

Model Theory and Proof Theory of Arithmetic

A Memorial Conference in honor of Henryk Kotlarski and Zygmunt Ratajczyk

Final Report

October 28, 2012

1 Summary

The conference took place 22-27th of July 2012 at the Banach Center of the Polish Academy of Sciences in Będlewo, Poland. It was organized by Zofia Adamowicz (Polish Academy of Sciences), Teresa Bigorajska (University of Cardinal Stefan Wyszyński in Warsaw), Leszek Kołodziejczyk (University of Warsaw), Roman Kossak (City University of New York), Roman Murawski (University of Adam Mickiewicz in Poznań), and Konrad Zdanowski (University of Cardinal Stefan Wyszyński in Warsaw).¹ There were 34 participants from Poland (14), USA (4), Belgium (3), UK (3), France (2), Italy (2), Spain (2), Brazil (1), Iran (1), Netherlands (1), and Sweden (1). Five days program consisted mostly of one hour research talks, followed by short discussions (see the full program below). There was a memorial session for Henryk Kotlarski and Zygmunt Ratajczyk including a presentation of their careers and research contributions given by Roman Murawski, and a prerecorded lecture by Victor Marek (University of Kentucky), the video is available at <http://www.youtube.com/watch?v=ZnkJG-k1n-k&feature=youtu.be>. There was one tutorial on full satisfaction classes given by Ali Enayat and Albert Visser. On the last day, the program included an open problems session, a series of contributed papers presentations given by three graduate students, and a special talk on *Number Theory, K-theory and their interaction* given by Matthew Morrow of University of Chicago. The full program, with abstracts and pdf files for most of the talks, is available at the conference website at http://www.impan.pl/~kz/KR/KR_Home.html. The website also includes the list of problems presented at the open problems session. The conference was supported by grants from ESF; Banach Center at Institute of Mathematics, Polish Academy of Sciences; Committee of Mathematics at Polish Academy of Sciences; Ministry of Science and Higher Education

¹Michał Krynicki, was the initiator of the conference and the first chair of the organizing committee. He passed away in October of 2011.

of Poland, program DUN; Adam Mickiewicz University in Poznań; University of Warsaw; and Cardinal Stefan Wyszyński University in Warsaw.

2 Description of the scientific content of and discussion at the event

The list below contains a rough classification of topics covered in the conference talks. Nonstandard models have rich structure that has been a subject of investigations for many years. They have been studied for their own sake, and they have applications to proof theoretic problems, to problems in computational complexity theory, and they have applications on other areas of mathematics. All those directions were represented. Despite the diversity of topics, the talks showed the unity of the discipline. There were many examples of interactions between proof-theoretic, model theoretic, and combinatorial aspects of first and second-order arithmetic and their fragments. Examples include the ordinal analysis of Ramsey type combinatorial principles independent of PA; applications of nonstandard satisfaction classes to problems concerning automorphism groups of nonstandard models; and the recent developments of the study of integer parts of real closed fields involving models of PA and its fragments and resplendent models. Many talks emphasized connections to reverse mathematics and subtheories of second order arithmetic.

1. Classification results in model theory of first-order arithmetic and its fragments: Richard Kaye, Ali Enayat, Tin Lok Wong.
2. Model theoretic techniques in proof theory: Andrés Cerdón-Franco, F. Félix Lara-Martn; F. Félix Lara-Martn; Zofia Adamowicz, Albert Visser.
3. Reverse mathematics and models of second order arithmetic: Michael Rathjen, Konrad Zdanowski, Tin Lok Wong.
4. Incompleteness phenomena in arithmetic and its fragments: Michael Rathjen, Lorenzo Carlucci, Zofia Adamowicz, Andrey Bovykin, Leszek Kołodziejczyk, Andreas Weiermann, Konrad Zdanowski.
5. Nonstandard satisfaction classes: Ali Enayat, Albert Visser, Roman Kossak.
6. Recursively saturated, resplendent, and saturated models of arithmetic: Paola D'Aquino, James Schmerl, Ermek Nurkaidarov, Roman Kossak.
7. Automorphisms and automorphism groups of models of arithmetic: James Schmerl, Ermek Nurkaidarov, Roman Kossak.
8. Nonstandard analysis and computability theory: Sam Sanders, Zenon Sadowski, Denis Richard, Marcin Mostowski.

9. General model theory: Aleksander Iwanow, Barbara Majcher-Iwanow, Marcin Mostowski.

3 Assessment of the results and impact of the event on the future direction of the field

In the opinion expressed by many participants, it was a very successful conference. In essence, despite diversity of topics, most talks presented different aspects of the study of one common area. It seems that from the period of many diverse developments, the discipline now enters an era of unification.

Many results presented at the conference were the result of joint work of several teams of participants (Carlucci-Zdanowski, Enayat-Visser, Kaye-Kossak-Tin-Lok-Wong, Schmerl-Nurkhaidarov, and other). The conference provided a very natural environment for follow-up discussions and further collaborations. There already have been results of those collaborations, for a example a new simplified approach to constructions of full satisfaction classes for recursively saturated models (Enayat-Visser-Schmerl) , a very interesting new paper by Visser influenced by the results on fragments of arithmetic by Cerdón-Franco and Lara-Martn. Dean and Zdanowski started collaboration in order to characterize functions computable in polynomial time within the universe of hereditarily finite sets. The project aims at giving the algebra of functions in Bellantoni-Cook style for hereditarily finite sets given in a usual notation.

While the talks given at the conference concentrated mostly on recent results, it should be noted that one of the main goals was to honor the memory of Henryk Kotlarski and Zygmunt Ratajczyk. In several talks, speakers made direct connections to Kotlarski and Ratajczyk's contributions of and during the memorial session Roman Murawski gave a full account of their accomplishments. Victor Marek, in his prerecorded lecture talked about the Warsaw school of mathematical logic in the 1960's and 1970's and the roles of Kotlarski, Ratajczyk, Michał Krynicki and Paweł Zbierski. It was very important, especially for the younger participants of the conference, to learn about this interesting period in history of mathematical logic in Poland.

4 Program

Monday, July 23rd

9:25-9:30 *Opening*

9:30-10:20 Andrés Cerdón-Franco, *On Local Induction and Collection Principles. (Part I: Basic notions and applications to reflection principles.)*

10:30-11:20 Michael Rathjen, *Well-ordering principles, omega models and beta models.*

Coffee break

11:50-12:40 Lorenzo Carlucci, *Three results related to the Paris-Harrington principle.*

Lunch break

15:00-15:50 Aleksander Iwanow, Barbara Majcher-Iwanow, *Polish G -spaces similar to logic G -spaces of continuous structures.*

16:00-16:50 Richard Kaye, *Counting external sets in models of arithmetic.*

Coffee break

17:20-18:10 Zofia Adamowicz, *Small initial segments and consistency.*

Tuesday, July 24th

9:30-10:20 F. Félix Lara-Martn, *On Local Induction and Collection Principles. (Part II: Inference rules and applications to parameter free induction.)*

10:30-11:20 James Schmerl, *Automorphism Groups of models of Peano Arithmetic.*

Coffee break

11:50-12:40 Paola D'Aquino, *Recursively saturated real closed fields.*

Lunch break

14:30 -16:00 Ali Enayat and Albert Visser, *Full Satisfaction Classes in a General Setting (Tutorial).*

16:10-16:40 Zenon Sadowski, *Characterizing the Existence of Optimal Proof Systems and Complete Sets for Promise Classes.*

Coffee break

17:00-18:30 *Memorial session for Henryk Kotlarski and Zygmunt Ratajczyk.*

Wednesday, July 25th

9:30-10:20 Ali Enayat, *Self-embeddings of Models of Arithmetic, Redux.*

10:30-11:20 Tin Lok Wong, *Axiom schema for a model of arithmetic with a cut.*

Coffee break

11:50-12:40 Andrey Bovykin, *New unprovability results in the Infinite-Dimensional Ramsey Theory and other recent developments in metamathematics.*

Lunch break

15:00-15:50 Albert Visser, *Local cut-interpretability in PA^- .*

16:00-16:50 Sam Sanders, *Nonstandard Analysis: a New Way to Compute.*

Coffee break

17:20-18:10 Leszek Kołodziejczyk, *A pesky fragment of bounded arithmetic.*

Thursday, July 26th

9:30-10:20 Denis Richard, *p -destinies for theory with 2 and 3 quantifiers.*

10:30-11:20 Andreas Weiermann, *Some applications of alpha large sets.*

Coffee break

11:50-12:40 Marcin Mostowski, *Truth in the limit.*

Lunch and excursion

Friday, July 27th

9:30-10:20 Konrad Zdanowski, *The strength of Ramsey theorem for coloring ω -large sets. (joint work with Lorenzo Carlucci)*

10:30-11:20 Ermeke Nurkhaidarov, *Properties of automorphisms of saturated models of arithmetic.*

Coffee break

11:50-12:40 Roman Kossak, *Automorphisms and cofinal extensions.*

Lunch break

15:00-16:00 *Contributed papers session*

Coffee break

16:30-17:20 Matthew Morrow *Number Theory, K-theory and their interaction*