

**Scientific Report on Workshop**  
**“Quantum Integrability and Gauge Theories”, Dublin**  
**28 March – 2 April 2011**

## **1. Introduction**

The relation between classical integrable systems and supersymmetric gauge theories in four dimensions was discovered long ago, in the mid-1990s. It was recently realised that the quantisation of exact same integrable systems, arising in Seiberg-Witten solution of  $N=2$  SYM theory in four dimension, could be achieved by considering the equivariant partition (with respect to Lorentz group) function of the four dimensional gauge theory. When one equivariant parameter (corresponding to one  $U(1)$  subgroup of Lorentz group) is zero, this partition function turns out to produce the Yang-Yang function of corresponding quantum integrable system (where the second, remaining, equivariant parameter plays the role of Planck constant). This observation led to many recent activities on the subject in both mathematics and physics. It also became obvious that it was related to the wall crossing formulae in counting the BPS states in the above-mentioned gauge theories, another topic which is being actively developed right now. The workshop aimed at bringing together experts working in this very active area to the Hamilton Mathematics Institute, Dublin, and at learning about its most recent developments. The speakers were selected accordingly. Kontsevich's topic was on integrability and wall-crossing; Lukyanov's was the Yang-Yang function in sine-Gordon theory; Jimbo's on the correlation functions in integrable models; Nekrasov's on the Geuge/Bethe correspondence; and Cecotti's on the classification of  $N=2$  theories. Most of the results presented at the workshop were based on yet unpublished work, while Cecotti's talk was based on the paper which appeared arXiv during the workshop.

## 2. Summary

The Workshop took place in the Hamilton Mathematics Institute (HMI), Trinity College Dublin, from March 28 to April 2, 2011. This is a traditional Spring workshop at HMI, its format being a five-day workshop with one speaker giving two lectures every day, 1 ½ hours in the morning and 1 ½ hours in the afternoon. This year's speakers were:

Sergio Cecotti (SISSA, Italy)

Michio Jimbo (Kyoto, Japan)

Maxim Kontsevich (IHES, France)

Sergey Lukyanov (Rutgers, USA)

Nikita Nekrasov (IHES, France)

The workshop was organised by Anton Gerasimov (ITEP Moscow and HMI, TCD), Fedor Smirnov (Jussieu Paris VI and HMI, TCD) and Samson Shatashvili (HMI TCD and IHES, France).

The workshop was attended by 25 participants out of which 10 were students from various European countries outside Ireland. The list of participants is attached below.

### 1. Description of the scientific content

The scientific content is well reflected in the title of the workshop. Each speaker was a leader in some aspect of the topic. Each presented a very modern and important point of view, valuable for other experts and students. There were about 3 to 4 hours available every day for informal discussions between and after the lectures. The HMI workshops are usually very popular in the discipline because they allow students to learn during the morning session a general viewpoint of the speaker and to focus on the details in the afternoon. Here is a brief review of each intervention:

**Maxim Kontsevich** spoke about his ongoing work with Ian Soibelman on wall-crossing. A large part of the morning session was devoted to the classical integrable systems. It is currently an important element in the developments of the quantum gauge theories and Maxim emphasized that even the real integrable systems (as opposed to algebraic, complex, integrable systems which appear in the description of the geometry of supersymmetric vacua in 4d gauge theories) play a key role in his work with Soibelman. During the afternoon session Maxim described, in the language of algebraic integrable systems, the relation between symplectomorphisms and BPS counting in corresponding supersymmetric gauge theories. The speaker devoted a large part of the afternoon session to a possible explanation of recently discovered quantization of these algebraic integrable systems in the approach of his work with Soibelman on wall-crossing.

**Sergey Lukyanov** talked about his ongoing work with Alexander Zamolodchikov on sine-Gordon theory. In the morning, he gave a general review of his previous work with Zamolodchikov, as well as a basic overview of Bethe ansatz. During the afternoon, he explained the appearance of Yang-Yang function in computation of correlation functions in quantum theory and the magic connection of quantum theory with another, classical, integrable system. The key point of the afternoon session was that the Yang-Yang function needs to be evaluated on its critical values (BA equation) in order to get an exact answer for correlators, and that sometimes this can be done exactly.

**Nikita Nekrasov** spoke about his recent work with Alexey Rosly and Samson Shatashvili. In the morning, Nikita described the Bethe/Gauge correspondence which appeared in the recent papers by Gerasimov, Nekrasov and Shatashvili. He talked about two classes of examples - those that appear as KK reduction from higher dimensional (e.g. 4d) theory, and those that appear as 2d  $\Omega$  background in 4d supersymmetric gauge theories. In the Bethe/Gauge correspondence, the Yang-Yang function plays a major role, and Nikita devoted the afternoon session on explaining the geometric meaning of the Yang-Yang function in the case of Hitchin integrable systems, the topic of his recent paper with Rosly and Shatashvili.

**Sergio Cecotti** talked about his yet unpublished work with Cumrun Vafa. He introduced the notion of a complete  $N=2$  supersymmetric theory in 4 dimensions as a UV complete theory for which all the BPS central charges can be arbitrarily varied as one varies its Coulomb branch parameters, masses, and coupling constants. During the afternoon session, he classified all such theories whose BPS spectrum can be obtained via a quiver diagram. This was done using the 4d/2d correspondence and by showing that such complete  $N=2$  theories map to quivers of finite mutation type. The list of such theories is given by the

(generalized) Gaiotto theories consisting of two 5-branes wrapping Riemann surfaces with punctures, as well as 11 additional exceptional cases, which he identified.

**Michio Jimbo** spoke about the results he obtained with Miwa and Smirnov on exact computation of correlation functions in sine-Gordon theory. During the morning session, he talked about the free fermion description of correlation functions, an approach they introduced during the last couple of years. During the afternoon session, he extended their previous construction and showed how to introduce two kinds of fermionic screening operators, in close analogy with conformal field theory with  $c < 1$ .

## 2. Assessment of the results and impact of the event

The goal of the HMI spring workshops is to learn about recent developments in a given, currently active, area of mathematics and theoretical physics in circumstances when both general talks are presented by leading experts, allowing students to catch up quickly with the topic, and more specialised talks to further develop the topic. The overall impression of the organizers is that the workshop was successful.

## 3. Brief comment on budget

Very briefly, the expenditure reads as follows:

Travel	€3,250
Accommodation	€4,020
Meals	€2,180
Administrative cost	€ 380
<b>Total</b>	<b>€9,830</b>

## 4. List of participants

Mr. Nicolas Behr,	Golm, (DE)
Professor Hermann Boos,	Wuppertal, (DE)

Mr. Stephen Britton,	Dublin, (IE)
Dr. Nigel Buttimore,	Dublin, (IE)
Mr. Dmitri Bykov,	Dublin, (IE)
Dr. Sergey Frolov	Dublin, (IE)
Dr. Michael Fry,	Dublin, (IE)
Dr. Dmitri Grigoriev,	Dublin, (IE)
Dr. Conor Houghton,	Dublin, (IE)
Mr. Nikos Karaiskos,	Patras, (GR)
Mr. Pavel Khromov,	Moscow, (RU)
Dr. Stefano Kovacs,	Dublin, (IE)
Dr. Marianne Leitner,	Dublin, (IE)
Dr. Liuming Liu,	Dublin, (IE)
Professor T. Miwa,	Kyoto, (JP)
Professor Werner Nahm,	Dublin, (IE)
Dr. Donal O'Donovan,	Dublin, (IE)
Dr. Michael Peardon,	Dublin, (IE)
Mr. Eoin Quinn,	Dublin, (IE)
Mr. Vidas Regelskis,	York, (UK)
Dr. Sinead Ryan,	Dublin, (IE)
Mr. Burkhard Schwab,	Golm, (DE)
Dr. David Simms,	Dublin, (IE)
Dr. Stefan Sint,	Dublin, (IE)
Dr. Dmitri Zaitsev,	Dublin, (IE)

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## Quantum Integrability and Gauge Theories

### HMI Workshop

March 28 - April 2, 2011

**Organizing Committee:**

Anton Gerasimov (ITEP/HMI/TCD)

Samson Shatashvili (HMI/TCD/IHES)

Fedor Smirnov (LPTHE/CNRS)

We are pleased to announce a workshop on Quantum Integrability and Gauge Theories. It is planned to have two lectures per day by same speaker, each for 1,5 hours. Thus we hope there will be enough time for non-formal discussions and improvised activities. The workshop will include lectures by leading international experts covering all aspects of the program. We hope that the lecture program will be of particular interest to young researchers in the field, including Ph.D. students and post-doctoral fellows.

**Invited lecturers:**

S. Cecotti (SISSA, Italy)

M. Jimbo (Rikkyo University, Japan)

M. Kontsevich (IHES, France)

S. Lukyanov (Rutgers University, USA)

N. Nekrasov (IHES, France)

[Schedule](#)

[Poster](#)

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Contact: [hmi@maths.tcd.ie](mailto:hmi@maths.tcd.ie). Last updated: 15/03/2011.

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