Final Report on the ITGP Science Meeting 3296
Topological Heterotic Strings and (0,2) Mirror Symmetry

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organized by

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1 Summary

The last few years have seen a revival of interest in the mathematical structure of heterotic string theories and its physical implications. This study is being undertaken by both mathematicians and physicists, and the goal of the workshop was to bring together a number of experts in the field to review recent progress, discuss current problems, and formulate some new directions of research. The format was deliberately not too lecture-intensive, leaving plenty of time for discussions, meetings and ongoing collaborations.

The meeting largely succeeded in its goals. There were many lively discussions initiated in the seminars and then continued later in the day. These included "cross-cultural" exchanges between the mathematicians and physicists, for example as a follow-up to Donagi's and Sharpe's presentations of a mathematically rigorous formulation of quantum sheaf cohomology; as well as researchers tackling related questions that were encountered by different groups, as in the discussions of torsional linear sigma models between Adams, Groot Nibbelink, and Quigley and Sethi (shortly after the workshop Nibbelink and collaborators, as well as Quigley and Sethi published closely related works on the subject).

There were many opportunities to track down "the expert" on some particular subject and ask questions about their work or related issues. These opportunities were clearly used by the participants during the breaks as well as after the regular sessions were over. As a result of these discussions a number of new collaborations are pursuing new lines of research.

2 Description of the scientific content of and discussion at the event

- Thorsten Rahn spoke about *Landscape study of target space duality of* $(0,2)$ *heterotic string models*. He discussed a target space duality of $(0,2)$ models which changes the Hodge numbers of the Calabi-Yau as well as the number of bundle moduli while keeping their sum constant. The duality is based upon a linear sigma model construction, where in certain phases it is possible to re-interpret some of the two-dimensional fields associated to the base manifold as fields associated to the gauge bundle. Although not all steps in this re-interpretation are completely understood, the numerical evidence, obtained by a systematic computer search, suggests that this duality is a property of many compactifications based on linear sigma models.

- Roberto Zucchini spoke about *A heterotic sigma model with novel target geometry*. He discussed sigma models whose targetspaces are Lie algebroids over Kähler manifolds. A Lie algebroid is a vector bundle equipped with a Lie bracket and a natural map to the tangent bundle of the base manifold. These structures were reviewed in some depth in the talk. Such models serve as laboratories for topological heterotic strings as

they have $(1,2)$ supersymmetry, making them potentially more tractable than their less symmetric $(0,2)$ relatives. It is possible to make two topological twists, leading to a pair of half-twisted models which have two BRST operators constructed from the $(0,2)$ and the $(1,0)$ components of the supersymmetries. The speaker furthermore commented on the chiral algebras and chiral rings in these sigma models — a subject related to the work of Meng-Chwan Tan reviewed below.

- Katrin Becker talked about *Disk amplitudes, picture changing and space-time actions.* She discussed higher derivative (fourth-order) corrections to D-brane actions. The existence of such corrections was motivated by using a T-duality argument, and an explicit calculation of such a correction in the linearized approximation was obtained by explicitly calculating a disk scattering amplitude with a RR-fields, a B-field and gravitons.

- Melanie Becker gave a review on *Heterotic flux, where do we stand?* — an overview of the space-time approach to construction of heterotic compactifications with fluxes, focusing in particular on models with torsional (i.e. non-Calabi-Yau) geometries. A duality between models with Kähler geometry and non-Kähler models with torsion was outlined and discussed for the example of $K3\times T^2$. The duality was explained via a common M-theory origin of the models. Further examples with $N = 0, 1, 2$ supersymmetry were discussed.

- Ilarion Melnikov's talk was titled *Tracking massless singlets in heterotic compactifications.* He addressed the issue of moduli in heterotic compactifications, focusing on the bundle moduli in the case where the gauge bundle is the tangent bundle. Due to classical obstructions which depend on the base manifold's complex structure, as well as quantum obstruction coming from worldsheet instantons, it is hard to determine the number of moduli. A subset of the bundle moduli is accounted for by $(0,2)$-deformations of the gauged linear sigma model. Several examples where the numbers of bundle moduli were counted at various points in moduli space were discussed.

- Jock McOrist spoke about *The $E_6 \to SO(10)$ Higgs mechanism in the linear sigma model*, focusing on the example of the quintic with gauge bundle $T_X \oplus \mathcal{O}$. He discussed issues that arise when one breaks the E_6 gauge group to $SO(10)$ by turning on VEVs linear sigma model operators that correspond to matter fields in spacetime. Although such a deformation exists in the underlying conformal field theory, it turns out to be a subtle matter to correctly implement it in the linear model: a naive form of the deformation leads to an inconsistent spectrum on the Landau-Ginzburg locus. Nevertheless, by using mirror symmetry it is shown how to correctly describe the deformation by taking into account maps in the Landau-Ginzburg BRST cohomology that violate the quantum symmetry of the orbifold. In the space-time picture this amounts to keeping track of the kinetic term normalizations.

- In his talk *Linear sigma models with torsion* Savdeep Sethi discussed how to define an extension of a gauged linear sigma model which leads to torsional geometries in the heterotic string. The construction turns on a two-dimensional realization of the Green-Schwarz mechanism: one considers a (0,2) supersymmetric linear sigma model with a holomorphic field-dependent Fayet-Iliopoulos coupling that fails to be gauge invariant; however, the non-invariance is chosen just right to cancel the one-loop gauge anomaly coming from the variation of the measure. By integrating out the gauge fields,

one obtains an effective description of the light degrees of freedom as a non-linear sigma model. The resulting geometry is a complete intersection in a non-Kähler generalization of a toric variety. The construction gives a potentially large class of smooth compact torsional geometries with a linear sigma model realization. The latter feature should be useful in studying quantum aspects of heterotic compactifications with flux.

- Stefan Groot Nibbelink discussed *Anomaly cancellation in* $(0,2)$ *heterotic orbifold resolutions.* He explained how (resolutions of) non-compact heterotic orbifold models with VEVs for twisted states can be embedded into a gauged linear sigma model. It turns out that solving the anomaly cancellation conditions of the GLSM implies that the Bianchi identities are satisfied for all resolutions of the orbifold singularity. In case one cannot find solutions to the anomaly cancellation conditions, one can implement a heterotic Green-Schwarz anomaly cancellation mechanism by introducing field-dependent FI-terms. Singularities of these FI-terms can be interpreted as NS5-branes in the target space.

- Eric Sharpe and Ron Donagi gave a joint session on *Quantum Sheaf Cohomology*, which is the $(0,2)$ generalization of the quantum cohomology of the topological A-model. Eric Sharpe gave a general introduction to the subject and discussed the type A half-twist of a $(0,2)$ theory based on a non-linear sigma model for a Calabi-Yau manifold equipped with a holomorphic gauge bundle satisfying the Green-Schwarz anomaly cancellation conditions. His talk focused on the classical spectrum and correlation functions. In order to define the quantum corrections due to world-sheet instantons, one must define the pull-back of the gauge bundle to the moduli space of maps from the worldsheet to the target. The resulting structure is in general a sheaf with certain regularity properties. In the second part of the talk Ron Donagi explained how to compute the quantum sheaf cohomology relations in the case where the targetspace is a compact smooth toric variety and the bundle is a deformation of the tangent bundle. These algebraic relations can be obtained as certain ideals in the space of polynomials with indeterminates corresponding to a basis of the Picard lattice of the toric variety. The computations are carried out by relating different instanton contributions to the correlators. The correlation functions in a particular instanton sector are encoded in the symmetric product of the Picard group of the associated toric space modulo an ideal determined by the primitive collection. Using techniques of toric geometry and Koszul resolution, the quatnum sheaf cohomology relations can be computed explicitly, as illustrated in the example of $\mathbb{P}^1 \times \mathbb{P}^1$. It was pointed out that the results in different instanton sectors are compatible and that there is agreement with results obtained earlier by physics methods.

- Meng-Chwan Tan gave a review entitled *A quasi-topological* $(0,2)$ *heterotic B-model, the mirror chiral de Rham complex, and twisted generalized mirror symmetry.* He discussed a B-twisted $(0,2)$ sigma model and its perturbation theory. After imposing anomaly cancellation conditions, one gets an infinite tower of states with holomorphic weights. The observables in the theory form a holomorphic chiral algebra whose ground states span a finite-dimensional chiral ring. This is the (0,2) analogue of the more familiar (a,c) and (c,c) rings of $(2,2)$ theories. For non-zero torsion the observables are described in terms of certain Čech cohomology groups. In the general case this structure in encoded in the theory of chiral differential operators, which reduces to the chiral de Rham complex at the $(2,2)$-locus. Locally this can be described in terms of a $bc - \beta\gamma$-

system. Obstructions to gluing local patches corresponds to the conformal anomaly in the sigma model, i.e. a non-trivial β function.

- Jacques Distler talked about *Quantized Fayet-Iliopoulos terms in $N = 1$ supergravity.* The talk began with a review of the constraints that a $D = 4$ $N = 1$ supersymmetric non-linear sigma model must satisfy in order to be consistently coupled to either gauge fields or supergravity. The former requires the existence of holomorphic isometries that can be represented by Hamiltonian vector fields; the latter requires the scalar manifold to be a Kähler-Hodge manifold. It was shown that putting these two requirements together leads to a quantization condition on the F-I terms — the integration constants appearing in the moment maps associated to $U(1)$ isometries. The procedure was illustrated by a $\mathbb{CP}^1$-example.

- Allan Adams discussed the *Landau-Ginzburg phases of linear sigma models with torsion.* It was shown that the Landau-Ginzburg phase of a torsional gauged linear sigma model is described by an orbifold with additional phase factors (not related to standard discrete torsion constructions), i.e. one where contributions of various twisted sectors to the partition function acquire non-trivial phase factors. These factors are the small-radius avatars of the non-trivial anomaly cancellation visible in the geometric phase.

- Anda Degeratu spoke about *Invariants of elliptically fibered Calabi-Yau threefolds.* She talked about heterotic/F-theory duality for heterotic stings on K3 vs. F-theory on elliptically fibered Calabi-Yau threefolds. It was shown that the charged massless hypermultiplet spectrum matched in the conjectured dual theories. In the heterotic string this number arises from an index related to the moduli spaces of antiselfdual representations of the gauge bundle whereas in F-theory the result is encoded in the singularity structure of the elliptic fibration. This provides another important check of the duality conjecture between six-dimensional type II and heterotic compactifications.

3 Assessment of the results and impact of the event on the future direction of the field

The workshop highlighted a number of new developments in the field, and both during the presentations themselves, as well as subsequent discussions, many new questions were posed. In this remaining section of the review, we will discuss a few of these future directions and the impact they might have on the field, as well as related larger areas of mathematics and string theory.

As was seen in a number of talks, we have by now a fairly good understanding of the ground ring in (0,2) theories that are obtained as deformations of (2,2) supersymmetric models. This leads to two obvious questions: can we generalize the known techniques to more general (0,2) theories? can we connect this work with the efforts, as reviewed by Tan, to understand the full infinite dimensional chiral ring? A quantitative understanding of the first issue would lead to methods to compute superpotential couplings in phenomenologically interesting heterotic models, as well as enable us to test generalizations of the mirror conjecture in the (0,2) setting. The second issue, while certainly more challenging, is also important: it would lead to a new sort of string theory — a half-way point between the familiar topological string and the physical string.

Another exciting set of developments presented at the workshop was a general construction of torsional gauged linear sigma models. Building on previous work of Adams et. al., S. Groot Nibbelink and collaborators, as well as Quigley and Sethi developed a more general framework for studying torsional geometries that should correspond to conformal (0,2) theories. The work of the former described non-compact models with NS5-brane sources; the latter work aims to build compact models by generalizing the familiar toric quotient to manifolds with torsion. These developments offer a potential to produce many new interesting geometries that can be used to test ideas about chiral rings and mirror symmetry, as well as to build interesting phenomenological models of the heterotic string.

The talks of Melnikov, McOrist, and Rahn produced some interesting questions regarding deformations of (0,2) theories. A common theme was the surprising existence of deformations, whether exact or even just first-order in many (0,2) theories. Understanding this deformation space and how obstructions can arise will certainly be an important direction to pursue in the future. It would be very interesting to study singularities that arise in these moduli spaces, especially since the existence of such features has important consequences for the global structure of the moduli space. It may well be that various quantization conditions, such as those described by Distler, may be modified once these singularities are taken into account.

As a final topic of future interest, we might mention the construction of non-supersymmetric vacua. As described by M. Becker and also mentioned by A. Adams, there are certain natural extensions of the known torsional constructions that yield $N = 0$ supersymmetry in space-time. Are these models still stable, i.e. do they correspond to conformal field theories? This, and related constructions, were discussed at some length during the workshop, and it is likely that this question will be pursued in future investigations.

A future workshop related to these investigations is already scheduled to take place at BIRS in Banff, Canada in December of 2012.

4 Final Programme of the meeting

The workshop began at 9am on Monday, June 20 and ended on the following Friday. The conference dinner was held on Wednesday evening. Below is the final program of the lectures.

	Monday	Tuesday	Wednesday	Thursday	Friday
09:00-09:30	Welcome				
09:30-10:30	Rahn	Melnikov	QSC I	Adams	Degeratu
10:30-11:00	Coffee				
11:00-12:00	Zucchini	McOrist	QSC II	FREE	
12:00-14:00	Lunch				discussion
14:00-15:00	K. Becker	Sethi	Tan		
15:00-15:30	Break			DAY	
15:30-16:30	M. Becker	Nibbelink	Distler		