

Scientific Report from the Young Set Theory Workshop 2010

19. April 2010

Summary

The Young Set Theory Workshop 2010 took place 14-19 February 2010 at Seminarzentrum Raach in Raach am Hochgebirge in Austria. The workshop had 67 participants, including 10 speakers. The participants represented at least 10 different European countries, the US, Canada, Israel, Colombia, Japan and Australia.

The goal of this workshop was to bring together postgraduates and post-docs in set theory in order to learn from senior researchers in the field, hear about the latest research and to discuss research issues in small focused groups. To this end, four senior researchers gave excellent and enlightening tutorials representing different branches of set theory, six outstanding post-doctoral researchers presented the latest trends in their areas of research and many students and post-docs presented or discussed their work in groups during the discussion sessions.

Another aim was to create a network of young researchers and senior researchers who support their work in order to establish working contacts and to better disseminate knowledge in the field. This has been the aim of the Young Set Theory Workshop series, of which this workshop in 2010 was the third. Due to their success, more such workshops are already in the planning for 2011 and 2012. One of the discussion sessions at this workshop dealt with the topic of 'The Future of Young Set Theory'. During this session, it was decided that there is the need and will for a more permanent network than just these workshops. Already, a website dedicated to this goal is being prepared with the support of the INFTY network. The workshop served as a springboard for the design discussions.

The discussion sessions themselves allowed participants the opportunity to find other young researchers interested in their work and to start projects with them or to find out the latest results in that area.

Description of the scientific content of and discussion at the event

The workshop focused on the interplay between forcing, inner models, descriptive set theory and applications. New results and new perspectives on classical results were presented in four tutorials, six post-doctoral talks and four discussion sessions.

Ralf Schindler gave a tutorial on “Mice and forcing absoluteness”. He presented new results by Woodin showing that if there is an iterable (countable) mouse with a measurable Woodin cardinal and if V and $V^{\mathbb{P}}$ both satisfy CH , then V and $V^{\mathbb{P}}$ have the same Σ_1^2 -theory. This result was obtained after he discussed the concept of a mouse, gave examples of mice and showed that any real can be made generic over mice with Woodin cardinals for the extender algebra with ω generators and that any subset of ω_1 can be made generic over mice with a measurable Woodin cardinal δ for the extender algebra with δ generators.

Greg Hjorth talked about “Effective descriptive set theory and admissible sets”. He discussed a circle of ideas which represent vital techniques in the theory of light faced Σ_1^1 , Π_1^1 and Δ_1^1 sets but whose representation in the existing literature of set theory is obscure. One of the motivating themes in modern descriptive set theory is that insight into certain complexity classes can be achieved by analyzing the appropriate inner model. For instance a set $A \subseteq \omega$ is Σ_2^1 if and only if it is Σ_1 definable over Gödel’s constructible universe L . In his lectures he was looking at sets far closer to ground and proved that $A \subseteq \omega$ is Π_1^1 if and only if it is Σ_1 -definable over $L_{\omega_1^{ck}}$. Using an effective version of the Kunen-Martin theorem he proved that ω_1^{ck} is the least ordinal $\alpha > \omega$ such that L_α is admissible. Moreover he gave a proof of the Lusin-Novikov theorem using the technology developed in the tutorial.

In his tutorial, Uri Abraham presented “Some classical ccc forcings”. He reproved Baumgartner’s classical result that it is consistent that $2^{\aleph_0} = \aleph_2$ and any two \aleph_1 -dense sets of reals with no end points are isomorphic. To do so, he introduced NA stating that whenever A, B are such sets then there exists another one called C and order-preserving $f : C \rightarrow A$, $g : C \rightarrow B$. Proofs of $Con(2^{\aleph_0} = \aleph_2 \text{ and } NA)$ and $MA_{\aleph_1} \not\equiv NA$ were given. The semi-open coloring axiom was introduced and its consistency with MA and $2^{\aleph_0} = \aleph_2$ was proved.

Justin Moore lectured on “The proper forcing axiom” and its astonishing applications. Recently found implications of PFA include: If X is a Banach space of density \aleph_1 , then X has a quotient with a basis of length ω_1 (Todorćević). If H is a separable Hilbert space then all automorphisms of $B(H)/K(H)$ are inner (Farah). Every uncountable linear order contains an isomorphic copy of ω_1 , $-\omega_1$, a Countryman line C , $-C$ or a set of reals of cardinality \aleph_1 (Moore). An emphasis was placed on how to construct a

proper partial order and how to verify its properness. He also discussed the open coloring axiom OCA and the P -ideal dichotomy PID . In particular, it was explained how PFA implies PID and why $PID \Rightarrow SCH$ as well as $PID \Rightarrow \neg \square(\kappa)$.

Inessa Epstein, Thomas Johnstone, Bart Kastermans, Wieslaw Kubis, Philipp Schlicht and Lyubomyr Zdomskyy were invited as outstanding post-docs to give talks on their research. Inessa Epstein gave a talk on “Descriptive set theory and measure preserving group actions”. She considered the space the space of free, measure preserving, ergodic actions of a countable group on a standard probability space. This space is of high importance in functional analysis. She introduced set theoretic aspects that are of interest as well as recent results concerning these group actions.

Thomas Johnstone discussed “The resurrection axioms”, a new class of forcing axioms, and explained that they are equiconsistent with an uplifting cardinal, a large cardinal notion that can exist in Gödel’s constructible universe L .

Bart Kastermans talked on “Formalizing set theory”. On the basis of examples from the theory of cofinitary groups, he explained how proofs are formalized in the Isar language, and how such proofs are checked with the Isabelle proof checker.

Wieslaw Kubis presented “Applications of elementary submodels to Banach spaces”. He showed how they can be used for constructing projections in Banach spaces and for studying the dual spaces of compact spaces.

Philipp Schlicht gave an overview of the “Descriptive set theory at uncountable cardinals”. There are several analogies to classical results in descriptive set theory for the spaces κ^κ , where κ is a regular uncountable cardinal with $\kappa^{<\kappa} = \kappa$. He put emphasis on regularity properties and definable equivalence relations on κ^κ .

Lyubomyr Zdomskyy presented his work on “Projective mad families”. He analyzed how low in the projective hierarchy one can consistently find mad subfamilies of $[\omega]^\omega$ or ω^ω considering the cases CH , $\neg CH$, $\mathfrak{b} = \omega_1$ and $\mathfrak{b} = \omega_2$.

During the discussion sessions various topics were discussed. In the following, four typical examples are given. Wieslaw Kubis discussed his work and ideas on finding universal objects for certain classes. Following the theory of Fraïssé limits of a class of first order structures, he developed a category-theoretic approach that gives much more freedom than the model-theoretic one.

Christoph Weiß discussed his versions of the tree property which relate to subtle and ineffable cardinals and to strongly compact and supercompact cardinals in the same way as the usual tree property relates to weakly compacts. So, in some sense, these principles are able to express that the ω_2 of a model was strongly compact or supercompact before carrying out a collapse.

Matteo Viale and Christoph Weiß explained how this can be used to show

that $\kappa = \omega_2^{V[G]}$ has to be strongly compact in the ground model V if PFA is obtained by any of the known forcing iterations.

In connection with this work, Thomas Johnstone shared his knowledge on strongly unfoldable cardinals, a notion of large cardinals very low in the hierarchy which can nevertheless be in many situations good substitutes for supercompact cardinals.

Luis Pereira posed the following question: In $L[U][G]$, where U is a normal ultrafilter and $G = \{\kappa_n\}_{n \in \omega}$ is a Prikry sequence, given an arbitrary *unary* function $f : \kappa \rightarrow \kappa$ are there infinite free sets $X = \{\gamma_n\}_{n \in \omega}$ whose elements are of the form $\gamma_n = \sup(N \cap \kappa_n^{+n})$, where N is an internally approachable submodel? This is related to the pcf conjecture. He conjectures that the fine structure of $L[U]$ should yield a negative answer.

There was a plenary discussion on the future of set theory and its relation to other fields in mathematics. In particular, its applicability in functional analysis was discussed, and the question of how much attention these set theoretic results receive there. Moreover, there was a discussion how the communication and the exchange within the (young) set theory community can be supported. This resulted in a wiki (<http://young-set-theory.net/>).

Results and impact of the event on the future direction of the field

Feedback from the participants have shown that the impact of the workshop content on future direction of the field is at least two-fold: both on the level of pure scientific content and on the level of facilitating scientific cooperation, building networks and disseminating knowledge - that is, on a pragmatic level - the young set theory series continues to contribute to setting very high standards.

Clearly, the workshop was very successful in providing an opportunity for young researchers to showcase their work, make their results known and thus lay the basis for cooperation. Young set theory has led to an unusual amount and depth of cooperation, in particular amongst young researchers, which has led to seminal results - just as an example, let us mention again the cooperation of Weiß and Viale leading to a major result on the consistency strength of PFA . In a similar vein, the workshop has succeeded in disseminating seminal results of senior researchers and led to a much deeper and quicker appreciation by a broader part of the community than can usually be expected. Just as an example, consider Ralf Schindler's tutorial, where new, important results were presented which would, in all likelihood, not be available to young set theorists for quite some time. On the other side, Uri Abraham's tutorial revitalized older results, making these useful techniques accessible and connecting them to modern results and concepts.

The organizers believe that this has a strong effect on the field, helping to

establish a new pace and much wider scope of dissemination. In particular, the intense working atmosphere and the amount of time devoted to discussion in small groups has had a profound role in this; the responses to and results of the workshop show that it has succeeded in bringing the in-depth and result oriented approach of high-end conferences like e.g. those in Oberwolfach etc. to a much wider and younger audience. It seems that a rather exceptional style of interactive and open-to-all discussion has been established, without compromising in any way the high level of discussion.

We would also like to stress the special attention given to applications and to the integration with other areas of mathematics. Farah and other's work in functional analysis was given special attention, as were recent developments in group actions (Epstein and others, again, just as examples). Also, we are happy that the future development of the field was addressed in plenary discussion and that the importance of making the work done in set theory known and useful to mathematicians working in other branches. Even inside our own field, a special appreciation of connections within and outside set theory could be felt, as in Moore's or Schindler's tutorial, which addressed deep issues of inner model theory or respectively, the theory of proper forcing, but centered on a topic which is relevant to many other branches of set theory or even applications of set theory to other branches of mathematics.

Again returning to the role of the workshop has played in the pragmatic aspect of establishing and strengthening cooperation and contacts, we are very happy that with the support of the INFTY network, we have succeeded in bringing to life a web-platform for researchers in set theory (the before-mentioned wiki). We hope that this platform will evolve to serve the same goals as the conference. The wiki will partly mimic the workshops in that registration will be open to all but will require researchers to publish their research statements. As a declarations of interests and a summary of results achieved, these research statements are a strong starting point for collaborations and for finding new areas of research. So far, there is nothing comparable in the realm of set theory, and we believe that we have truly succeeded in opening up a space for a new and open network furthering collaboration and dissemination in our field, with an underlying orientation also towards the broader mathematical community. The financial and institutional support of this project by the INFTY can be seen as a direct outcome of the discussion at the workshop.

Schedule Young Set Theory Workshop 2010

Time	Monday	Tuesday	Wednesday	Thursday	Friday
9:00 – 9:50	Tutorial: Ralf Schindler	Tutorial: Uri Abraham	Tutorial: Uri Abraham	Tutorial: Justin Moore	Tutorial: Justin Moore
10:00 – 10:50					
10:50 – 11:10	Coffee break	Coffee break	Coffee break	Coffee break	Coffee break
11:10 – 12:00	Bart Kastermans	Tutorial: Ralf Schindler	Lyubomyr Zdomskyy	Tutorial: Greg Hjorth	Tutorial: Greg Hjorth
12:10 – 13:00	Inessa Epstein		Lunch + EXCURSION (Raxalpe)		
13:00 – 14:30	Lunch	Lunch		Lunch	Lunch
14:30 – 17:30	Discussions	Discussions		Discussions	Discussions
	with coffee at 16:00	with coffee at 16:00		with coffee at 16:00	with coffee at 16:00
17:30 – 18:20	Thomas Johnstone	Philipp Schlicht		Wiesław Kubiś	Leave for Vienna
19:00 - ?	Dinner	Dinner	Dinner	Dinner	Dinner in Vienna 20:00