Final report on the CAST/ESF Workshop on Contact and Symplectic Topology, with a focus on open problems

September 18, 2011

1 Summary

The Workshop on Contact and Symplectic Topology has been held in Łódź, Poland, on September 12-15, 2011, as a special session of the Joint Meeting of the Israel Mathematical Union and the Polish Mathematical Society. Leading experts in contact and symplectic topology have given talks on a wide variety of topics. The talks have led to a fruitful exchange of ideas between the participants and strengthened scientific cooperation between them.

2 Description of the scientific content of and discussion at the event

The workshop has attracted about 35 participants from many countries, among them 16 speakers who presented their recent results and discussed related open problems and possible directions for future research. Here is a brief description of the talks given at the workshop:

Mohammed Abouzaid (MIT, USA) discussed a way to capture the structure of Fukaya categories.

Miguel Abreu (IST, Portugal) discussed a geometric construction which turns out to be very helpful in the study of Lagrangian intersections in symplectic toric manifolds.

Urs Frauenfelder (Seoul National U., South Korea) presented a new version of Rabinowitz action functional on very negative line bundles over a symplectic manifold and its possible applications to Arnold's conjecture.

Kenji Fukaya (U. of Kyoto, Japan) explained a calculation of Floer homology of some Lagrangian tori in a symplectic toric surface obtained by deforming an orbifold singularity in the surface.

Swiatosław Gal (U. Wien, Austria) discussed a cocycle on a class of groups of homeomorphisms, in particular, on groups of Hamiltonian diffeomorphisms of symplectically aspherical manifolds, and its applications to the study of algebraic properties of the diffeomorphisms and of the fundamental groups of the manifolds.

Richard Hind (U. of Notre Dame, USA) discussed constructions and obstructions to symplectic embeddings of ellipsoids in other domains in dimension greater than four.

Jarek Kedra (U. of Aberdeen, UK, and U. of Szczecin, Poland) presented a construction of quasi-isometric embeddings of various discrete groups into the group of volume-preserving diffeomorphisms of a manifold and other similar groups.

François Lalonde (U. of Montreal, Canada) discussed the structure of moduli spaces appearing in the construction of Floer homology.

Klaus Niederkrueger (U. Paul Sabatier - Toulouse III, France) proposed a new definition for weak fillability of higher-dimensional contact manifolds and discussed its relation with other similar notions.

Kaoru Ono (Hokkaido U., Japan) presented a new lower bound for the displacement energy of Lagrangian torus fibers of symplectic toric manifolds using Lagrangian Floer homology.

Yaron Ostrover (Tel-Aviv U., Israel) addressed the question of existence of Finsler-type bi-invariant metrics on the group of Hamiltonian diffeomorphisms that are essentially different from Hofer's metric and presented a new result on the subject.

Andrés Pedroza (U. de Colima, Mexico) presented his proof of the bounded isometry conjecture of F.Lalonde and L.Polterovich for a large class of closed symplectic manifolds.

Yasha Savelyev (CRM-Montreal, Canada) discussed a Floer-theoretic, variationally complete length functional on the path spaces of $Ham(M, \omega)$, inducing the spectral norm.

Egor Shelukhin (Tel-Aviv U., Israel) showed how to construct a non-trivial homogeneous quasimorphism on the group of Hamiltonian diffeomorphisms of an arbitrary closed symplectic manifold.

Michael Usher (U. of Georgia, USA) showed how to use a Floer-theoretic quantity called the boundary depth to prove infinite diameter (and moreover the existence of infinite-dimensional quasi-isometrically embedded normed vector spaces) for Hofer's metrics on groups of Hamiltonian diffeomorphisms or spaces of Lagrangian submanifolds in a variety of cases.

Jean-Yves Welschinger (U. Lyon 1, France) presented an upper bound on the expected total Betti number of a random real hypersurface of a smooth real projective manifold.

Some of the talks at workshop have attracted prominent participants from other sections of the Joint Meeting – e.g. Semyon Alesker (Tel Aviv U., Israel), an expert in convex, integral and complex geometry, and Wojciech Domitrz (Warsaw University of Technology, Poland), an expert in singularity theory, to name a few.

3   Assessment of the results and impact of the event on the future direction of the field

The workshop has served as a stage for presentation of top quality new results in the field and discussion of approaches to further research. It has strengthened scientific cooperation between the participants and led to an important exchange of ideas between them. This will be certainly of great help in their future research and for the general progress in contact and symplectic topology.

In particular, we see the following promising directions of research discussed at the workshop:

* Getting a better (hopefully, complete) description of non-displaceable toric fibers of symplectic toric manifolds by combining Floer homology techniques and symplectic reduction.

* Understanding Hofer geometry on the group of Hamiltonian diffeomorphisms by embedding in it various "large" groups in a quasi-isometric way by means of Floer-homology and geometric techniques.

* Finding out whether the class of symplectic manifolds for which Pedroza has proved the Lalonde-Polterovich bounded isometry conjecture includes all symplectic manifolds and, if not, describing the simplest manifold for which the conjecture is still open.

4   Final programme of the meeting

Monday, September 12, 2011

  9:00-14:30 Activities at the Joint Meeting of the IMU and the PMS

  14:30-15:20 Michael Usher, *Hofer's metric and the boundary depth*

  15:30-16:00 Egor Shelukhin *A quasimorphism for every symplectic manifold*

  16:00-16:30 COFFEE BREAK

  16:30-17:20 Jean-Yves Welschinger *What is the total Betti number of a random real hypersurface?*

  17:30-18:20 Mohammed Abouzaid *Generating Fukaya categories*

  18:30-19:30 Kenji Fukaya *Deformation of surface singularity*

and Lagrangian Floer homology

Tuesday, September 13, 2011

8:30-9:15 Klaus Niederkrueger, *Weak fillability for higher dimensional contact manifolds*

9:20-10:00 Światosław Gal, $dK(g) = g^*\alpha - \alpha$

10:00-14:00 Activities at the Joint Meeting of the IMU and the PMS

14:00-14:50 Yaron Ostrover, *On the uniqueness of Hofer's geometry*

15:00-16:00 Francois Lalonde, *Moduli spaces in cluster homology*

16:00-16:30 COFFEE BREAK

16:30-17:20 Richard Hind *Symplectic embeddings of ellipsoids*

17:30-18:20 Miguel Abreu *Remarks on Lagrangian intersections in toric manifolds*

Wednesday, September 14, 2011

8:30-9:00 Andres Pedroza, *Bounded hamiltonian symplectomorphisms*

9:10-10:00 Kaoru Ono, *Displacement energy of Lagrangian submanifolds and the torsion of Lagrangian Floer cohomology*

From 10:00 on - Activities at the Joint Meeting of the IMU and the PMS

Thursday, September 15, 2011

8:30-9:20 Jarek Kedra *Quasi-isometric embeddings into diffeomorphism groups*

9:00-14:00 Activities at the Joint Meeting of the IMU and the PMS

14:00-14:50 Yasha Savelyev *Quantum corrections to Hofer length and diameter*

15:00-15:50 Urs Frauenfelder *Bubbles and onis*