FINAL REPORT: QUANTIZATION OF SINGULAR SPACES – SCHOOL AND CONFERENCE

1. Organizing Committee

- Jørgen Ellegaard Andersen (QGM, Aarhus University)
- Hans-Christian Herbig (QGM, Aarhus University)
- Markus J. Pflaum (University of Colorado at Boulder)

2. Summary

The school and conference *Quantization of Singular Spaces* was held December 8–10 and 13–17, 2010, respectively, at the *Centre for Quantization of Moduli Spaces*, which is part of the Department of Mathematical Sciences of Aarhus University. The school was adressed to junior researchers (i.e. postdocs and PhD students) coming mostly from Europe and the United States. It comprised of 3 courses, 3 times 2 academic hours each, which served as an introduction to the themes of the conference to follow. During the conference 19 one-hour-talks were given. The participants of the conference were mainly European and US based researches working in quantization theory, non-commutative geometry, gauge theory, representation theory, complex and microlocal analysis and mathematical physics. The funding was provided by the Centre for Quantization of Moduli Spaces (QGM), by the National science foundation (NSF) and by the ESF Research Network ITGP. In total, there have been 25 participants in the school and 49 participants in the conference.

3. Scientific Content

A main motivation to study the commutative and noncommutative geometry of singular phase spaces comes from the attempt to gain a full mathematical understanding of gauge theory. This is a central theme in modern physics, since gauge theory provides the common language for the description of all fundamental interactions except gravity. Since the work of Witten, Donaldson and many others gauge theory techniques have been used heavily in low dimensional topology and global analysis. Furthermore, the problem of quantization of singular phase spaces is closely related to central topics in representation theory and microlocal analysis.

For the description of singular phase spaces there are essentially three approaches, which are of course interrelated. Firstly, the geometric one which describes singular phase spaces, such as singular sympletic quotients or moduli spaces of flat connections on Riemann surfaces, as stratified symplectic spaces. This point of view goes back to a work Sjamaar and Lerman from 1991, and has been further developed by many people ever since. Secondly, there is the categorical approach which describes singular phase spaces as orbit spaces of Lie groupoids, or more generally, using the language of stacks. This approach in recent years has turned out to be quite fruitful for understanding the commmutative and noncommutative geometry of orbifolds, and lead to interesting results like, e.g., a new proof of the Kawasaki index theorem. Thirdly, there is the commutative, differential graded algebraic point of view, which

plays an important role in the study of complex spaces using Tjurina resolvents and in the BRST-formalism.

Similarly, there are several approaches for the axiomatization of the quantization problem. Firstly, there is the theory of geometric quantization originating from works of Kirillov, Kostant and Souriau from the seventies. Geometric quantization has been successfully used for the construction of representations of Lie groups and among other things employed by Witten for the quantization of Chern-Simons gauge theory. Secondly, there is the approach of deformation quantization, which puts more emphasis on the algebra of quantum observables. In the last decade, there has been a lot of activity in formal deformation quantization triggered by Kontsevich's proof of his formality conjecture. One should also mention strict deformation quantization in the sense of Rieffel, which is analytically more refined but less universal than the formal approach. Deformation quantization is also closely related to algebras of differential operators on quotient varieties occuring in representation theory and to quantum groups. A third approach, which lies somewhat in between the aforementioned ones, is Berezin-Toeplitz quantization. This quantization method has been proposed by Bordemann-Meinrenken-Schlichenmaier in the early nineties, and works best for compact Kähler manifolds having an ample line bundle. Among other things Berezin-Toeplitz quantization has been used by J. E. Andersen to prove asymptotic faithfulness of quantum representation of the mapping class group.

The idea of the conference has been to take an unbiased point of view, and bring together researchers working on the mathematical theory of quantization of singular phase spaces in any of its incarnations. Since the last meetings addressing the singular case took place several years ago, the new results in the field appear to be scattered and have to be made accessible for non-experts. A main purpose of the conference was to facilitate communication between the competing philosophies, with the idea that this will lead to new substantial insights and future results. It was intended that the event helps to establish and maintain a community of scientists interested in the problem of quantization of singular phase spaces, and to draw the attention of young researchers to this interesting field of research.

4. Lectures and Talks

4.1. **School Lecturers.**

- **Johannes Huebschmann** (Université des Sciences et Technologies de Lille): *Poisson-Kähler geometry of stratified Kähler spaces and quantization*
- **Eugene Lerman** (University of Illinois at Urbana-Champaign): *Differential characters and prequantization*
- **Hessel Posthuma** (Korteweg-de Vries Institute for Mathematics, University of Amsterdam), *Noncommutative models for singular spaces*

4.2. **Conference Speakers.**

- **Daniel Sternheimer** (Rikkyo University (Tokyo) and Université de Bourgogne): *A singular view on quantization*
- **Johannes Huebschmann** (Université des Sciences et Technologies de Lille): *The moduli space of semi-stable holomorphic vector bundles on a curve revisited as a stratified Kähler space: The singular structure*

- **Hessel Posthuma** (University of Amsterdam): *The higher index theorem on orbifolds.*
- **Tudor Ratiu** (École polytechnique fédérale de Lausanne): *Singular reduction, quantum reduction, and coherent states quantization*
- **Alexander Karabegov** (Abilene Christian University): *Star products with separation of variables admitting a smooth extension*
- **Xiang Tang** (Washington University in St. Louis): *Mackey Machine and Duality of Gerbes on Orbifolds*
- **Giuseppe Dito** (Université de Bourgogne): *Star-products as a pseudo BCH-formula*
- **Alejandro Uribe** (University of Michigan): *The spectral function of a Riemannian orbifold*
- **Martin Bordemann** (Université de Haute Alsace): *Classical BRST cohomology for general graded commutative algebras*
- **Mark Gotay** (University of British Columbia): *Quantization via Prequantization*
- **Eugene Lerman** (University of Illinois at Urbana-Champaign): *Orbifolds and (pre)quantization*
- **Victor Palamodov** (Tel Aviv University): *Associative deformation and quantization in analytic geometry*
- **Gerald Schwarz** (Brandeis University): *Real double coset spaces and their invariants*
- **Markus Pflaum** (University of Colorado at Boulder): *On the geometry and topology of orbit spaces of proper Lie groupoids*
- **Klaas Landsman** (Radboud University Nijmegen): *Functoriality of quantization: an operator-algebraic approach*
- **Hans-Christian Herbig** (QGM, Aarhus University): *On Deformations of Singular Poisson Algebras*
- **Marius Crainic** (Utrecht University): *Prequantization and the integrability of Lie brackets*
- **Vladimir Fock** (IRMA Strasbourg): *Partial compactification of cluster varieties and quantization*
- **Martin Schlichemaier** (Université de Luxembourg): *Berezin symbols and Berezin transformations revisited*

5. Results and Impact

The organizers believe that the school and conference *Quantization of Singular Spaces* has been a substantial step towards establishing a research commutity for the subject. The lively discussions in between the talks indicate that the idea of bringing researchers together, which otherwise rarely have the chance to interact, has been a success. It is expected, that the new perspectives on the field will lead to new collaborations and publications in the near future. Moreover, a number of talented young researchers have been exposed and introduced to the material, and had the chance to get acquainted with the senior experts in the field and with their recent results. The organizers obtained very positive feedback from the young researchers. It was mentioned several times that having a school before the conference was an efficient preparation for the research talks at the conference. Moreover,

the organizers had the impression that the junior participants appreciated the international atmosphere of the school and conference. As the video recordings and slides of lectures and talks are permanently available at the conference homepage `http://qgm.au.dk/events/dec-2010-conference-quantization-of-singular-spaces/`, the material is accessable for a wider audience.

6. List of participants

6.1. **Participants of the School.**

- Harold Williams, UC Berkeley
- Romão, Jagiellonian University
- Matthew Young, Stony Brook University
- Peter Samuelson, Cornell University
- Pedro Frejlich, IST, Lisbon
- Graeme Wilkin, University of Colorado, Boulder
- Eitan Angel, University of Colorado, Boulder
- Ryan Caroll, Rhodes College
- Magnus Goffeng, Chalmers University of Technology, University of Gothenburg
- Yassir Dinar, University of Khartoum
- John Hower, University of Colorado, Boulder
- Jens-Jakob Kratmann Nissen, Aarhus University
- Jenny Santoso, University of Stuttgart
- Karl Leicht, University of Lille
- Marcel Bökstedt, Aarhus University
- Alexei Venkov, Aarhus University
- Johan Martens, Aarhus University
- Douglas LaFountain, Aarhus University
- Amit De, Aarhus University
- Yusuke Kuno, Hiroshima University
- Kiyonori Gomi, Kyoto University
- Benjamin Himpel, Aarhus University
- Mario Garcia Fernandez, Aarhus University
- Jens Kristian Egsgaard, Aarhus University
- Therese Søby Andersen, Aarhus University

6.2. **Participants of the conference.**

- Daniel Sternheimer, Rikkyo University (Tokyo) and Universit de Bourgogne
- Johannes Huebschmann, Université des Sciences et Technologies de Lille
- Hessel Posthuma, University of Amsterdam
- Tudor Ratiu, École polytechnique fédérale de Lausanne
- Alexander Karabegov, Abilene Christian University
- Xiang Tang, Washington University in St. Louis
- Giuseppe Dito, Université de Bourgogne
- Alejandro Uribe, University of Michigan):
- Martin Bordemann, Université de Haute Alsace
- Mark Gotay, University of British Columbia at Vancouver
- Eugene Lerman, University of Illinois at Urbana-Champaign
- Victor Palamodov, Tel Aviv University

- Gerald Schwarz, Brandeis University
- Klaas Landsman, Radboud University Nijmegen
- Marius Crainic, Utrecht University
- Vladimir Fock, IRMA Strasbourg
- Martin Schlichemaier, Université de Luxembourg
- Harold Williams, UC Berkeley
- Nuno Romão, Jagiellonian University
- Matthew Young, Stony Brook University
- Peter Samuelson, Cornell University
- Pedro Frejlich, IST, Lisbon
- Graeme Wilkin, University of Colorado, Boulder
- Eitan Angel, University of Colorado, Boulder
- Ryan Caroll, Rodes College
- Yassir Dinar, University of Khatoum
- John Hower, University of Colorado, Boulder
- Magnus Goffeng, Chalmers University of Technology, University of Gothenburg
- Jakob Blaavand, Aarhus University
- Søren Fuglede Jørgensen, Aarhus University
- Christopher Rogers, University of California, Riverside
- David Martinez Torres, IST, Lisbon
- Greg Muller, LSU, USA
- Christopher Seaton, Rhodes College
- William Kirwin, University of Cologne
- Yael Fregier, Luxembourg University
- Florian Schätz, IST. Lisbon
- Marcel Bökstedt, Aarhus University
- Alexei Venkov, Aarhus University
- Johan Martens, Aarhus University
- Douglas LaFountain, Aarhus University
- Yusuke Kuno, Hiroshima University
- Kiyonori Gomi, Kyoto University
- Benjamin Himpel, Aarhus University
- Reza Rezazadegan, Aarhus University
- Amit De, Aarhus University
- Jens-Jakob Kratmann Nissen, Aarhus University
- Sergey Arkhipov, Aarhus University
- Svend Heerullf