

**3rd International School and Conference on
GEOQUANT
Geometry and Quantization
University of Luxembourg, Mathematics Research Unit
31.8. – 11.9.2009**

ESF Networking Programme HCAA, Meeting 2327

ESF Networking Programme ITGP, Meeting 2580

FNR Lux /08/AM3/36

Scientific Report

Summary

The 3rd International School and Conference on Geometry and Quantization took place at the Mathematics Research Unit (RMATH) of the University of Luxembourg (UL) August 28 till September 11, 2009.

The scientific program of the School-Conference was concentrated around the following main topics:

1. algebraic-geometric and complex-analytic-geometric aspects of quantization
2. geometric quantization and moduli space problems
3. asymptotic geometric analysis
4. infinite-dimensional geometry
5. relations with modern theoretical physics.

These fields are directions of ongoing intensive mathematical research with numerous open challenging problems waiting to be taken up by researchers.

The first week of the School-Conference was a school for young scientists, while the second part was a scientific conference on the above topics. During the first part (the school), newcomers to the field got acquainted with the basics, needed to get ready to listen to the scientific talks on the subject, presented during the second part (the conference). It was an international conference. Participants and speakers from all over Europe, Russia, Japan, USA, Canada, China, and elsewhere came.

The school was attended by 65 registered participants, the conference by 83 registered participants. A follow-up School-Conference is foreseen for the year 2011 either in China or in Japan.

The **Scientific Organization Committee** consisted of Joachim Hilgert (Paderborn University, Germany), Ryoichi Kobayashi (Nagoya University, Japan), Martin Schlichenmaier (Luxembourg University, Luxembourg), Armen Sergeev (Steklov Mathematical Institute, Moscow, Russia), Oleg Sheinman (Steklov Mathematical Institute, Moscow, Russia), Tilmann Wurzbacher (University of Metz, France).

The **Local Organization Committee** consisted of Martin Schlichenmaier (Main responsible), Yael Fregier, Laurent Hofer, Marie-Amelie Lawn, Martin Olbrich, Norbert Poncin, Boris Shoikhet, Anton Thalmaier

The conference was **sponsored** by DFG-IRTG Geometry and Analysis of Symmetries, University of Paderborn; ESF research networking programme HCAA; ESF research networking programme ITGP; FNR Luxembourg; Mathematics Research Unit of the University of Luxembourg; Mathematics Laboratory of the University of Metz.

Description of the Scientific content of and discussions at the event

The scientific field of the conference and school belongs to Modern Mathematics which is partly inspired by developments in Mathematical and Theoretical Physics.

In the last couple of years quite a number of challenging mathematical problems came from theoretical physics. The conference centered on some of the mathematical theory and mathematical problems in this context. Clearly as this is a huge field we had to put the focus of the conference to certain points.

The following fields were addressed: Concepts of Algebraic Geometry and Complex Analytic Geometry in relation to quantization, in particular geometric quantization, relations to moduli spaces, their geometry and quantization, asymptotic geometric analysis, infinite-dimensional Kähler geometry with its relation to quantum field theory and other directions in theoretical physics, heat equation techniques, infinite-dimensional Lie algebras of geometric origin.

To allow newcomers to the field, Phd students etc. to enter these very active fields of mathematical research a school for young scientists was held. The second part was a scientific conference on the above topics.

The School

The school took place in the first week, i.e. from 31.8. till 5.9.2009. The program included lecture courses on the subject of the School-Conference, delivered by experts:

The list of lecturers was

- **Jorgen Andersen** (Aarhus University, CTQM, Denmark), *Quantization of moduli spaces of bundles and the mapping class group*
- **Johannes Hübschmann** (University of Lille, France): *Stratified Kähler spaces*
- **Armen Sergeev** (Steklov Institute, Moscow, Russia): *Quantization of universal Teichmüller space*
- **Dmitry Talalaev** (Institute for Theoretical and Experimental Physics, Moscow, Russia) *Quantum integrable systems and Langlands program*
- **Tatsuya Tate** (University of Nagoya, Japan) *Problems on asymptotic analysis over convex polytopes*
- **Tilman Wurzbacher** (University of Metz, France): *Geometry of loop spaces*
- **Weiping Zhang** (Nankai Institute of Mathematics, Tianjing, China) *Analytic aspects of geometric quantization*

Each speaker gave 3 lectures each lasting 1 hour.

In addition there was an introductory talk by Martin Schlichenmaier on Berezin-Toeplitz Quantization and several discussion sessions.

The Scientific Conference

The Scientific Conference (7.9. – 11.9. 2009) consisted of talks of either 60 minutes or 30 minutes lengths (including discussions).

The following is a list of the talks presented at the conference:

Andersen, Jorgen, Aarhus, Denmark *The Hitchin connection, Toeplitz operators and TQFT*

Baier, Thomas, Lisbon, Portugal *Geometric quantization of families of degenerating complex structures*

Bondal, Alexey, Aberdeen, UK *Orthogonal decomposition of Lie algebras and a problem of quantum information*

Charles, Laurent, Paris, France: *Quantization of polygon space*

Dai, Xianzhe, Santa Barbara, USA: *The asymptotic expansion of Bergman kernel*

Driver, Bruce, San Diego, USA: *Holomorphic functions and subelliptic heat kernels over Lie group*

Englis, Mirek, Prague, Tschech Republic: *Toeplitz quantization on real symmetric domains*

Fujita, Hajime, Tokyo, Japan: *Torus fibrations and localization of index*

Gorodentsev, Alexey, Moscow, Russia: *Functorial A infinity coproduct of combinatorial simplicial chains that induces itself under barycentric subdivision*

Huebschmann, Johannes, Lille, France: *Line bundles on moduli and related spaces*

Kaledin, Dmitry, Moscow, Russia: *Cotangent bundles of complex Grassmanians and representation theory*

Karabegov, Alexander, Abilene, USA: *Formal symplectic groupoids with separation of variables and Bergman idempotent*

Kori, Tosiaki, Tokyo, Japan: *Geometric pre-quantization of $SU(N)$ flat connections on 3-manifolds*

Khudaverdian, Hovhannes, Manchester, UK: *Differential forms in an odd symplectic geometry and Pfaffians*

Ma, Xiaonan, Paris, France *Bergman kernel and geometric quantization*

Mano, Toshiyuki, Kyoto, Japan: *The Riemann-Wirtinger integral and its application*

Marinescu, George, Kln, Germany: *Toeplitz operators on complex and symplectic manifolds*

Moriyama, Takayuki, Kyoto, Japan *Deformations of transverse Calabi-Yau structures on foliated manifolds*

Natanzon, Sergey, Moscow, Russia: *Topological string theory*

Nohara, Yuichi, Nagoya, Japan: *Toric degenerations of Gelfand-Cetlin systems and potential functions*

Osipov, Denis, Moscow, Russia: *Formal groups arising from formal punctured ribbons*

Paoletti, Robert, Milano, Italy: *Local asymptotics for Toeplitz operators*

Shadrin, Sergey, Amsterdam, The Netherlands: *An explicit construction of BCOV cohomological field theory*

Schottenloher, Martin, Munich, Germany *The geometric phase in QED*

Sheinman, Oleg, Moscow, Russia: *Lax operator algebras and integrable systems*

Shoikhet, Boris, Luxembourg *Bialgebras and n -monoidal categories*

Talalaev, Dmitry, Moscow, Russia: *Bethe Ansatz and Isomonodromic transformations*

Tate, Tatsuya, Nagoya, Japan: *An asymptotic Euler-Maclaurin formula for Delzant polytopes*

Treschev, Dmitry, Moscow, Russia: *Non-commutative structures and operators on Hilbert spaces*

Tyurin, Nikolai, Dubna, Russia: *Non toric lagrangian fibrations of toric (and non toric) Fano varieties*

Upmeyer, Harald, Marburg, Germany: *Generalized Fock spaces on Jordan varieties*

Yoshida, Takahiko, Japan: *Torus fibrations and localization of index -application to locally toric Lagrangian fibrations*

Wendland, Katrin, Augsburg, Germany: *Asymptotic analysis for unitary conformal field theories*

Further details, including abstracts and lecture notes can be found on the web-page of the conference: <http://math.uni.lu/geoquant>.

Assessment of the results and impact of the event on the future direction of the field

Geometric quantization is a field having mathematical and physical aspects. Furthermore quite a number of mathematical directions (algebraic geometry, Kähler geometry, complex analysis, representation theory, number theory, etc.) are involved. It is important that researchers working in these directions meet each other on a semi-regular basis to present recent results, to learn what else has been done or is under investigation in neighbouring directions relevant to the field, and to discuss further research plans. The conference succeeded in its goal to supply this forum of interaction. Based on the discussions further research ideas were created and further research directions opened.

Also the school was a success. Young scientists got acquainted with a very active and challenging research topic in mathematics and mathematical physics. The field offers a number of interesting problems to be solved by them.

We got an excellent feedback from the participants. This feedback showed the need of a 4th edition of GEOQUANT. It will be held 2011 most likely either in China or in Japan.

Final program of the meeting

Schedule for the School GEOQUANT

at the University of Luxembourg

31.8.2009 – 5.6.2009

Lectures

- **Jorgen ANDERSEN** (Aarhus University, CTQM, Denmark), *Quantization of moduli spaces of bundles and the mapping class group*
- **Johannes HÜBSCHMANN** (University of Lille, France): *Stratified Kähler spaces*
- **Armen SERGEEV** (Steklov Institute, Moscow, Russia): *Quantization of universal Teichmüller space*
- **Dmitry TALALAEV** (Institute for Theoretical and Experimental Physics, Moscow, Russia) *Quantum integrable systems and Langlands program*
- **Tatsuya TATE** (University of Nagoya, Japan) *Problems on asymptotic analysis over convex polytopes*
- **Tilmann WURZBACHER** (University of Metz, France): *Geometry of loop spaces*
- **Weiping ZHANG** (Nankai Institute of Mathematics, Tianjing, China) *Analytic aspects of geometric quantization*

Each speaker will give 3 lectures each lasting 1 hour.

In addition there will be an introductory talk by Martin Schlichenmaier on Berezin-Toeplitz Quantization, several discussion sessions, dinner and excursion.

<http://math.uni.lu/geoquant/school>

The **time slots** are as follows:

10.00 - 11.00 : slot 1
11.00 - 11.30 : coffee break
11.30 - 12.30 : slot 2
12.30 - 14.00 : common lunch at the cafeteria of the university
14.00 - 15.00 : slot 3
15.10 - 16.10 : slot 4
16.10 - 16.40 : coffee break
16.40 - 17.40 : slot 5

The lectures will take place in the Batiment des Sciences in the lecture hall **BS1.03**.

Registration will be necessary and will take place Monday, 31.8.2009 from 8.30 to 9:30 in the Batiment des Recherches B (BRB) in the meeting room, ground floor. Those participants who want to join the dinner on Wednesday and/or the excursion on Saturday have to declare this during the registration.

Schedule

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
	9.30 Wellcome					9.00 Start
sl1	Schlichenmaier	Tate	Hübschmann	Hübschmann	Talalaev	Excursion
sl2	Wurzbacher	Sergeev	Talalaev	Zhang	Andersen	Excursion
sl3	Sergeev	Zhang	Andersen	Talalaev	Sergeev	Excursion
sl4	Zhang	Wurzbacher	Wurzbacher	Andersen	Tate	Excursion
sl5	Hübschmann	Discussion	Discussion	Tate	Discussion	Excursion
	18.00 Reception		20.00 Dinner			20.00 End

Date: 19.8.2009

Schedule for the conference GEOQUANT

at the University of Luxembourg

7.9.2009 – 11.9.2009

Monday, September 7, 2009

- 8.30 - 9.30 : **Registration**
- 9.30 - 10.00 : Welcome
- 10.00 - 10.50 : *Andersen, Jorgen, Aarhus, Denmark:*
The Hitchin connection, Toeplitz operators and TQFT
- 11:00 - 11.30 : **Coffee break**
- 11.30 - 12.20 : *Dai, Xianzhe, Santa Barbara, USA:*
The asymptotic expansion of Bergman kernel
- 12.30 - 12.55 : *Osipov, Denis, Moscow, Russia:*
Formal groups arising from formal punctured ribbons
- 13.00 - 14.00 : **Lunch break**
- 14.00 - 14.50 : *Marinescu, George Cologne, Germany*
Toeplitz operators on complex and symplectic manifolds
- 15.00 - 15.50 : *Ma, Xiaonan, Paris, France:*
Bergman kernel and geometric quantization
- 15.50 - 16.20 : **Coffee break**
- 16.20 - 17.10 : *Wendland, Katrin, Augsburg, Germany:*
Asymptotic analysis for unitary conformal field theories
- 17.20 - 17.45 : *Schottenloher, Martin, Munich, Germany:*
The geometric phase in QED
- 17.50 - 18.30 : *Sheinman, Oleg, Moscow, Russia:*
Lax operator algebras and integrable systems
- 18.30 - 20.00 : **Reception**

Tuesday, September 8, 2009

- 9.30 - 10.20 : *Driver, Bruce, San Diego, USA:*
Holomorphic functions and subelliptic heat kernels over Lie group
- 10.30 - 11.00 : **Coffee break**
- 11.00 - 11.50 : *Englis, Mirek, Prague, Tschech Republic:*
Toeplitz quantization on real symmetric domains
- 12.00 - 12.50 : *Upmeyer, Harald, Marburg, Germany:*
Generalized Fock spaces on Jordan varieties
- 13.00 - 14.00 : **Lunch break**
- 14.00 - 14.50 : *Gorodentsev, Alexey, Moscow, Russia:*
Functorial A infinity coproduct of combinatorial simplicial chains
that induces itself under barycentric subdivision
- 15.00 - 15.50 : *Tyurin, Nikolai, Dubna, Russia:*
Non toric lagrangian fibrations of toric (and non toric) Fano varieties
- 15.50 - 16.20 : **Coffee break**
- 16.20 - 17.10 : *Fujita, Hajime, Tokyo, Japan:*
Torus fibrations and localization of index
- 17.20 - 17.45 : *Yoshida, Takahiko, Japan:*
Torus fibrations and localization of index – application to
locally toric Lagrangian fibrations
- 17.50 - 18.40 : *Shoikhet, Boris, Luxembourg:*
Bialgebras and n-monoidal categories

Wednesday, September 9, 2009

- 9.30 - 10.20 : *Huebschmann, Johannes, Lille, France:*
Line bundles on moduli and related spaces
- 10.30 - 11.00 : **Coffee break**
- 11:00 - 11.50 : *Kori, Tosiaki, Tokyo, Japan:*
Geometric pre-quantization of $SU(N)$ flat connections on 3-manifolds
- 12.00 - 12.50 : *Kaledin, Dmitry, Moscow, Russia:*
Cotangent bundles of complex Grassmanians and
representation theory
- 13.00 - 14.00 : **Lunch break**
- 14.00 – : **Excursion**
- 20.00 - 23.00 : **Conference dinner: Restaurant Last Supper**

Thursday, September 10, 2009

- 9.30 - 10.20 : *Charles, Laurent, Paris, France:*
Quantization of polygon space
- 10.30 - 11.00 : **Coffee break**
- 11:00 - 11.50 : *Paoletti, Robert, Milano, Italy:*
Local asymptotics for Toeplitz operators
- 12.00 - 12.50 : *Shadrin, Sergey, Amsterdam:*
An explicit construction of BCOV cohomological field theory
- 13.00 - 14.00 : **Lunch break**
- 14.00 - 14.50 : *Tate, Tatsuya, Nagoya, Japan:*
An asymptotic Euler-Maclaurin formula for Delzant polytopes
- 15.00 - 15.50 : *Karabegov, Alexander, Abilene, USA:*
Formal symplectic groupoids with separation of variables
and Bergman idempotent
- 15.50 - 16.20 : **coffee break**
- 16.20 - 17.10 : *Nohara, Yuichi, Nagoya, Japan:*
Toric degenerations of Gelfand-Cetlin systems and potential functions
- 17.20 - 17.45 : *Moriyama, Takayuki, Kyoto, Japan:*
Deformations of transverse Calabi-Yau structures on foliated manifolds
- 17.50 - 18.15 : *Baier, Thomas, Lisbon, Portugal:*
Geometric quantization of families of degenerating complex structures

During the whole day there will be the **poster presentation** on the first floor of the Batiment des Sciences building

Friday, September 11, 2009

- 9.30 - 10.20 : *Talalaev, Dmitry, Moscow, Russia:*
The elliptic quantum spin model
- 10.30 - 11.00 : **Coffee break**
- 11:00 - 11.50 : *Natanzon, Sergey, Moscow, Russia:*
Topological string theory
- 12.00 - 12.50 : *Treschev, Dmitry, Moscow, Russia:*
Non-commutative structures and operators on Hilbert spaces
- 13.00 - 14.00 : **Lunch break**
- 14.00 - 14.50 : *Bondal, Alexey, Aberdeen, UK :*
Orthogonal decomposition of Lie algebras and
a problem of quantum information
- 15.00 - 15.50 : *Mano, Toshiyuki, Kyoto, Japan:*
The Riemann-Wirtinger integral and its application
- 15.50 - 16.20 : **Coffee break**
- 16.20 - 17.00 : *Khudaverdian, Hovhannes, Manchester, UK:*
Differential forms in an odd symplectic geometry and Pfaffians
- 17.00 : **Closing**