G.E.S.T.A. 2010 SCIENTFIC REPORT

1. Summary

G.E.S.T.A. 2010 has been a meeting in symplectic and contact topology and related areas. 48 participants, 28 of them young researchers, have gathered to follows courses by three outstanding senior researchers, Miguel Abreu, Michael Entov and Viktor Ginzburg, who lectured on "Contact and Lagrangian Floer homologies of toric manifolds", "Quasi-morphisms, quasi-states and their applications in symplectic topology" and "Symplectic topology of coisotropic submanifolds" respectively.

An important feature of the meeting has been that the 6 remaining talks where proposed and delivered by young researchers.

As for the topics discussed, they reflect the rough division in symplectic/contact topology between hard techniques (those based on P.D.E. and analysis methods), and soft techniques (those closer to differential topology and O.D.E.'s methods). Most of the talks and courses have been based on hard methods, mostly around Hamiltonian Floer homology, Lagrangian Floer homology and quantum homology (Abreu, Entov, Ginburg, Mazzucchelli, Maydanskiy and Amorim). Soft methods such as reduction, surgeries, and geometry of area forms have also appeared (Mandini, Rieser and Zapolsky).

Overall, the results has been to introduce to researchers to some of the most recent results in symplectic and contact topology, to allow young researchers to present their research, and to foster interaction between researchers working in the same or related areas of contact and symplectic topology.

2. Scientific content

G.E.S.T.A. 2010 has been a meeting in contact and symplectic geometry, and related areas, directed mainly to young researchers, including graduate students. To this end we had seminars of two types:

(1) Three 3 hours mini courses, given by renowned specialists and intended to introduce young researchers to active topics in symplectic and contact topology.
(2) Six 1 hour seminars, all delivered by young researchers, so they could present the results of their research.

The schedule was arranged so that there was time for discussion between seminars.

The topics discussed reflect roughly the division in symplectic geometry between hard techniques (those based on P.D.E. and analysis methods), and soft techniques (those closer to differential topology, O.D.E.'s methods).

The majority of the talks where based on hard methods, mostly around Hamiltonian Floer homology, Lagrangian Floer homology and quantum homology. This is something to be expected when looking at problems dealing with Lagrangians intersections and Hamiltonian displeceability. This was for example the case in the talks by Maydansky, Amorim, Mazucchelli and the courses by Entov, Abreu and Ginzburg.

Soft methods such as reduction, (relative) surgeries (blow-up, blow-down), geometry of area forms and geometric measure theory, appeared in the talks by Mandini, Rieser, Mazzucchelli and Zapolsky.

We now describe in more detail the contents of the expositions, and of some of the discussions they gave rise to.

2.1. **Mini courses.**

- **Michael Entov** (Technion, Haifa) delivered a course on *Quasi-morphisms, quasi-states and their applications in symplectic topology.* He described certain "almost homomorphisms" (quasi-homomorphisms) on groups of Hamiltonian symplectomorphisms and then their related infinitesimal version, some "almost linear" functionals (quasi-states) on the infinite dimensional Lie algebra of the group of Hamiltonian symplectomorphisms. This already generated some questions, as to the existence of interesting quasi-homomorphisms for some Lie groups, so these examples could illustrate a bit the infinite dimensional theory being presented. Then applications of quasi-morphisms to the group of area-preserving (compactly supported) homeomorphisms of the 2-disk were given. Entov also discussed some applications of quasi-states to symplectic topology, namely, to rigidity of intersections in symplectic manifolds, to existence of integral trajectories of certain Hamiltonian vector fields connecting certain disjoint sets, and to C^0-rigidity of Poisson brackets. This last topic was also related to a talk of another of the speakers, Frol Zapolski. The lectures finished with a rather spectacular description of a quasi-state in the 2-sphere.
- **Miguel Abreu** (I.S.T., Lisbon) lectured on *Contact and Lagrangian Floer homologies of toric manifolds.* He started by recalling classification of contact toric manifolds, and then described how contact homology could be described just in term of the combinatorics defining the toric contact manifold. This led to a description of infinitely many non-isotopic contact structures in the same homotopy class in $S^2 \times S^3$. Questions about problems to get higher dimensional results were raised. The second topic that Abreu described an application of Lagrangian Floer homology to the non-displeaceability of the monotone torus fiber and the real part of a Fano toric manifold. This second topic was also complemented by the talk presented by Lino Amorim, and a discussion about the precise relation took place.
- **Viktor Ginzburg** (UC Santa Cruz) gave a course on *Symplectic topology of coisotropic submanifolds.* Coisotropic submanifolds include as a special case Lagrangian ones. There is an enormous amount of research on the latter, and Ginzburg explained how to find natural analogues of the Lagrangian intersection property and the Liouville and Maslov class rigidity for coisotropic submanifolds. Interestingly enough he described an emerging picture enabling to treat non-existence of exact Lagrangian embeddings and the existence of closed characteristics on a contact hypersurface as particular cases of one phenomenon.

2.2. **Seminars.**

- **Alessia Mandini** (I.S.T., Lisbon) presented her results on *Hyperpolygon spaces.* Polygon spaces can be constructed by Kahler reduction, and Hyperpolygon spaces are their hyper-Kahler counterparts. She described how a description of the latter spaces using Higgs bundles enabled one to understand the wall crossing phenomenon. There where natural question about how to get similar results for reductions of coadjoint orbits of compact semisimple Lie groups different form $SU(2)$ (the polygon space case).
- **Antonio Rieser** (University of Montreal) lectured on his research on *Lagrangian submanifolds, blow-ups, and real packing.* He described how to construct version of blow up and blow down relative to a Lagrangian, and

how to extend anti-symplectic involutions in blow ups. This was used to present results on real packing numbers for real Lagrangian submanifolds in symplectic four manifolds.

- **Marco Mazzucchelli** (Max Planck Institute) presented his results on *The Conley conjecture for Tonelli systems*. Tonelli Lagrangians are certain type of time periodic functions $\mathbb{R}/\mathbb{Z} \times TM \to \mathbb{R}$ with global Euler-Lagrange flow, and Mazuchelli showed the existence of infinitely many periodic orbits with a priory bounded mean, of which infinitely many have period 1, or the period is unbounded. There was further discussion about the minimal regularity needed for his proofs, and the methods in his proof mimicking the approximation of the infinite dimensional space of geodesics by the finite dimensional spaces of broken geodesics, but where geodesics where replaced by the Euler-Lagrange solutions of the Tonelli Lagrangian in question.
- **Maksim Maydanskiy** (MSRI/University of Cambridge) gave a talk on *A criterion for emptiness*. This was a presentation on how to know whether a symplectic manifold is empty, meaning that it contains no exact lagrangians. He described his fundamental tool, wrapped Floer Homology. This was used to proof how for Lefschetz fibrations a condition on the vanishing cycles in the wrapped Fukaya category of the fiber, implied the emptiness of the space. In particular exotic structures on cotangent bundles of spheres where described.
- **Lino Amorim** (University of Wisconsin-Madison) lectured on *Floer cohomology in Fano toric manifolds*. The topic here was to give conditions under which the Floer cohomology of the torus fiber and the real part of a toric Fano manifold where defined. As Amorim explained, one need to search for Maurer-Cartan elements in a (curved) A_∞-algebra associated to moduli spaces of holomorphic disks with boundary conditions to produce such Floer theories. He used that to present a combinatorial description of the Floer complex -very much related to that of Abreu- and applied it to study non-displeceability and minimum number of intersections under Hamiltonian isotopy.
- **Frol Zapolsky** (Max Planck Institute) presented his results on *Almost Poisson commutativity in dimension two*. He looked at the question of extendability of the Poisson bracket to C^0-limits of Poisson commuting functions. Zapolski showed how in a surface with the Poisson bracket induced by a symplectic form, arguments from geometric measure theory show that this extension is possible. An application to the construction of quasi-states in surfaces was given. Natural questions, namely, about how to extend such results to general symplectic manifolds and to general Poisson structures on surfaces, led to an interesting discussion.

3. Assessment of the result and impact

There were several goals that the Organizing and Scientific committees had in mind when preparing the G.E.S.T.A. meeting. Briefly, we wanted a school with outstanding senior lecturers, which attracted enough interest form young researchers, in which they had both the opportunity to know about the state of the art in several important topics in symplectic and contact geometry, and to present their own research, and in which new contacts could be made and new projects could be started. We believe these objectives have been attained.

More specifically, we will argue about the goals achieved:

- **Gathering a group of outstanding senior speakers.** Miguel Abreu, Michael Entov and Viktor Ginzburg are undoubtedly outstanding researchers

in the fields of toric geometry, structure of the group of Hamiltonian diffeomorphisms, and Hamiltonian systems respectively. They agreed on delivering mini-courses in G.E.S.T.A., we think also attracted by the idea of this being a school aimed at young researchers.

- **Having enough young researchers interested.** Among the 48 seven people that attended G.E.S.T.A. 2010, 28 of them are graduate students or post-doctoral researchers which graduated no more than 2 years ago. This is a large number if we have into account the fact that there was another workshop in Turkey with a 1 day overlap with G.E.S.T.A. 2010, and that 2009/2010 has been a year in symplectic and contact geometry at M.R.S.I. (thus lowering the budget for traveling to conferences). We understand that the key for these was the quality of the senior lecturers and the possibility of applying for delivering a talk.
- **Having participants from different from different countries.** Of the 48 participants 16 were Portuguese, 21 European non-Portuguese, and 11 non-European. We believe this diversity was beneficial since it allowed scientific interaction among people for which meeting is not that easy.
- **Offering high quality courses and seminars.** The topics of the three courses of G.E.S.T.A. 2010 are the subject of recent or current research. Results may be found in papers in mathematical journals, but there are no textbooks available right know. Therefore, G.E.S.T.A. offered the only possibility for a young researcher to get to know about the state of the art in such topics. As for the seminars delivered by the young researchers, much of the material presented has been or is been published in very high quality journals. We also mention we had more proposals than slots available and the Scientific Committee had a hard time in choosing among them.
- **Fostering interaction and shaping new research projects.** There were plenty of discussions around the subject of the talks (this we believe was facilitated by a schedule which allowed a at least fifteen minutes pause between talks). Also having had this meeting may potentially lead to new research: an example of this is the interaction between Miguel Abreu and Lino Amorim, and the corresponding comparison between their two approaches to the problem of displeceability of the monotone torus fiber and the real part of a Fano toric manifold.

4. Final Programme

	Wednesday 9th
9.00 – 9.30	Registration
9.30 – 11.00	Entov I
11.00 – 11.30	Coffee Break
11.30 – 12.30	Mandini
12.30 – 14.00	Lunch
14.00 – 15.30	Abreu I
15.30 – 16.00	Coffee Break
16.00 – 17.00	Rieser
17.15 – 18.15	Mazzucchelli

	Friday 11th
9.30 – 11.00	Ginzburg I
11.00 – 11.30	Coffee Break
11.30 – 12.30	Maydanskiy
12.30 – 14.00	Lunch
14.00 – 15.30	Entov II
15.30 – 16.00	Coffee Break
16.00 – 17.30	Abreu II

	Saturday 12th
9.30 – 11.00	Ginzburg II
11.00 – 11.15	Break
11.15 – 12.15	Zapolsky
12.15 – 12.30	Break
12.30 – 13.30	Amorim