Report on the workshop
YEP VIII 2011
(Young European Probabilists)

Stochastic Models for Population Dynamics

Date: March 14-18, 2011
Place: EURANDOM, The Netherlands

Organizers:

Prof. Dr. Matthias Birkner
Institute of Mathematics
Johannes Gutenberg University of Mainz
Germany

Prof. Dr. Jochen Blath Institute
of Mathematics Technical
University of Berlin Germany

Subject: Stochastic Models for Population Dynamics

Summary:
The main focus of the workshop was on "Population Dynamics", more precisely on stochastic models for population dynamics in ecology and evolution, particularly with a view towards spatial models.

Population biology has long been a fruitful source of research problems and inspiration for probabilists, starting with classical branching processes, their diffusion approximations, ranging over interacting particle systems up to measure-valued processes. Processes in biological populations are often very complex and stochastic by nature, thus requiring and inspiring probabilistic modeling. The situation becomes especially intriguing when the role of space is taken into account, where this can refer to geographical space, with the two-dimensional space being biologically most interesting and also posing the toughest mathematical challenges, or to a more abstract 'type space' or possibly to a combination of both. Examples of important questions that mathematical models can help to attack are in how far spatial distribution can promote survival of a population or coexistence of various types, how the populations tend to arrange themselves in space, or how new species can emerge through the interplay of mutations and competition of different types.

In this workshop we focused on such mathematical questions addressing challenging problems at the interface between probability theory and population dynamics.

Contributions:

The workshop consisted of three courses, two problem sessions and 13 contributed talks. The courses were given in three, respectively two lectures of each 120 minutes:

Course 1: Limit theorems for voter model perturbations
Speaker: Ted Cox (Syracuse, US)

Abstract: We present joint work with Rick Durrett and Ed Perkins on limit theorems for a class of interacting particle systems (spatial competition models) we call voter model perturbation. This work includes a measure-valued limit theorem and a hydrodynamic type limit theorem. In each case it is possible to "invert" the limit and prove survival and/or coexistence results for several interesting competition models.

Course 2: Population dynamics and evolution in the trait space
Speaker: Amaury Lambert (Paris, F)

Abstract: We will start with a brief study of some well-known deterministic models for populations structured by types. Then we will set up a general framework for probabilistic models of structured populations. In these models, all types, but one, eventually become extinct, so we will (prefer to) assume the presence of mutations. Our goal is to understand the evolution of such populations, seen as the sequential substitution of ancient types by recent (often more fit) types. We will first study different sorts of large population limits and explain how they are used by biologists, in addition to simplifying our problem. We will also display different concepts of a nearly neutral population. The last part will consist in showing how the recent field of adaptive dynamics uses these tools to model evolution.

Course 3: Temporal and spatial scales in geographically structured population models
Speaker: Amandine Véber (Paris, F)

Abstract: The aim of this course is to review some classical models of geographically structured populations, with a particular view towards questions of population genetics. We shall first consider the island and stepping stone models, which assume that the global population is split into discrete communities connected through a constant exchange of migrants. Then, we shall present a rather new model for evolution in a continuum. For all these frameworks, our main interest will be to understand the space- and time-scales on which the action is taking place, in order to describe the pattern of genetic correlations they produce and to find appropriate criteria to discuss their relevance to some "real" situations.

In addition, there were two problem sessions (each 60 minutes), headed by senior scientists who presented important open problems and directions for future research. The first emphasized aspects of mathematical modelling for biological questions, the second focussed more on questions concerning the mathematical analysis of specific models from population biology.

Problem session 1: Stochastic models in Ecology and Evolution
Speaker: Patsy Haccou (Leiden)

Abstract: We will consider models of the dynamics of small populations (extinction and establishment), effects of spatio-temporal variation on population dynamics, bethedging & evolution in stochastic environments, game theory & mixed strategies. If there is time, we will also consider quasi-species dynamics. The main focus will be on the modeling part, i.e. how to translate biological processes and questions into mathematical models, rather than the mathematical analysis.

Problem session 2: Problems in population genetics involving selection, recombination and migration
Speaker: Feng Yu (Bristol)

Abstract: In this problem session, I will start with the classic model of a population of constant size under the influence of selection and mutation. Then I will go onto Petry's island/continent model involving migration, selection and recombination. I will show that in the large population limit, effective migration rate to the island is reduced due to effects of recombination and selection, and give Petry's approximate formula for effective migration rate. This approximation formula is not accurate for finite populations, and finding one is a challenging open problem. This leads naturally to the final topic of discussion, which is the genic view of parapatric speciation, where selection, recombination and migration all play important roles. I will present only intuition on the final topic, which still awaits proper mathematical analysis.

Furthermore, there were 13 contributed talks from young participants on topics of their own research.

The **contributed talks** with abstracts were as follows:

Speaker: Agnes Backhausz
Title: A random model of publication activity

Abstract: A discrete model, inspired by publication activity, is introduced. It includes an increasing number of objects (authors) equipped with positive weights, which also increase with time. We start with one author with random initial weight. At each step a randomly chosen group of authors produces a new publication. The weights of the members of this group are increased by random quantities. Then a new author comes with random initial weight. The probability distributions are chosen such that this becomes a preferential attachment model. It is proven that the empirical weight distribution converges weakly with probability 1, and the limit law has a regularly varying tail. Methods of martingale theory and renewal theory are applied in the proofs [1, 2]. References:
[1] F. Chung and L. Lu, Complex graphs and networks, CBMS Regional Conference Series in Mathematics, 107, Washington, DC, 2006.
[2] Neveu, J. Discrete-parameter martingales. North-Holland Publishing Co., Amsterdam-Oxford, 1975.

Speaker: Roman Berezin
Title: Survival and Extinction of the Contact Random Walk

Abstract: We study a nearest neighbour contact process fused with an independent random walk in $d \geq 3$. It is known that the critical value of the birth rate of the contact process

starting with a single particle, necessary for inde nite survival approaches 1 as a suitable scaling parameter approaches in nity. The main point of interest is to nd sharp asymptotic for how close this critical value is to 1. This result give further a clue to improving the results of Konno (1995) for the Contact Processes with Rapid Stirring.

Speaker: Andrej Depperschmidt
Title: Ancestry in the face of competition: Directed random walk on the directed percolation cluster

Abstract: The spatial embeddings of genealogies in models with fluctuating population sizes and local regulation are complicated random walks in a space-time dependent random environment (RWRE). Such RWRE are presently not well understood. We consider the supercritical discrete time contact process on Z^d which is the simplest non-trivial example of a locally regulated population model. We study the RWRE performed by the ancestral lineage of an individual sampled from the invariant distribution. We prove a LLN and an annealed CLT via a regenerative approach. Based on joint work in progress with M. Birkner, J. Cerny and N. Gantert.

Speaker: Leif Döring
Title: Limit Properties of Mutually Catalytic Branching Models

Abstract: A 2-parameter class of mutually catalytic branching models interpolating between various probabilistic models such as the voter process, the stepping stone model, and a parabolic Anderson model shall be discussed. The parameters govern correlations and strength of the underlying branching mechanism. Based on an observation taken from a recent work with Jochen Blath and Alison Etheridge, the focus lies on negative correlations. Surprisingly, this allows an "in a nutshell" presentation of results and proofs for limit results for finite and infinite branching rate processes.

Speaker: Sandra Kliem
Title: Convergence of Rescaled Competing Species Processes to a Class of SPDEs

Abstract: We construct a sequence of rescaled perturbations of voter processes in dimension d=1 whose approximate densities are tight. We combine both long-range models and fixed kernel models in the perturbations. In the case of long-range interactions only, the approximate densities converge to continuous space time densities which solve a class of SPDEs (stochastic partial differential equations), namely the heat equation with a class of drifts, driven by Fisher-Wright noise. As an example we show that the results cover the stochastic spatial Lotka-Volterra model for parameters approaching one.

Speaker: Mickaël Launay
Title: Interacting Urn Models

Abstract: The aim of this paper is to study the asymptotic behavior of strongly reinforced interacting urns with partial memory sharing. The reinforcement mechanism considered is as follows: draw at each step and for each urn a white or black ball from either all the urns combined (with probability p) or the urn alone (with probability $1-p$) and add a new ball of the same color to this urn. The probability of drawing a ball of a certain color is proportional to w_k where k is the number of balls of this color. The higher the p, the more memory is shared between the urns. The main results can be informally stated as follows: in the exponential case $w_k = p^k$, if $p \geq 1/2$ then all the urns draw the same color after a finite

time, and if $p < 1/2$ then some urns fixate on a unique color and others keep drawing both black and white balls.

Speaker: Tobias Müller
Title: Random geometric graphs

Abstract: If we pick points $X_1, \ldots, X_n$ at random from d-dimensional space (i.i.d. according to some probability measure) and fix a $r > 0$, then we obtain a random geometric graph by joining points by an edge whenever their distance is $< r$. I will give a brief overview of some of the most important results on random geometric graphs and then describe some of my own work on Hamilton cycles, the chromatic number, and the power of two choices in random geometric graphs.

Speaker: Francesca Nardi
Title: An upper bound for front propagation velocities inside moving populations

Speaker: Leonid Petrov
Title: Infinite-dimensional diffusions related to the two-parameter Poisson-Dirichlet distributions

Abstract: The one-parameter infinitely many neutral alleles diffusion model was introduced by Ethier and Kurtz in 1981. This is a family of infinite-dimensional diffusion processes preserving the one-parameter Poisson-Dirichlet distributions $PD(\theta)$. It is known that these diffusions are approximated by certain random walks on partitions (a Moran-type discrete population model). A natural algebraic combinatorial interpretation of these random walks is given which allows to extend the infinite-dimensional diffusions to the case of the two-parameter Poisson-Dirichlet distributions $PD(\alpha, \theta)$. The Markov generator for the two-parameter family of infinite-dimensional diffusions is explicitly computed.

Speaker: Lorenz Pfeifroth
Title: Frogs in a random environment

Abstract: In the first part in this talk we consider the frog model on the integers where the underlying random walk, which every active frog performs, is an arbitrary nearest neighbor Markovian random walk on Z with drift to the right. The question, we are interested in, is if the origin is visited infinitely often by active frogs with probability 1 or not. We give a necessary and sufficient condition that this will happen. Also we present a 0-1 law for this model. In the second part we consider the frog model in a random environment which is the mixture of the normal frog models. We give recurrence criteria for such a model and derive 0-1 law too and show that the recurrence of such a model only depends on the distribution of the starting configuration of frogs and not on the distribution of the jumping probability of the underlying random walk.

Speaker: Thomas Rippl
Title: Pathwise uniqueness for the stochastic heat equation with Holder continuous coefficient: the colored-noise-case

Abstract: We consider the stochastic heat equation in \R_+ \times \R^d with multiplicative noise. The noise is white in time and colored in space with correlation kernel $k(x,y) \leq const (|x-y|^{- \alpha } +1)$ for a fixed $\alpha \in (0, 2 \wedge d)$ and $x,y \in \R^d$. We prove that if the noise coefficient is Holder-continuous of order \gamma and

satisfies $\alpha < 2(2\gamma -1)$ then the equation has a pathwise unique solution. This was conjectured by Mytnik and Perkins in 2009.

Speaker: Pieter Trapman
Title: SIR epidemics on random intersection graphs

Abstract: We consider a model for the spread of a stochastic SIR (Susceptible --> Infectious --> Removed) epidemic on a network of individuals described by a random intersection graph. The number of cliques a typical individual belongs to follows a mixed-Poisson distribution, as does the size of a typical clique. Infection can be transmitted between two individuals if and only if they belong to the same clique. An infinite-type branching process approximation (with type being given by the length of an individual's infectious period) for the early stages of an epidemic is developed and made fully rigorous by proving an associated limit theorem as the population size tends to infinity. This leads to a threshold parameter R*, so that in a large population an epidemic with few initial infectives can give rise to a large outbreak if and only if R* > 1. A law of large numbers for the size of such a large outbreak is proved by exploiting a single-type branching process that approximates the susceptibility set of a typical individual. This talk is based on joint work with Frank Ball and David Sirl.

Speaker: Yinna Ye
Title: Asymptotic behavior of the survival probability for a critical branching process in markovian environments

Programme

MONDAY, MARCH 14

09.30-10.30	Ted Cox	Limit theorems for voter model perturbations
10.30-11.00	*Coffee/tea break*	
11.00-12.00	Ted Cox	Limit theorems for voter model perturbations
12.00-12.30	Mikaël Launay	Interacting Urn Models
12.30-13.00	Leonid Petrov	Infinite-dimensional diffusions related to the two-parameter Poisson-Dirichlet distributions
13.00-15.00	*Lunch, informal discussion*	
15.00-16.00	Amaury Lambert	Population dynamics and evolution in the trait space
16.00-16.30	*Coffee/tea break*	
16.30-17.30	Amaury Lambert	Population dynamics and evolution in the trait space
17.30-18.30	*Social get-together*	at EURANDOM front desk

TUESDAY, MARCH 15

09.30-	Amandine Véber	Temporal and spatial scales in geographically structured

10.30		population models
10.30-11.00	*Coffee/tea break*	
11.00-12.00	Amandine Véber	Temporal and spatial scales in geographically structured population models
12.00-12.30	Roman Berezin	Survival and Extinction of the Contact Random Walk
12.30-13.00	Andrej Depperschmidt	Ancestry in the face of competition: Directed random walk on the directed percolation cluster
13.00-15.00	*Lunch, informal discussion*	
15.00-16.00	Problem session Patsy Haccou	Stochastic models in Ecology and Evolution
16.00-16.30	*Coffee/tea break*	
16.30-17.00	Francesca Nardi	An upper bound for front propagation velocities inside moving populations
17.00-17.30	Lorenz Pfeifroth	Frogs in a random environment

WEDNESDAY, MARCH 16

09.30-10.30	Amaury Lambert	Population dynamics and evolution in the trait space
10.30-11.00	*Coffee/tea break*	
11.00-12.00	Amaury Lambert	Population dynamics and evolution in the trait space
12.00-13.00	Problem session Feng Yu	Problems in population genetics involving selection, recombination and migration
13.00-15.00	*Lunch, informal discussion*	
15.00-16.00	Ted Cox	Limit theorems for voter model perturbations
16.00-16.30	*Coffee/tea break*	
16.30-17.30	Ted Cox	Limit theorems for voter model perturbations
18.30	CONFERENCE DINNER	

THURSDAY, MARCH 17

09.30-10.30	Amandine Véber	Temporal and spatial scales in geographically structured population models
10.30-11.00	*Coffee/tea break*	
11.00-12.00	Amandine Véber	Temporal and spatial scales in geographically structured population models
12.00-12.30	Sandra Kliem	Convergence of Rescaled Competing Species Processes to a Class of SPDEs
12.30-13.00	Thomas Rippl	Pathwise uniqueness for the stochastic heat equation with Holder continuous coefficient: the colored-noise-case
13.00-15.00	*Lunch, informal discussions*	

15.00-15.30	Yinna Ye	Asymptotic behavior of the survival probability for a critical branching process in markovian environments
15.30-16.00	Leif Döring	Limit Properties of Mutually Catalytic Branching Models
16.00-16.30	*Coffee/tea break*	
16.30-17.00	Peter Trapman	SIR epidemics on random intersection graphs

FRIDAY, MARCH 18

09.30-10.30	Ted Cox	Limit theorems for voter model perturbations
10.30-11.00	*Coffee/tea break*	
11.00-11.30	Agnes Backhausz	A random model of publication activity
11.30-12.00	Tobias Müller	Random geometric graphs
12.00-13.00	Ted Cox	Limit theorems for voter model perturbations