described a recent construction of Alberto Abbondandolo and Markus Kunze that does not use Floer homology, but "only" the tools in Floer's proof of the degenerate Arnold conjecture.

In the afternoon, the participants were devided into groups, and worked with a mentor on problems that gave them a hands-on feel for the methods of the field. The afternoon groups were related to the topics discussed in morning lectures. Participants choosed their working group in advance when they were accepted for participation.

In these groups, the mentors explained ideas and set problems, which the participants then discussed, tried to understand and worked out. Throughout the afternoon, the mentors lectured for no more than 30-40 minutes total. In the remaining time, the participants were discussing in smaller groups, worked out examples and details of proofs, and presented the results to each other. The mentor presented to guide the discussion, helped the subgroups and explained material that wasn't clear. The subgroups got together in the evenings to continue the discussion, or to prepare a presentation for the next day.

3) Assessment of the results and impact of the event on the future direction of the field

The summer school provided an opportunity for the exchange of ideas between Ph.D. students and postdocs coming from diverse backgrounds, some more analytical, some more geometric. Many had little prior exposure to questions of celestial mechanics and were excited about the directions for research in this area. We expect that this will lead to an increased activity in this area. One immediate outcome was new stimulus for the book project on the geometric foundations of celestial mechanics by one of the organizers (H.G.); Cambridge University Press has already expressed a strong interest in this proposal.

4) Final programme of the meeting

Time	Friday 26.07.	Saturday 27.07.	Sunday 28.07.	Monday, 29.07.2013	Tuesday, 30.07.2013	Wednesday, 31.07.2012	Thursday, 01.08.2013	Friday, 02.08.2013
08.15-09.00				REGISTRATION In front of Hörsaal C				
08.45- 09.00				WELCOME Hörsaal C				
09.00 -10.00				Talk by Prof. Féjoz Hörsaal C	Talk by Prof. Schlenk Hörsaal C	Talk by Prof. Ortega Hörsaal C	Talk by Prof. Féjoz Hörsaal C	Talk by Prof. Ortega Hörsaal C
10.00-10.30				*coffee/tea break I* *vor Hörsaal C*	*coffee/tea break I* *vor Hörsaal C*	*coffee/tea break I* *vor Hörsaal C*	*coffee/tea break I* *vor Hörsaal C*	*coffee/tea break I* *vor Hörsaal C*
10.30-11.30				Talk by Prof. Ortega Hörsaal C	Talk by Prof. Féjoz Hörsaal C	Talk by Prof. Schlenk Hörsaal C	Talk by Prof. N.N. Hörsaal C	Talk by Prof. Schlenk Hörsaal C
11.30-14.00				*Lunch 11.30 – 14.00*	*Lunch 11.30 – 14.00*	*Lunch 11.30 – 14.00*	*Lunch 11.30 – 14.00*	*Lunch 11.30 – 14.00*
14.00-16.00				Afternoon session Seminargebäude Prof. Féjoz: S14 Prof. Ortega: S15 Prof. Schlenk: S16	Afternoon session Seminargebäude Prof. Féjoz: S14 Prof. Ortega: S15 Prof. Schlenk: S16		Afternoon session Seminargebäude Prof. Féjoz: S14 Prof. Ortega: S15 Prof. Schlenk: S16	Afternoon session Seminargebäude Prof. Féjoz: S14 Prof. Ortega: S15 Prof. Schlenk: S16
16.15-16.30				*coffee/tea break II* *Flur 1.2, Seminargebäude*	*coffee/tea break II* *Flur 1.2, Seminargebäude*		*coffee/tea break II* *Flur 1.2, Seminargebäude*	*coffee/tea break II* *Flur 1.2, Seminargebäude*
16.30-∞				Homework	Homework	as of 16.00h Guided Tour over the High Roof of Cologne Cathedrale and through the excavation	Homework	
17:00h				17.00 Party Flur 1.2, Seminargebäude			19.00 Dinner Gilden im Zims Heumarkt 77, 50667 Köln	

5) List of participants

Name	Adresse
Alessandro Arsie	University of Toledo (Ohio), USA
Kilian Barth	University of Cologne
Maryam Beygmohammadi	University of Cologne
Marcel Braukhoff	University of Paderborn
Gustavo de Oliveira	University of Bonn, Germany
Max Dörner	University of Cologne, Germany
Sebastian Durst	University of Cologne, Germany
Christian Evers	University of Cologne, Germany
Jacques Féjoz	University Paris-Dauphine, France
David Frenkel	Institut de Mathématiques Neuchâtel, Switzerland
Stephan Gareis	University of Cologne, Germany
Hansjörg Geiges	University of Cologne, Germany
Robert Gollan	University of Cologne, Germany
Jean Gutt	Université Libre de Bruxelles, Belgium
Kerstin Hastrich	University of Cologne, Germany
Hien Minh Huynh	University of Cologne, Germany
Veronica Istrate	University of Cologne, Germany
Riccardo Danilo Jadanza	Politecnico di Torino, Italy
Dominic Jänichen	University of Cologne, Germany
Jungsoo Kang	University of Münster, Germany & Seoul National University, Korea
Elena Kantonistova	Moscow State Univesity, Russian Federation
Marc Kegel	University of Cologne, Germany
Florian Krügel	University of Cologne, Germany
Markus Kunze	University of Cologne, Germany
Christian Lange	University of Cologne, Germany
Chiara Leonhardt	University of Cologne, Germany
Arun Maiti	Max-Plank Institute for Mathematics in the Sciences, Leipzig, Germany
Vito Mandorino	University Paris-Sud 11, France
Stefano Marò	University of Torino, Italy
Timur Mashkin	University of Cologne, Germany
Michael H. Mertens	University of Cologne, Germany
Matthias Nagel	University of Cologne, Germany
Rafael Ortega	University of Granada, Spain
Carolin Pomrehn	University of Cologne, Germany
Marcelo Ribeiro de Resende Alves	Libre de Bruxelles, Bruxelles, Belgium
Thomas Rot	VU University Amsterdam, The Netherlands
Hironori Sakai	University of Münster, Germany
Felix Schlenk	University of Neuchâtel, Switzerland
Jan Philipp Schröder	Ruhr-University Bochum, Germany
Patanjali Sharma	Banasthali University, India
Oldrich Spacil	University of Aberdeen, Scotland, United Kingdom

Marcin Styborski	Gdansk University of Technology, Poland
Arjun Sudan	VU University Amsterdam, The Netherlands
Stefan Suhr	University of Hamburg, Germany
Jagna Wisniewska	VU University Amsterdam, The Netherlands
Raphael Wullschleger	Institut de Mathématiques Neuchâtel, Switzerland
Brent Young	University of Cologne, Germany
Guowei Yu	University of Minnesota (Minneapolis), USA
Manuel Zamora	University of Granada, Spain
Kai Zehmisch	University of Cologne, Germany
Raphael Zentner	University of Cologne, Germany
Lei Zhao	Paris Diderot University, France
Elizabeth Zollinger	St. Joseph's College Brooklyn (New York), USA