

Summer School on the Topology of Manifolds

Budapest, August 15-19, 2011

Report

1 The lectures

The Summer School on *Topology of Manifolds* was organized by András Stipsicz (Rényi Institute of Mathematics, Budapest, Hungary) and András Szűcs (Eötvös University, Budapest, Hungary). The Summer School was built on lectures of Matthias Kreck on his method of classifying smooth, closed manifolds. (The rough outline of the lectures can be deduced from the titles of the individual talks given below.) The lectures were complemented by Problem sessions, led by Diarmuid Crowley. In these sessions 10 problems were handed to the participants, and after an hour of individual work, some solutions were discussed. Also, participants were encouraged to 'sign up' for particular problems to hand in typed solutions, which will be collected and put on the web page of Diarmuid Crowley (<http://www.dcrowley.net/teaching.html>). We found this format very effective in teaching the material of the main lectures well. In addition, designated note-takers took careful notes of the lectures, and we hope to have an edited version of these notes shortly on the above mentioned web page, together with the solutions.

On Wednesday three special lectures on 4-manifolds were given by A. Stipsicz and Z. Szabó. In these lectures, besides the topological and smooth aspects of manifold theory, symplectic questions (such as the symplectic rational blow down procedure, and the symplectic Luttinger surgery operation) were also discussed.

2 The schedule

Each day begun with a series of three 45 minute lectures (9:15-10:00, 10:30-11:15 and 11:30-12:15) which introduced the main concepts for that day. After lunch there was an extended exercise session (2:30-3:30) and then the discussion of some of the solutions (4:00-4:25). The day ended with a further less formal lectures (4:30-5:30) to elaborate and clarify the main points for that day. Wednesday departed from this model by leaving the afternoon free. Unless otherwise indicated, the lectures were given by Matthias Kreck.

- Monday
 - 1 Overview: main problems and the statement of the s -cobordism theorem
 - 2 Normal structures: the normal k -type and normal bordism groups [K, §2], [MAP]
 - 3 Surgery below the middle dimension [M, §1-3], [K, §3]
- E1 First exercise session

4 Informal lecture

- Tuesday

- 1 The stable classification of $2q$ -manifolds [K, §2]
 - 2 Application: the stable classification of 4-manifolds, bordism of automorphism and stably unique smooth structures
Goal: The statement of Freedman's classification of closed, simply connected topological 4-manifolds [F, Theorem 1.5]
 - 3 The main theorem for odd-dimensional bordisms: the obstruction monoid $l_{2q+1}(\pi)$ and the group $L_{2q+1}(\pi)$ [K, §6]
- E2 Second exercise session
- 4 Informal lecture

- Wednesday (*Lectures by Zoltán Szabó and András Stipsicz about smooth 4-manifolds*)

- 1 (Stipsicz) Basic invariants, examples and constructions of smooth 4-manifolds
 - 2 (Szabó) Smooth 4-manifold invariants
 - 3 (Szabó) Exotic structures
- * Afternoon free

- Thursday

- 1 The main theorem for even-dimensional bordisms: the obstruction monoid $l_{2q}(\pi)$ and $L_{2q}(\pi)$ [K, §5]
 - 2 The computation of $L_{2q+1}(e)$ [K-M, §6] and a report on some results for $L_{2q+1}(\pi)$
 - 3 The h-cobordism classification of closed simply connected 4-manifolds
- E3 Third exercise session
- 4 (Crowley) Cancellation of $S^q \times S^q$ and $l_{2q+1}(\pi)$ [C-S]

- Friday

- 1 (Crowley) A short review of the classical surgery exact sequence [W, Ch 10]
 - 2 On the classification of complete intersections [K, §8]
 - 3 On the classification of non-simply connected 4-manifolds [H-K-T]
- E4 Fourth exercise session

References

- [C-S] D. Crowley and J. Sixt, *Stably diffeomorphic manifolds and $l_{2q+1}(\mathbb{Z}[\pi])$* , Forum Math., **23** (2011), no.3, 483–538.
- [F] M. H. Freedman, *The topology of four-dimensional manifolds*, J. Differential. Geom. **17** (1982), no.3, 357–453.

- [H-K-T] I. Hambleton, M. Kreck and P. Teichner, *Topological 4-manifolds with geometrically two-dimensional fundamental groups*, J. Topol. Anal. **1** (2009), no. 2, 123–151.
- [K-M] M. A. Kervaire and J. W. Milnor, *Groups of homotopy spheres I*, Ann. of Math. **77** (1963), 504–537.
- [K] M. Kreck, *Surgery and Duality*, Ann. of Math. **149** no.3 (1999), 707–754.
- [MAP] Manifold Atlas Project, *B-Bordism*, <http://www.map.him.uni-bonn.de/B-Bordism>
- [M] J. Milnor, *A procedure for killing homotopy groups of differentiable manifolds*, Proc. Sympos. Pure Math., Vol. III (1961), 39–55.
- [P] T. Panov, *Bordism*, Bull. Man. Atl. (2011), 23–29. <http://www.boma.him.uni-bonn.de/articles/31>
- [W] J. H. C. Whitehead, *On simply connected, 4-dimensional polyhedra*, Comment. Math. Helv. **22** (1949,) 48–92.

3 List of participants

The Summer School was very successful to attract a large number of highly motivated graduate students and postdoctoral researchers. We were particularly happy to have so many participants from Hungary, hence the School helped to disseminate this branch of topology in Hungary.

- Dr. Anna Abczynski Bonn, (DE)
- Mr. Spiros Adams-Florou Edinburgh, (UK)
- Miss Irida Altman Coventry, (UK)
- Dr. Lukasz Bak Krakow, (PL)
- Miss Agata Barecka Wroclaw, (PL)
- Professor Refik Inanc Baykur Bonn, (DE)
- Dr. Ahmet Beyaz Ankara, (TR)
- Professor Maciej Borodzik Warsaw, (PL)
- Dr. Roger Casals Madrid, (ES)
- Dr. DongHeon Choe Seoul, (KR)
- Dr. HAKHO CHOI Seoul, (KR)
- Dr. Peter Csorba Budapest, (HU)
- Dr. Andrzej Czarnecki Krakow, (PL)
- Mr. Andrew Donald Glasgow, (UK)

- Dr. Peter Frenkel Geneve, (CH)
- Professor Paolo Ghiggini Nantes, (FR)
- Mr. Adam Gyenge Budapest, (HU)
- Mr. Marton Horvath Budapest, (HU)
- Mr. Kristof Huszar Budapest, (HU)
- Mrs. Eموke Imre Budapest, (HU)
- Dr. Andras Juhasz Cambridge, (UK)
- Dr. Tamas Kalman Tokyo, (JP)
- Dr. Boldizsar Kalmar Budapest, (HU)
- Dr. Marek Kaluba Poznan, (PL)
- Dr. Daniel Kasproowski Munster, (DE)
- Dr. Hee Jung Kim Binghamton, (US)
- Mr. Tomasz Kolodziejski Krakow, (PL)
- Dr. Ju A Lee Seoul, (KR)
- Mr. Francesco Lin Pisa, (IT)
- Dr. Johannes Nordstrom London, (UK)
- Mr. Patrick Orson Edinburgh, (UK)
- Dr. Mehmetcik Pamuk Ankara, (TR)
- Dr. Semra Pamuk Ankara, (TR)
- Professor Jongil Park Seoul, (KR)
- Dr. Heesang Park Seoul, (KR)
- Professor Krzysztof Pawalowski Poznan, (PL)
- Miss Aleksandra Perisic Lisbon, (PT)
- Mr. Gergo Pinter Budapest, (HU)
- Dr. Wojciech Politarczyk Poznan, (PL)
- Dr. Mark Powell Hayling Island, (UK)
- Dr. Khaled Qazaqzeh Irbed, (JO)
- Mr. Bela Andras Racz Princeton, (US)

- Dr. Rares Rasdeaconu Nashville, TN, (US)
- Dr. Antonio Rieser Haifa, (IL)
- Dr. Dongsoo Shin Daejon, (KR)
- Dr. Wolfgang Steimle Bonn, (DE)
- Mr. Balazs Strenner Madison, WI, (US)
- Dr. Ioana Suvaina Nashville, TN, (US)
- Mr. Ferenc Szurdoki Budapest, (HU)
- Dr. Tamas Terpai Geneve, (CH)
- Dr. Rafael Torres Ruiz Oxford, (UK)
- Dr. Vera Vertesi Cambridge, (US)
- Dr. Russ Woodroffe St. Louis, (US)
- Dr. Ki-Heon Yun Seoul, (KR)
- Professor Diarmuid Crowley Bonn, (DE)
- Professor Matthias Kreck Bonn, (DE)
- Professor Zoltan Szabo Princeton, (US)
- Professor Andras Stipsicz Budapest, (HU)
- Professor Andras Szucs Budapest, (HU)

4 Budget

The total budget of the Summer School was 15,749 Euros. The *Lendület program* of the Hungarian Academy of Sciences covered 9,258 Euros (travel support for graduate students and postdocs, and the hotel expenses of professors and graduate students) while 6,491 Euros was used from the support of the CAST network to cover further travel supports (750 Euros), the hostel of graduate students (2,875 Euros), and the expenses of lunch and coffee for all the participants (2866 Euros).

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