SCIENTIFIC REPORT: HOFER 20 – SYMPLECTIC GEOMETRY AND TRANSFROMATION GROUPS EDINBURGH, JULY 5-9, 2010

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1. SUMMARY

This meeting marked the 20th anniversary of the publication of the paper "The topology of symplectic maps" by Helmut Hofer, where he introduced a remarkable metric on the group of Hamiltonian diffeomorphisms of a symplectic manifold. This metric is now known as the Hofer metric and is a classical notion of symplectic geometry. Hofer geometry has been extensively investigated for the last 20 years, but still there are many open problems.

Invited Speakers: Miguel Abreu, Augustin Banyaga, Misha Bialy, Yasha Eliashberg, Misha Entov, Helmut Hofer, Boris Khesin, Francois Lalonde, Kaoru Ono, Yaron Ostrover, Pierre Py, Matthias Schwarz, Jean-Claude Sikorav, Claude Viterbo.

Organisers: Kai Cieliebak, Jarek Kędra, Dusa McDuff, Leonid Polterovich.

2. SCIENTIFIC CONTENT

There were 17 one-hour talks given by invited speakers. Although the main theme of conference was the Hofer geometry, the topics of the talks ranged throughout the whole of symplectic geometry and the relevant parts of transformation groups. Apart from the talks, there was enough time for informal discussions which were an integral part of the workshop.

The list of talks with abstracts:

(1) Abreu, Miguel – *Contact homology of good toric contact manifolds*

   Abstract: In this talk I will introduce toric contact manifolds and present their classification due to Banyaga-Molino, Boyer-Galicki and Lerman (2003).

   I will then show that any good toric contact manifold has well defined cylindrical contact homology and describe how it can be combinatorially computed from the associated moment cone.

   As a particular application, I will compute the cylindrical contact homology of a family of good toric contact structures arising in the work of Gauntlett-Martelli-Sparks-Waldram (2004), a group of mathematical-physicists, showing that it contains infinitely many new inequivalent contact structures on $S^2 x S^3$.

   This is joint work with Leonardo Macarini.

(2) Banyaga, Augustin – *The Hofer topology on the group of all symplectic diffeomorphisms.*

   Abstract: We construct a Hofer-like metric on the group $\mathrm{Symp}(M, \omega)$ of symplectic diffeomorphisms of a compact symplectic manifold and show that the resulting metric topology is intrinsic and the group of Hamiltonian diffeomorphisms is a closed subgroup. We get this way a new metric on the group of Hamiltonian diffeomorphisms.

(3) Bialy, Misha – *Integrable geodesic flows on surfaces.*

   Abstract: It is a very classical problem going back to Liouville, Darboux and Poincare to describe those Riemannian metrics on surfaces which have integrable geodesic flows. I'd like to discuss new ideas and methods for this problem.

(4) Eliashberg, Yasha – *More about Legendrian surgery.*

   Abstract: I will discuss new joint work with F. Bourgeois and T. Ekholm; results which extend our paper about effects of Legendrian surgery on symplectic and contact invariants.

(5) Entov, Michael – C^0-*function theory of Poisson brackets.*

Abstract: The Poisson bracket of two smooth functions on a symplectic manifold depends on their first derivatives. Nevertheless, it turns out that the Poisson bracket (as well as certain expressions involving iterated Poisson brackets of two functions) is rather sensitive to C^0-small perturbations of the functions. I will survey the C^0 functional theory of the Poisson brackets and an important role played in it by the Hofer metric and symplectic quasi-states. I will also discuss a relation between this theory and dynamics which has been found in our recent joint work with L.Buhovsky and L.Polterovich.

(6) Hofer, Helmut – *Polyfolds and SFT.*

Abstract: In this talk I describe some of the ideas of the polyfold approach to SFT.

(7) Khesin,Boris – *Dynamics on diffeomorphism groups: Burgers vs. Euler*

Abstract: We describe a relation between curvatures of the group of volume-preserving diffeomorphisms (responsible for Lagrangian instability of ideal fluids) and the generation of shocks for potential solutions of the inviscid Burgers equation (important in mass transport). For this we characterize focal points of the group of volume-preserving diffeomorphisms, regarded as a submanifold in all diffeomorphisms and the corresponding conjugate points along geodesics in the Wasserstein space of densities. We also present the Hamiltonian framework for the corresponding mass transport problem as an infinite-dimensional Hamiltonian reduction on diffeomorphism groups, and describe its subriemannian version on a manifold with a bracket-generating distribution.

(8) Lalonde, Francois – *Homological Lagrangian monodromy.*

Abstract: Consider a Lagrangian submanifold in a symplectic manifold and its group of monodromies defined as the time-one maps of all Hamiltonian isotopies starting and ending at L. Using a relative form of the Seidel morphism, we will show that this group, at the homological level, is trivial when L is weakly exact.

(9) Long, Yiming – *Multiple closed geodesics on compact simply connected manifolds.*

Abstract: The problem of closed geodesics is a traditional and important topic in dynamical systems and differential geometry. There is a long standing conjecture that there exist infinitely many distinct closed geodesics on every compact Riemannian manifold. The current interest on this problem is on compact simply connected manifolds, specially spheres. So far not much is known on the multiplicity of closed geodesics on such manifolds when their dimensions are at least 3. Recently, Dr. Huagui Duan and myself proved the following Theorem: There exist at least two distinct closed geodesics on every compact simply connected Riemannian (or Finsler) manifold whose dimension is at least 2.

In this lecture, I shall give a survey on the study of the problem of closed geodesics and explain some ideas in the proof of the above theorem.

(10) Ono, Kaoru – *Bulk deformations in Lagrangian Floer theory.*

Abstract: Based on the joint work with K. Fukaya, Y.-G. Oh, H. Ohta, I plan to explain bulk deformations in Lagrangian Floer theory and its applications. If time allows, I would also like to include implications for Lagrangian torus fibers in compact toric manifolds.

(11) Ostrover, Yaron – *On the uniqueness of Hofer's geometry.*

Abstract: In this talk we address the question whether Hofer's metric is unique among the Finsler-type bi-invariant metrics on the group of Hamiltonian diffeomorphisms. The talk is based on a recent joint work with Lev Buhovsky.

(12) Py, Pierre – *Quasi-morphisms and Hofer's metric.*

Abstract: In this talk we will survey some of the recent constructions of quasi-morphisms on groups of Hamiltonian diffeomorphisms (Floer theoretic as well as topological ones). In particular we will discuss some open problems related to Hofer's metric: continuity of Calabi quasi-morphisms on surfaces of positive genus, existence of C^0-robust bounds on Hofer's metric, and description of the mixed action-Maslov quasi-morphism. Along the way, we will survey some of the recent works of M. Entov and L. Polterovich, M. Rosenberg, F. Zapolsky, as well as of the speaker.

(13) Sandon, Sheila – *An integer valued bi-invariant metric on the group of contactomorphisms of* $\mathbb{R}^{2n} \times S^1$.

Abstract: I will describe a generalization to the contact case of the Viterbo metric on the Hamiltonian group of $\mathbb{R}^{2n}$, and discuss how it is related to the theory of orderability and contact non-squeezing of Eliashberg, Kim and Polterovich.

(14) Savelyev, Yasha – *On Hofer geometry and quantum classes.*

Abstract: We describe quantum characteristic classes and their natural applications in Hofer geometry. One of the main classical tools will be Bott periodicity for the unitary group.

(15) Schwarz, Matthias – *Floer homology for cotangent bundles.*

Abstract: Floer homology on cotangent bundles for Hamiltonians equivalent to the geodesic flow is isomorphic to loop space homology. We show how this isomorphism extends to further structures such as loop product and cup product by chain level constructions.

(16) Sikorav, Jean-Claude – *Hofer distance of Lagrangian submanifolds in a cotangent bundle.*

Abstract: We study the Hofer distance on the space of Lagrangian submanifolds of T^*M which are Hamiltonian isotopic to the zero section, where M is a closed manifold. We prove a Hamilton-Jacobi differential inequality for a suitable generating function of the isotopy, which we use to prove some lower bounds on the distance.

(17) Viterbo, Claude – *Non convex Mather theory.*

Abstract: Using recent results and methods of symplectic homogenization, we shall show how one can construct invariant measures for Hamiltonian flows in non-convex situations.

3. Assessment and impact

The conference gathered the world top researchers working in symplectic geometry as well as young talented researchers in the beginning of their careers. The talks reported the state of the art of

the subject, indicated new areas of research and presented new important results in the field. Two breakthroughs made by young researchers presented at the conference deserve a special mentioning: Sheila Sandon discovered a completely unexpected bi-invariant metric on the group of compactly supported contactomorphisms of $\mathbb{R}^{2n} \times S^1$, and Yaron Ostrover (jointly with Lev Buhovsky) proved a long standing conjecture on the uniqueness of Hofer's metric among non-degenerate bi-invariant Finsler metrics on groups of Hamiltonian diffeomorphisms.

Symplectic geometry has always been a broad subject containing a variety of techniques from algebraic geometry, analysis, topology etc. It is clear after the conference that the modern symplectic geometry has further branched out to geometric group theory and functional analysis. In particular Hofer's EMS-lecture, which was one of the highlights of the conference, presented foundations of the theory of polyfolds, a novel functional analytic technique designed for handling moduli spaces of solutions of elliptic PDE's arising in symplectic and contact topology.