SCIENTIFIC REPORT

Conference

3Quantum: Algebra, Geometry, Information

10 – 14 July 2012

Tallinn University of Technology (TUT), Tallinn, Estonia

Summary

This conference was a satellite meeting to the 6th European Congress of Mathematics (Krakow, 2 – 7 July 2012), to the 12th International Congress of Mathematical Physics (Aalborg, 6 – 11 August 2012), and to the 13th Marcel Grossmann Meeting on General Relativity (Stockholm, 1 – 7 July 2012).

The organizers of the conference were E. Paal (Tallinn), A. Fialowski (Budapest), J. Fuchs (Karlstad), M. Schlichenmaier (Luxembourg), V. Shumeiko (Göteborg), and A. Stolin (Göteborg).

The conference was attended by 98 participants from 27 countries.

The topics of the conference were covered by 20 plenary talks and by 57 talks in parallel sessions. The precise schedule, as well as abstracts of all talks, are provided at the web page `www.agmp.eu/qqq/pro.php`.

Scientific content

The basic purpose of the conference was to stimulate and promote interactions between three major research areas: (quantum) algebra, (quantum) geometry, and (quantum) information theory. These areas were covered by the plenary talks and parallel sessions.

As an illustration, here is a collection of some of the specific topics that were discussed at the meeting:

— deformation of algebraic structures, deformation quantization

— integrable systems and quantum inverse scattering

— categorification of three-manifold invariants

— the relation between Turaev-Viro invariants and Reshetikhin-Turaev invariants

— motivic Donaldson-Thomas invariants

— the Grothendieck-Teichmüller group

— higher categorical structures, algebras in categories, module categories

— Hopf algebras, weak Hopf Hopf algebras, quantum groups

— co-Frobenius Hopf algebras
— differential graded Lie algebras
— Jordan algebras
— operads, permutads and shuffles
— the cohomology of flag varieties
— modular forms and mock modular forms in string theory
— characteristic properties of quantum channels in information theory
— the quantum marginal problem in quantum information
— gauge fields on noncommutative manifolds
— geometric and topological aspects of quantum gravity

Results and impact

In the opinion of the organizers, the conference achieved its scopes very well. The plenary talks, as well as most of the talks at the parallel sessions, were of high quality and informative, and there was a lot of interaction between the speakers and their audience. A major virtue of this meeting was to allow the participants to get in touch with a large number of recent results and developments, some of which presumably had so far been absorbed only in much smaller communities. In the meeting these advances could be presented to a broad audience with a variety of mathematical provenance. As a result, one can be confident that the meeting made a considerable impact on several of the fields that were represented in the talks, and in particular on the interdisciplinary areas connecting them.

There was also room for discussions on a more personal level, which was greatly appreciated by the participants. One way in which this was promoted was by having pre-arranged lunches and dinners, for all participants, in close proximity with the conference lecture hall. Indeed many talks led to lively discussions, both concerning issues within the individual presentations and the connection with other developments. As an illustration, the use of Gray 3-categories as a tool for the study of three-dimensional quantum gravity that was advocated in Barrett's talk gave rise to a more general discussion on the virtues and drawbacks of different versions of weakness for 3-categories.

As compared with similar previous conferences that had already brought together some of the participants, a particular feature of the present meeting was to include quantum information as a key topic. While it is still too early to make definite statements about the transfer of knowledge from this area to other fields that were represented at the meeting, at the very least it can be stated that the speakers, who are among the leading specialists in their field, made a great effort to present their results in a form well accessible to non-experts. As a consequence, each of the four talks on this subject was much appreciated by the physicists and mathematicians in the audience alike.

Budget

The budget can be summarized as follows:

Expenditures	*Total*		*ITGP grant*	
Travel invited speakers	6 600	EUR	985	EUR
Accommodation invited speakers	5 300	”	4 195	”
Meals (coffee, $4 \times$ lunch, $3 \times$ dinner)	6 900	”	5 620	”
Materials, printing	1 600	”	—	
Secretary / administration TUT	2 100	”	1 000	”
Rent lecture halls	2 000	”	—	
Administration Karlstad University	500	”	—	
Proceedings	5 500	”	—	
Income:	*Total*		*ITGP grant*	
ITGP	11 800	EUR	11 800	EUR
Swedish Research Council	5 700	”		
TUT Faculty of Science	5 000	”		
Registration fees	8 000	”		
Total expenditures / Total income	**30 500**	**EUR**		

APPENDIX

Meeting Programme

There were 20 plenary talks, each of of 45 minutes length. These were complemented by 8 parallel sessions on special topics, having on the average 7 speakers (the minimum number was 4, the maximum 11). At each time, either three or four of theses sessions were running in parallel. The duration of session talks was 30 minutes, except for a few longer talks in the session *Geometry and topology in quantum gravity*, like the one by John Barrett. For more details, in particular the precise schedule, see the conference web site.

List of speakers

The plenary speakers were:

1. Anna Beliakova (Zürich)
2. Matthias Christandl (Zürich)
3. Nilanjana Datta (Cambridge)
4. Tony Dorlas (Dublin)
5. Ludwig Faddeev (St. Petersburg)
6. Alice Fialowski (Budapest)
7. Alexander Kirillov (Stony Brook)
8. Petr Kulish (St. Petersburg)
9. Giovanni Landi (Treste)
10. Andrey Lazarev (Leicester)
11. Olav Arnfinn Laudal (Oslo)
12. Yuri Manin (Bonn)
13. Sergei Merkulov (Stockholm)
14. Bengt Nilsson (Göteborg)
15. María Ronco (Talca, Chile)
16. Yan Soibelman (Kansas State)
17. Vitaly Tarasov (Indiana)
18. Stefan Waldmann (Erlangen, Germany))
19. Sara Westreich (Ramat-Gan, Israel)
20. Andreas Winter (Bristol)

The speakers at the parallel sessions were:

Session *Deformation theory and quantization* (coordinated by A. Siqveland):

1. Dmitry Artamonov (Moscow)
2. Martin Bernd (Cottbus)
3. Thomas Ernst (Uppsala)
4. Valentina Golubeva (Moscow)
5. Olav Gravir Imenes (Drammen, Norway)
6. Arvid Siqveland (Kongsberg, Norway)
7. Sergiu Vacaru (Iasi, Romania)

Session *Geometry and topology in quantum gravity* (coordinated by C. Meusburger):

8. Benjamin Bahr (Cambridge)
9. Angel Ballesteros (Burgos, Spain)
10. Fernando Barbero Madrid)
11. John Barrett (Nottingham)
12. João Faria Martins (Caparica, Portugal)
13. Kirill Krasnov (Nottingham)
14. Gregor Schaumann (Erlangen)
15. Marko Vojinovic (Belgrade)
16. Derek Wise (Erlangen)

Session *Gravitation and cosmology* (coordinated by P. Kuusk):

17. Alexander Burinskii (Moscow)
18. Dmitry Galtsov (Moscow)
19. Stefan Groote (Tartu, Estonia)
20. Manuel Hohmann (Hamburg)
21. Laur Järv (Tartu)
22. Rein-Karl Loide (Tallinn)
23. Zbigniew Oziewicz (Izcalli, Mexico)
24. Erik Randla (Tartu)
25. Margus Saal (Tartu)
26. Hardi Veermäe (Tartu)

Session *Hopf algebra, quantum algebra* (coordinated by S. Caenepeel):

27. Nadezhda Bazunova (Tallinn)
28. Olga Bershtein (Tallinn)
29. Daniel Bulacu (Bucharest)
30. Stefaan Caenepeel (Brussels)
31. Juan Cuadra (Almeria, Spain)

32. Fernando Fantino (Paris)
33. Ramon Gonzalez Rodriguez (Vigo, Spain)
34. Stefan Kolb (Newcastle)
35. Martin Mombelli (Hamburg)
36. Carl Stigner (Karlstad, Sweden)
37. Yinhuo Zhang (Hasselt, Belgium)

Session *Jordan structures in mathematical physics* (coordinated by R. Iordanescu):

38. Leron Borsten (London)
39. Guowu Meng (Hongkong)
40. Sophie Morier-Genoud (Paris)
41. Guy Roos (St. Petersburg)

Session *Modularity and non-perturbative strings* (coord. by B. Nilsson & D. Persson):

42. Sergei Alexandrov (Montpellier)
43. Axel Kleinschmidt (Potsdam, Germany)
44. Samuel Monnier (Paris)
45. Sameer Murthy (Amsterdam)
46. Roberto Volpato (Potsdam, Germany)

Session *Nonassociative and Hom-algebras* (coord. by A. Makhlouf & S. Silvestrov):

47. Thomas Ernst (Uppsala)
48. Alar Leibak (Tallinn)
49. Abdenacer Makhlouf (Mulhouse, France)
50. Johan Richter (Lund, Sweden)
51. Sergei Silvestrov (Västerås, Sweden)
52. Gregory Wene (San Antonio, US)

Session *Operads, noncommutative algebra and geometry* (coordinated by N. Iyudu):

53. Askar Dzhumadildaev (Almaty, Kazakhstan)
54. Natalia Iyudu (Belfast)
55. Dmitri Piontkovski (Moscow)
56. Pasha Zusmanovich (Tallinn)
57. Aleksy Tralle (Olsztyn, Poland)

Description of plenary talks

Here is a short description of most of the plenary talks, in some cases with a mentioning of related session talks. More detailed abstracts of all talks are provided at the web page `www.agmp.eu/qqq/pro.php`. In the list the talks are grouped under specific subject headers, but it should be appreciated that, in accordance with the interdisciplinary nature of the meeting, quite a few of them could have equally well be listed under a different heading.

Quantum groups and integrable systems

Ludwig **Faddeev** described aspects of the representation theory of the so-called modular double of the quantum group $\mathrm{SL}(2,R)_q$. The focus was on the decomposition of the tensor product of two representations from the continuous series.

Petr **Kulish** gave an overview of representation theoretic applications of integrable spin systems that can be obtained by the quantum inverse scattering method. A specific subclass gives rise to 'non-quasiclassical' quantum groups, which are related to special R-matrices of Hecke type.

Vitaly **Tarasov** talked about geometric and algebraic structures arising in the study of Yangian symmetries. He showed how to identify the equivariant cohomology of the cotangent bundle of a partial flag variety with the image of the Gelfand-Zetlin subalgebra of the Yangian $Y(gl_N)$ acting on a space of vector-valued polynomials. He also conjectured a description of the small quantum equivariant cohomology algebra of the cotangent bundle of a partial flag variety as the image of the Bethe subalgebra of this Yangian.

Algebraic structures

Andrey **Lazarev** described formal-geometric and operadic structures governing the Maurer-Cartan equation, or master equation in physics terminology, and its solutions. The Maurer-Cartan equation for odd elements in a differential graded Lie algebra is ubiquitous in e.g. differential and algebraic geometry, deformation theory, rational homotopy theory, and topological conformal field theory.

María **Ronco** talked about joint work with the recently deceased Jean-Louis Loday on shuffle products and permutads. A permutad is an algebra over the monad that is formed by the surjective maps with substitution, and can be presented with partial operations that are labeled by shuffles. Ronco showed that the minimal model for the permutad is given by permutahedra, in an analogous way as the minimal model for the non-symmetric operad of associative algebras is given by associahedra.

Sara **Westreich** discussed how one can extend the notion of various group-theoretical concepts, like characters, conjugacy classes, class sums and character tables, from group algebras to general Hopf algebras. She also presented several applications, e.g. an integrality property for the decomposition of the product of two class sums of a semisimple quasitriangular Hopf algebra, and the characterization of normal left coideal subalgebras of a semisimple Hopf algebra via their character table. A related session talk by Juan **Cuadra** described the relation between Haar measures and finiteness conditions on Hopf

algebras, presenting e.g. a proof of the conjecture by Andruskiewitsch and Dascalescu that the coradical filtration of a co-Frobenius Hopf algebra is finite. Further session talks with close connections to Westreich's presentation were those by Martin **Mombelli** on a tensor product of bimodule categories over Hopf algebras and by Carl **Stigner** on an application of Hopf algebra characters in conformal field theory.

Topological field theory

Anna **Beliakova** reported on efforts (joint with K. Habiro) towards categorifying a unified invariant that provides a generating function for the Reshetikhin-Turaev invariants of any homology three-sphere at all roots of unity. As a crucial step towards categorification, they have constructed a bicomplex whose Euler characteristic is the ribbon element of quantum sl(2). The categorification theme was also taken up in the session talk by João **Faria Martins**.

Alexander **Kirillov** discussed his work with B. Balsam, as well as the alternative approach by Turaev and Virelizier, which establish the equality of the Turaev-Viro invariant based on a spherical tensor category $\mathcal{C}$ and the Reshetikhin-Turaev invariant based on its Drinfeld center $\mathcal{ZC}$. An important aspect was the connection between these results and aspects of extended topological field theories in the sense of Lurie. The latter is also of interest for understanding topological field theories that allow for manifolds containing defects, and thereby related to the topics of the session talks by John **Barrett** on the use of Gray 3-category diagrams in the study of three-dimensional quantum gravity, and by Gregor **Schaumann** on traces on module categories over monoidal categories.

At this place it also fits to mention the talk by Sergei **Merkulov**, who in his introduction summarized how the Grothendieck-Teichmüller group arises in the theory of moduli spaces of algebraic curves, as well as in deformation quantization and in other areas, He reported how, using recent results by T. Willwacher, one can show that the Grothendieck-Teichmüller group acts non-trivially on Poisson cohomologies of Poisson manifolds.

Quantum physics and quantum information

Matthias **Christandl** described the so-called quantum marginal problem – the question when a set of local density matrices describing states of a quantum system arises from a joint global state. This issue plays a role for many aspects of quantum theory, ranging from quantum chemistry to quantum information theory. He presented some remarkable relations of the quantum marginal problem with the representation theory of symmetric groups as well as with the P-vs-NP conjecture of computational complexity.

Nilanjana **Datta** and Andreas **Winter** discussed properties of quantum channels in information theory. Two characteristic quantities of a quantum channel are its capacity – the maximum rate at which information can be reliably transmitted – and its strong converse rate – the rate threshold above which information transmission fails with certainty. A channel for which these two quantities are equal is said to have the strong converse property. Datta showed how to decide whether a quantum channel possesses this property, giving examples both for channels which do and for channels which don't. Winter

discussed relations with strong sub-additivity, a fundamental inequality satisfied by the quantum mechanical entropy function.

The presentation by Yuri **Manin** formed a bridge between this group of talks and those on algebra as well as the ones on quantum field theory. He discussed graphs as a universal tool providing a combinatorial skeleton for computations in theoretical computer science, quantum physics and various operadic formalisms in abstract algebra. As an application, he argued that the renormalization programme of quantum field theory has a counterpart in treating infinities that arise in classical computation theory.

String theory and quantum field theory

Bengt **Nilsson** started with an overview over problems and successes of quantum field theory. This was followed by a discussion of recent developments in the study of superconformal Chern-Simons-matter theories, which e.g. in string theory arise in connection with M2-branes. He emphasized the role of duality relations for the understanding of these theories.

Giovanni **Landi** discussed the geometrical structure of gauge fields on noncommutative manifolds, describing the latter, as well as vector bundles over them, as natural deformations of their classical counterparts.

Stefan **Waldmann** reported on recent progress, made in a Fréchet-algebraic context, which allows one to understand the passage between two different approaches to deformation quantization – the framework of formal calculus and the C*-algebraic setting, respectively. He also discussed implications for several specific examples of deformation quantization.

Yan **Soibelman** gave a review of his joint work with M. Kontsevich on motivic Donaldson-Thomas invariants and described applications of the theory of Donaldson-Thomas invariants to cluster algebras, to integrable systems of Hitchin type and to wall-crossing formulas for moduli spaces. Such wall-crossing formulas have close connections with number theory, and in particular with mock modular forms, which e.g. arise when counting the number of quarter-BPS dyonic states in superstring theory. A structure theorem for mock modular forms, obtained in work with A. Dabholkar and D. Zagier, was presented in the session talk by Sameer **Murthy**.

Country list

The participants came from 27 different countries (abbreviations as used by the ESF):

AE	1	IE	2	RO	3
BE	2	IL	1	RS	1
CH	3	IT	1	RU	8
CL	1	KZ	1	SE	10
DE	12	LU	1	SG	1
EE	15	MX	1	UK	9
ES	4	NL	1	US	5
FR	6	NO	3		
HK	1	PL	3		
HU	1	PT	1		

Thus 67 of the 98 participants, and 11 of the 20 plenary speakers, came from countries which participate in the ITGP network.