Constructions of Nonequilibrium Statistical Mechanics

School supported by the ESF programme
Random Geometry of Large Interacting Systems and Statistical Physics

Summary: The school prepared for mathematical and theoretical research in current themes of nonequilibrium mesoscopic physics. The formalism used standard Markov processes for interacting particles in contact with a thermodynamic environment. That environment consists of different macroscopic equilibrium reservoirs driving the system of interest in a nonequilibrium regime under the condition of local detailed balance. The notions of entropy production and of dynamical activity were introduced as a function of the system's state trajectory, with special emphasis on their role in a fluctuation theory for steady states. In the same context new nonequilibrium variational principles and fluctuation-response relations were explained. We concentrated on general results and on physical ideas that are described in a simple but precise mathematical language, and which could be inspiring for different scientific disciplines where fluctuations, dissipation and activity combine.

Lectures:

Chrisian Maes (K.U. Leuven)

Constructions of nonequilibrium statistical mechanics

Nonequilibrium phenomena are ubiquitous and range from cosmological effects to life processes and nano-scale transport phenomena. Since about 15 years a unifying mathematical framework has been developing which combines dynamical and statistical theories. Fluctuations are especially important for small systems and the tools of probability theory and of stochastic processes prove relevant for unraveling their structure. As it was the case for equilibrium statistical mechanics it is expected that now also nonequilibrium physics will positively stimulate new directions in probability theory and mathematical analysis, enabling for example more easy access to mathematical formulations of questions and answers in the life sciences. The study of nonequilibrium problems starting even from simple models of transport and dissipation in physics but hopefully extending to spatially extended systems of nontrivially interacting particles and combined with precise mathematical formulations, appears very timely and even urgent for serious discussions on complex dynamical phenomena in the natural and the human sciences.

Course program:

- Introduction to stochastic processes: jump and diffusion processes with emphasis on the meaning of the Master/Fokker-Planck equation, detailed balance (reversibility), the notion of generator and semigroup, probabilistic ergodicity;
- Path-integral formalism with Girsanov formula and elements of stochastic calculus (Stratonovich versus Ito integral, Feynman-Kac formula);
- Examples and mathematical formulation of nonequilibrium processes and interacting particle systems (Langevin dynamics, open exclusion processes, coupled oscillators);
- Introduction of path-dependent analogues of energy, heat, work and entropy fluxes in the context of processes with local detailed balance (first law and Clausius heat theorem);
- Fluctuation symmetries for the entropy fluxes, such as known under the names of Jarzynski (transient) and Gallavotti-Cohen (steady), and some extensions;
- Definition of the entropy production functional and its main properties;
- Definition of dynamical activity and fluctuation theory for the occupation times;
- Entropy production principles and their possible extension beyond the linear regime;
- Joint large deviations for currents and occupations;
- Fluctuation-response relations and extensions of the (equilibrium) fluctuation-dissipation theorem;
- Review of open problems and interdisciplinary topics.

Kazimierz Sobczyk (IPPT and MIMUW)

Systems with incomplete information: inference via maximum entropy method

This lecture consists of two parts. The first one is concerned with reconstruction of unknown probability distribution for the basic characteristics of random polycrystalline microstructures. The method is inspired by the Gibbsian ensemble method known in classical statistical physics/mechanics. Among all possible distributions of microstructural features we are looking for the distributions which are compatible with the available macroscopic information.
The method of maximum informational entropy is used. In the second part of the lecture it will be shown how the idea of maximum entropy principle can be extended to stochastic dynamical systems (described by stochastic differential equations). Assuming that the available (incomplete) information about the system response is given by equations For statistical moments the approximate probability density of the solution is derived via maximization of entropy. Specific examples will be shown to illustrate the goodness of the method.

Assessment of the results

The school was attended by students, PhD students, and post-docs, from both Mathematics and Physics. There were many interdisciplinary discussions. Especially an approach of Kazimierz Sobczyk (inference via maximum entropy method) and Christian Maes (nonequilibrium statistical mechanics) were confronted. Problems of nonequilibrium statistical mechanics were exposed to a wider audience of mathematicians. During his course, Christian Maes presented many exercises of different levels of difficulties. Some of these exercises were solved by students and submitted as reports from the school. Christian Maes prepared over 20 open problems which were distributed among participants of the school. One of the future research plan is to extend result obtained in the finite state Markov-Chain set-up to infinite spaces.

Final programme of the school

11 – 14 April, 16:15 – 18:15

Christian Maes (K.U. Leuven)

Constructions of nonequilibrium statistical mechanics

Wednesday, 13 April, 14:15 - 15:45

Kazimierz Sobczyk (IPPT and MIMUW)

Systems with incomplete information:
inference via maximum entropy method

Short talks by students and post-docs

Thursday, 14 April

14:00 - 14:35 Alberto Salazar Martínez (K.U. Leuven)
Fluctuation theorem for currents and the spinning Lorentz gas

14:40 - 15:15 Renato Soares dos Santos (Leiden University)
Mixing conditions for random walks in dynamic random environments

15:20 - 15:55 Florian Völlering (Leiden University)
Concentration inequalities for Markov processes