



Science Meeting – Scientific Report

The scientific report (WORD or PDF file - maximum of seven A4 pages) should be submitted online within two months of the event. It will be published on the ESF website.

Proposal Title: SFT 7 - Function theory on symplectic manifolds

Application Reference N°: Science Meeting 5385

1) Summary (up to one page)

The workshop series SFT (=Symplectic Field Theory) is regular event at German universities which aims at presenting recent developments of high importance and impact in the field of symplectic geometry. The main target group is graduate students and young postdocs but also more senior researchers. The format of the SFT series consists of one main speaker which gives several lectures on the main topic of the workshop. Roughly 10 researchers present related research results in typical 60 minute lectures. During the weekend before the workshop a precourse on basic notions, concepts and results is offered in order to make the workshop even more accessible to graduate students.

The 7th iteration *SFT 7 - Function theory on symplectic manifolds* followed this general scheme. The main lecturer was Leonid Polterovich from Tel Aviv University, one of the founders of and the main contributor to *Function theory on symplectic manifolds*. Other speakers are listed under Annex 4b).

The majority of the participants (not counting the speakers) were graduate students and young postdocs. In total 60 people meet at the workshop including 13 speakers. The precourse was well received, roughly 30 attended.

The lecture course by Polterovich was excellent and accompanied by several exercise and example sessions, again mainly for graduate students and young postdocs. On Thursday afternoon we included a moderated open problem session which lasted for several hours. The full program can be found in Annex 4a).

Students and young postdoc were exposed to a rapidly developing new branch of symplectic geometry: Function theory on symplectic manifolds. The research talks exhibited and underlined connections to other directions within and beyond symplectic

geometry. Moreover, participants had ample opportunity to establish new contacts, exchange ideas and to start and continue collaborations.

2) **Description of the scientific content of and discussions at the event (up to four pages)**

The main theme of the workshop was *Function theory on symplectic manifolds*. This new development in symplectic geometry is based on the fundamental work by Entov and Polterovich.

One of the main themes in symplectic geometry is flexibility vs. rigidity. All basic notions in symplectic geometry such as symplectic form, symplectomorphisms etc. are differentiable (smooth) in nature. The rigidity question underlying function theory on symplectic manifolds is whether one can say something meaningful by considering the space of continuous functions equipped with the C^0 -topology. To give a very concrete theorem: The C^0 -norm of the Poisson bracket of two smooth functions lower semi-continuous. This surprising rigidity result is one of the starting points of Polterovich's lectures. The following is summary of the five lectures.

Lecture 1: Hamiltonian chords and Poisson brackets

Based on the above mentioned rigidity result for Poisson brackets on symplectic manifolds Polterovich exhibits applications to the existence of Hamiltonian chords. Existence results for Hamiltonian chords lie at the heart of symplectic dynamics. Finally he presents Buhovsky's proof of the rigidity result for Poisson brackets.

Lecture 2: Symplectic quasi-states with applications

Symplectic quasi-states are introduced and serve as one of the main symplectic tools in function theory on symplectic manifolds. First basic applications include rigidity of partitions of unity and generic growth in Hofer's geometry. Existence of symplectic quasi-states is deferred to lecture 3.

Lecture 3: Subadditive spectral invariants and symplectic quasi-states

The main tool (with the exception of dimension 2) for constructing symplectic quasi-states is Floer homology. The basics of this technically demanding theory were recalled during the precourse. With help of Floer homology subadditive spectral invariants can be defined and give rise to the so-called spectral pseudo norm. The main observation is that, if and only if the spectral pseudo norm is bounded the spectral invariants give rise to a symplectic quasi-state. Examples are discussed.

Lecture 4: More on symplectic quasi-states

This lecture brings all the previous lectures together and proves a number of theorems and applications mentioned in earlier lectures.

Lecture 5: Quantum Stories

In his last lecture Polterovich reports on a very exciting new line of research which grew out of the study of function theory on symplectic manifolds. It concerns applications of symplectic rigidity results to quantum mechanics. This is very surprising since symplectic geometry can be interpreted as the geometry of *classical* mechanics. Nonetheless, via

Berezin-Toeplitz quantization rigidity results from symplectic geometry can be brought to bear.

Through his five lectures Polterovich presented a new development in symplectic geometry to a broad audience and made it accessible. His enthusiastic, clear and engaging lecturing style was recognized by long lasting applause.

During the 10 research talks (see detailed program in Annex 4a)) various topics of Polterovich's lecture course were treated in more detail resp. were complemented by current developments.

In the open problem session 9 speakers explained in short presentations open questions, problems and directions lead to several detailed discussions and hopefully trigger future research.

3) Assessment of the results and impact of the event on the future directions of the field (up to two pages)

The main goal of this workshop was to present a emerging new line of research in symplectic geometry to a broad audience with a focus on graduate students and young postdocs. The precourse recalls and lays necessary foundations for the lecture course by Polterovich which is complemented by research lectures. In addition exercise and example courses and an open problem session were offered.

The workshop was a big success, foremost due to Polterovich's clear and well prepared lectures. The precourse and the exercise and example courses were very well attuned to the main lecture course. Finally the supporting research talks were all in all of very high quality. During the workshop week participants had a lot of discussion at and around the workshop, including junior participants!

Short term impact of this conference is that several researchers in ongoing collaborations used the opportunity to exchange and discuss ideas. Mid and long term impact will come from the open problem session and, of course, through graduate students and young postdocs which are influenced by the exciting mathematics presented at this workshop but also through the enthusiasm and dedication shown by Polterovich and other speakers of the conference.

4) Annexes 4a) and 4b): Programme of the meeting and full list of speakers and participants

Annex 4a: Programme of the meeting

Precourse August 23 & 24 by Urs Frauenfelder & Kai Zehmisch

Saturday

9-10:30 Frauenfelder lecture 1

11-12:30 Zehmisch lecture 1

15-16:30 Frauenfelder lecture 2

Sunday

9-10:30 Zehmisch lecture 2

11-12:30 Frauenfelder lecture 3

15-16:30 Zehmisch lecture 3

Workshop August 25 - 29

Monday

9-10 Polterovich lecture 1

11-12 Opshtein

12:15-13 Exercise/example session

15-16 Borman

17-18 Seyfaddini

Tuesday

9-10 Polterovich lecture 2

11-12 Schlenk

12:15-13 Exercise/example session

15-16 Merry

17-19 Q/A session with Strom Borman, Daniel Rosen, and Frol Zapolsky

Wednesday

9-10 Polterovich lecture 3

11-12 Buhovski

12:30-13:30 Zapolsky

Thursday

9-10 Polterovich lecture 4

11-12 Kang

12:15-13 Exercise/example session

15-16 Open problem session part 1 - moderated by Kai Cieliebak

17-19 Open problem session part 2 - moderated by Kai Cieliebak

Friday

9-10 Polterovich lecture 5

11-12 Khanevsky

12:15-13:15 Entov

15-17 Q/A session with Strom Borman, Daniel Rosen, and Frol Zapolsky

Annex 4b: Full list of speakers and participants

Speakers:

- 5-hour minicourse by Leonid Polterovich (Tel Aviv University)
- 1-hour lectures by Strom Borman (Stanford University), Lev Buhovski (Tel Aviv University), Michael Entov (Technion Haifa), Jungsoo Kang (Münster University), Michael Khanevsky (University of Chicago), Will Merry, (ETH Zürich) Emmanuel Opshtein (IRMA Strasbourg), Sobhan Seyfaddini (Ecole Normale Supérieure Paris), Felix Schlenk (Université de Neuchâtel), Frol Zapolsky (Haifa University)
- Precourse: 3 lectures by Urs Frauenfelder (Augsburg University) and Kai Zehmisch (Münster University)

Participants:

1. Albers, Peter (Münster)
2. Barth, Kilian (Münster)
3. Borman, Strom (Stanford)
4. Buhovski, Lev (Tel Aviv)
5. Castellano, Robert (New York)
6. Cieliebak, Kai (Augsburg)
7. Dahinden, Lucas (Neuchatel)
8. Diogo, Luís (New York)
9. Doicu, Alexandru (Augsburg)
10. Duretic, Jovana (Belgrad)
11. Entov, Michael (Haifa)
12. Espina, Jacqui (London)
13. Fauck, Alexander (Berlin)
14. Fuchs, Urs (Bochum)
15. Frauenfelder, Urs (Augsburg)
16. Gadbled, Agnès (Porto)
17. Ganor, Yaniv (Tel Aviv)
18. Gerstenberger, Andreas (Hamburg)
19. Haug, Carsten (Tübingen)
20. Hein, Doris (Freiburg)
21. Helmsauer, Kathrin (Augsburg)
22. Hohloch, Sonja (EPFL Lausanne)
23. Horvath, Ramon (Uppsala)
24. Hwang, Taekgyu (Seoul)
25. Kang, Jungsoo (Münster)
26. Katic, Jelena (Belgrad)
27. Kawasaki, Morimichi (Tokyo)
28. Kegel, Marc (Köln)
29. Khanevsky, Michael (Chicago)
30. Kirchner, Fabian (Hamburg)
31. Lanzat, Sergei (Geneva)
32. Latschev, Janko (Hamburg)
33. Lee, JunYoung (Augsburg)
34. Maydankiy, Maksim (Paris)
35. Meiwes, Matthias (Münster)
36. Membrez, Cedric (ETH Zürich)
37. Merry, Will (ETH Zürich)
38. Milinkovic, Darko (Belgrad)
39. Mohnke, Klaus (Berlin)
40. Ojeda Santana, Juan (Bochum)
41. Opshtein, Emmanuel (Strasbourg)

42. Orita, Ryuma (Tokyo)
43. Peiffer-Smadja, Amiel (Paris)
44. Piwatz, Anna (München)
45. Polterovich, Leonid (Tel Aviv)
46. Rosen, Daniel (Tel Aviv)
47. Schlenk, Felix (Neuchatel)
48. Schulte, Ingo (Münster)
49. Seongchan, Kim (Augsburg)
50. Shelukhin, Egor (Jerusalem)
51. Siburg, Karl Friedrich (Dortmund)
52. Sieftring, Richard (Leipzig)
53. Spacil, Oldrich (London)
54. Sugiyama, Satoshi (Tokyo)
55. Seyfaddini, Sobhan (Paris)
56. Uebele, Peter (Augsburg)
57. Wisniewska, Jagna (Amsterdam)
58. Zapolski, Frol (Haifa)
59. Zehmisch, Kai (Münster)
60. Zhao, Jingyu (New York)