1) Summary.

The international Spring School on Geometry and Quantum Topology took place on 20-25 March 2011 in Les Diablerets, Switzerland. Overall, there were 76 participants including 24 students and 17 speakers. There were 23 lectures, each of duration 50 min. The discussed topics included quantum invariants of knots and 3-manifolds, 3-dimensional hyperbolic geometry, knot theory, Dehn surgery, Teichmueller Space, Volume and AJ Conjectures, Chern-Simons Theory.

2) Description of the scientific content and discussion at the event.

The scientific content was partitioned into classical geometry, topology, and number theory (30%) and quantum topology (70%). Most of the lectures contained an introductory part (15-20 min.) accessible for students as well as discussions of some of the latest developments in the subject. The lectures took place in morning sessions (3 lectures) and late afternoon sessions (2 lectures). There were long brakes of 5 hours in between, where participants could have long private discussions.

Classical geometry, topology, and number theory

Cameron Gordon gave two lectures about exceptional Dehn surgeries. Each such surgery is represented by a pair (K,r) where K is a hyperbolic knot in 3-sphere and r is a rational number, such that r-Dehn surgery on K gives a non-hyperbolic 3-manifold. Every hyperbolic knot has only finitely many exceptional surgeries, and most hyperbolic knots have none at all. On the other hand infinitely many exceptional Dehn surgeries do exist. After having introduced to this subject, he described progress towards the goal of classifying of all such pairs (K,r).

Robert Penner gave two lectures, where, starting from first principles, he reviewed the basic combinatorial description of Teichmueller space in terms of fatgraphs. Concentrating on surfaces with one boundary component, in order to set the stage for Jean-Baptiste Meilhan's lectures, he explained several combinatorial algorithms for fatgraphs, including the Ptolemy groupoid which is extremely important in quantum Teichmueller theory. He also sketched the fundamentals of Torelli-Johnson-Morita theory in order to exhibit the first Johnson homomorphism as an example of extending homomorphisms to the Ptolemy groupoid.

Michel Boileau in his two lectures discussed the geometry and topology of complete hyperbolic 3-manifolds with finite volume. In particular, he gave a characterization of these 3-manifolds in terms of some volumes inequalities. This characterization has been shown to be seen as a step in Perelman's proof of the Geometrisation Conjecture.

François Gueritaud talked about the Schlaefli formula which describes how the volume of hyperbolic objects change in terms of their dihedral angles under smooth deformations. He explained this formula in full detail and outlined a perspective on how a quantum version of it

might help to prove a "volume conjecture" for objects which have a large deformation space such as polyhedra.

Don Zagier talked about q-series, modularity, and their manifestation in quantum topology. In particular, in the context of the volume conjecture, he showed interesting results on numerical calculations of the Kashaev invariant for the figure-eight knot, exhibiting validity of the volume conjecture at different roots of unity.

Quantum topology

Roland van der Veen gave two introductory lectures on a special class of fatgraphs called spin networks which originate in the classical theory of angular momentum and are closely related to the colored Jones polynomial of knots. He discussed properties of their evaluations, asymptotics in the context of the volume conjecture and connections to geometry.

Vladimir Turaev in his lecture outlined two constructions of 3-dimensional TQFTs from fusion categories - via surgery and via state sums. Then he formulated his with Alexis Virelizier recent theorem relating these constructions.

Jean-Baptiste Meilhan gave two lectures, where he explained basics of the finite type invariants of links and 3-manifolds, and some of the main objects of these theories, namely the Kontsevich integral and the LMO invariant. Following a joint work with J.E. Andersen, A.J. Bene and R.C. Penner, he defined a universal finite type invariant of homology cylinders over once borderred surfaces that depends only upon the choice of a fatgraph. He gave several results relating this construction to the Ptolemy groupoid, introduced in Bob Penner's lectures.

Stavros Garoufalidis talked about the “slope conjecture” which relates the degree of the colored Jones polynomial, a quantum invariant, to slopes of incompressible surfaces in the knot complement, a classical invariant. In the case of alternating knots, he discussed what lies beyond the degree of the colored Jones polynomial, namely the stability but not modularity of its coefficients. In particular, he gave a local state-sum formula for alternating knots which uses 2 tetrahedra per crossing, and which is an infinite, but yet convergent sum. This may be mysteriously related to recent work of Witten on Knots and Fivebranes.

Thang Le in his two lectures reviewed the AJ conjecture, the theory of Kauffman bracket skein module, the theory of q-holonomic functions, and gave a proof of the AJ conjecture for some class of knots.

Gregor Masbaum discussed the finite-dimensional representations of mapping class groups of surfaces coming from the Witten-Reshetikhin-Turaev TQFT, in the special case of the TQFT constructed from the skein theory of the Kauffman bracket.

Joanna Kania-Bartoszynska described her recent joint work with Charles Frohman on several invariants of knots in the 3-sphere and discussed relationships between them. The invariants

include A-ideal, Witten-Reshetikhin-Turaev invariant, Dubois torsion, and the Reidemeister torsion of the double of the knot complement.

Michael Polyak discussed a combinatorial construction of 3-manifold invariants by counting certain subdiagrams in a diagram of a surgery link. This construction may be interpreted as counting maps of surfaces of a fixed genus. For the Casson-Walker invariant this counts maps of a punctured torus.

Nathan Geer discussed the Turaev-Viro construction which leads to information about the topology of a 3-manifold from one of its triangulations. This construction is based on algebraic tools which are 6-parameter quantities called 6j-symbols. First he recalled the Turaev-Viro invariant of 3-manifolds arising from restricted quantum sl(2) at a root of unity. The underlying category of modules associated to this invariant is semi-simple and all the simple modules have non-vanishing quantum dimension. Then he explained his joint work with B. Patureau and V. Turaev on how the Turaev-Viro invariant can be modified to fit the context of non-restricted quantized sl(2) at a root of unity. Here the underlying category is not semi-simple and many of the simple modules have vanishing quantum dimensions. This modified Turaev-Viro invariant is closely related to Kashaev's invariant for which the volume conjecture first has been stated.

Christian Blanchet gave an introduction to TQFT in dimension 2 and explained the examples used in link homology. Then he defined Khovanov homology and some of its variants and discussed the functoriality properties.

Tudor Dimofte discussed some new perspectives on gluing along boundaries in three-dimensional TQFT, and in particular in Chern-Simons theory. By applying these gluing methods to the tetrahedra in ideal triangulations of knot complements, he showed how the A-polynomial can be explicitly quantized (leading to the recursion relations for colored Jones polynomials found by Garoufalidis and Le) and how a state integral model that encapsulates the asymptotics of the colored Jones polynomials can be constructed.

Hitoshi Murakami first explained Ekholm’s proof of the volume conjecture for the figure-eight knot, and then from the asymptotic behavior of an evaluation of the colored Jones polynomial of the figure-eight knot he showed how can one extract the Chern-Simons invariant and the twisted Reidemeister torsion associated with a representation of the fundamental group of the knot complement to SL(2;C).

3) Assessment of the results and impact of the event on the future direction of the field.

The principal goal of the school was to create favorable circumstances for addressing the basic problem of understanding the geometrical nature of quantum invariants which by now still remains largely open. Thus, the main idea was to bring together world leading experts working in 3-dimensional geometry and topology from one hand side and quantum topology on the

other, in order they could share their expertise between themselves as well as with students and young researchers. The general impression of the organizers is such that the school was successful. Many participants expressed the opinion that most of the lectures combined a high quality material with reasonable level of difficulty. The topics were interesting and motivating, especially taking into account the fact that the school has succeeded in attracting many students and young researchers (postdocs). The organizers expect, that the school will trigger new collaborations and research projects involving new doctoral students.

4) Final programme of the meeting.

	Monday 21	Tuesday 22	Wednesday 23	Thusday 24	Friday 25
8:30 - 9:20	Gordon	Gordon	Boileau	Boileau	Blanchet
9:50 - 10:40	van der Veen	van der Veen	Le	Le	Dimofte
11:00 - 11:50	Penner	Penner	Meilhan	Meilhan	Murakami
17:00 - 17:50	Turaev	Garoufalidis	Masbaum	Polyak	
18:00 - 18:50	Gueritaud	Zagier	Kania-Bartoszynska	Geer	

Christian Blanchet: TQFT and link homology.
Michel Boileau: Hyperbolic 3-manifolds and volumes.
Tudor Dimofte: Gluing tetrahedra in TQFT, quantum A-polynomials, and Chern-Simons theory.
Stavros Garoufalidis: Beyond the Slope.
Nathan Geer: The Turaev-Viro invariant and some of its relatives.
Cameron Gordon: Exceptional Dehn surgeries.
François Gueritaud: Towards a quantum Schlaefli formula.
Joanna Kania-Bartoszynska: A tale of two torsions.
Thang Le: Introduction to the AJ conjecture.
Gregor Masbaum: On quantum representations of mapping class groups.
Jean-Baptiste Meilhan: Fatgraphs and Finite Type Invariants 3, 4.
Hitoshi Murakami: The colored Jones polynomial, the Chern-Simons invariant, and the Reidemeister torsion of a knot.
Robert Penner: Fatgraphs and finite type invariants 1, 2.
Michael Polyak: Invariants of 3-manifolds via counting surfaces.
Vladimir Turaev: TQFTs in dimension 3.

Roland van der Veen: Classical and quantum spin networks.
Don Zagier: q-series, modularity, and quantum topology.