INFTY @ ESSLLI 2012

Scientific and financial report

Applicants. Prof. Dr. Benedikt Löwe
Universiteit van Amsterdam

Prof. Dr. Andreas Weiermann
Universiteit Gent

As a continuation of the INFTY efforts to be involved in the *European Summer School for Logic, Language and Information*, we organized two courses at ESSLLI 2012 in Poland (following the INFTY activities in Ljubljana in 2011).

We see these events as an important step towards re-injecting mathematical logic into the programme of the ESSLLI schools: In recent years, ESSLLI has seen a shift from foundational studies to less mathematical courses and workshops. Several ESSLLIs had so little mathematical content that it was not useful for PhD students in mathematical logic to attend this otherwise very important networking event. It is in the interest of the mathematical logic community that these summer schools remain interesting for students working in the foundations of mathematics; furthermore, it is of interest for us to make sure that mathematical logic remains an important part of the community represented at ESSLLI.

INFTY@ESSLLI2012 was part of the *Alan Turing Year* and was INFTY's contribution to the worldwide celebrations of Alan Turing. Dr Robert Lubarsky and Professor Larry Moss taught two courses at ESSLLI 2012, entitled *Models of Computation* and *Circularity*. In the appendices of this report, we give the official course descriptions from the ESSLLI 2012 website.

Models of Computation. This course was an introduction to the theory of computability. While it could function as such an introduction for mathematicians, at the same time the topics were chosen to serve the needs of philosophers and linguists.

The first lecture described Turing machines (after a brief biography of Turing, in honor of Alan Turing Year), the Church-Turing thesis, universal machines, the unsolvability of the halting problem and other natural questions, and the distinctions between extensionality and intensionality and between computable and computably enumerable, among other topics. The second day was dedicated to machine-based proofs of Gödel's Incompleteness Theorems. The third discussed other models of computation, mostly ones equivalent with Turing's (such as Herbrand-Gödel systems of equations, Kleene recursion, Church's lambda-calculus), which were compared with various programming paradigms (imperative, declarative, functional), and also one that isn't (BSS computability). For the fourth talk, I went through restricted notions of computation: primitive recursion, ordinals and the various function hierarchies (fast-growing, slow-growing), ε_0 and other computable ordinals, automata, and time and space bounds, including the P vs. NP problem. The last day was about extended notions of computation, including oracle computability and iterated jumps, both finite and transfinite, infinite time and space Turing machines, ordinal register machines, recursion in higher types and E- (i.e. set-) recursion, and α-computability theory.

Lubarsky tied the course together by having some themes appear several times, for reinforcement. For instance, ordinals, likely not well understood the first time somebody sees them, were introduced to help analyze restricted computability, and then used to next to to support extended computability. Also, register machines, first mentioned on day 3 as another model of Turing-like computation, were used for very different purposes later that day for BSS-computability, and on the last day for extended computability in the form of ordinal register machines.

Student interest was strong: about 50 to 70 students every day except for the last when the number dipped below 50. Lubarsky had conversations with participants and handed out homework-like questions provided on handouts. The attendees included both students, young researchers and even a senior colleague.

It is quite clear how the topics of the course relate to the *computational aspects of infinity* that are part of the focus of the INFTY network.

Circularity. Larry Moss's class was very well-attended throughout (roughly 50-75 participants) and received very positively (one student told the teacher that she "wanted to live in the course forever because it touched on topics she always was interested in but never saw in any depth"). In addition to the course itself, there was a growing informal lunch discussion group with the teacher of exceptionally interested students (about 10). The lectures can be found at

`http://www.indiana.edu/~iulg/moss/ESSLLI2012`

The course supported research on aspects of infinity in Europe due to the topics. The second lecture mentioned the ZFC axioms in some detail, and it was aimed partly at clarifying the Foundation Axiom vs. the Anti-Foundation Axiom. That lecture was concerned with the Hypergame Paradox, a paradox related to the Burali-Forti Paradox. A later lecture covered self-writing computer programs (related to the Kleene (second) Recursion Theorem). The last two lectures were on algebra and coalgebra, presented as a way to unify the phenomena of the previous days. All in all, the mathematical level of the course was pretty high, certainly for ESSLLI.

Acknowledgement of funding. The funding of INFTY was clearly acknowledged on the website and in the programme of ESSLLI 2012.

Financial Aspects. We covered the travel and accommodation expenses of the two lecturers.

	Lubarsky	Moss	*Total*
Registration:	EUR 0	EUR 0	**EUR 0**
Travel:	EUR 1678.64	EUR 771.15	**EUR 2449.79**
Accommodation:	EUR 719.41	EUR 438.41	**EUR 1157.82**
Total	**EUR 2398.05**	**EUR 1209.56**	**EUR 3607.61**

Appendix 1.

Models of Computation
Teacher: Dr. Robert Lubarsky
Week 1, 14:00-15:30, Aula B
Area: Logic and computation
Type: Foundational
http://www.esslli2012.pl/index.php?id=97

Abstract:

This foundational course teaches the basics of the mathematical theory of computation to students from all backgrounds. It links the theory of computation to Gödel's incompleteness phenomenon, and discusses alternative models of computation, including infinitary computation. The mathematical theory of computation goes back to Alan Turing (1912-1954) whose centenary we celebrate in 2012.

Content of the course:

Computability is fundamental to all of the areas covered by ESSLLI. It is what computer science is all about: the study of computation. In philosophy, it is of central importance to the mind-body problem and the nature of consciousness. Are our brains just computers, and are we ultimately just machines? In order to address these issues, one needs a sense of what computability is, and of its possible variants. A particular aspect of this is language. As a fairly well defined area of mental activity, it is easier to study what is needed for language, mechanical or not, and to examine human, living non-human, and computer linguistic abilities. All of this needs a foundation of computability theory.

Tentative outline

Lecture 1. Alan Turing, notion of an algorithm, relations to Hilbert problems and decision procedures for logic, Turing machines, halting problem, undecidability and nonexistence of algorithms.

Lecture 2. Gödel's first incompleteness theorem as a corollary of the undecidability of the halting problem.

Lecture 3. Other models of computation: register machines, recursion, lambda calculus; the Church-Turing thesis.

Lecture 4. More restricted notions of computation (primitive recursive, elementary, possibly time/space constraints on machines).

Lecture 5. Hypercomputation, computing the halting problem using an infinite amount of time, infinite time Turing machines, infinite time register machines.

Appendix 2

Circularity
Teacher: Prof. Dr. Larry Moss
Week 2, 11:00-12:30, held in Aula A
Area: Logic and computation
Level: Introductory
`http://esslli2012.pl/index.php?id=113`

Content of the course:

This course will cover a number of LLI topics having to do with circularity. On the theoretical side, it will present the basics of non- wellfounded sets, modal logic, and self-replicating computer programs. Computer science applications will be taken from streams and corecursion; linguistics/AI applications could come from modeling the semantic paradoxes and also common knowledge.

The course will give students experience with concepts like bisimulation and coinduction. This will strengthen their understanding of ordinary induction. The course will also open people up to category theory in a gentle way. In work on circularity and logics of knowledge, the key ideas of modal logic and of the game-theoretic semantics of logic will come up. On a different pedagogic note, students will see what it is to do LLI work which both has mathematical content and leads to interesting applications.

As a feature of my proposal, I will tie into computablity theory with one lecture on textual self-reference, including self-reproducing register machines, the Kleene Recursion Theorem, and connections to (of all things) computer viruses. This lecture will be 'interactive' in the sense that students will learn enough to write their own self-reproducing programs in a simple programming language.

The overall idea is to provide background and stimulation to people who are not specialists in any area of mathematics but rather are typical members of the set of ESSLLI students with enough math background to follow a series of technical lectures.

Tentative outline

Lecture 1 would be based on `http://cs.indiana.edu/cmcs/circintro.pdf`, covering general circular phenomena, especially ones coming from streams and trees.

Lecture 2 would be from `http://cs.indiana.edu/cmcs/categories.pdf`, covering the basics of category theory. I would probably do less of the mathematics than is in that lecture, and instead I would start in on non-wellfounded sets and the treatments of the Liar Paradox that come from that.

Lecture 3 would be on non-wellfounded sets themselves.

Lecture 4 would be the computability theory topic that I mentioned above, based on `http://www.indiana.edu/~iulg/trm/self.shtml.`

Lecture 5 could consider fractals, universal type spaces from game theory, or other topics.

Sources

The course will be based in part on the following sources:

1. "Vicious Circles" by Jon Barwise and Lawrence S. Moss.

2. Non-wellfounded Set Theory:

`http://plato.stanford.edu/entries/nonwellfounded-set-theory/`

3. 1#: a Text Register Machine Introduction to Computability Theory:

`http://www.indiana.edu/~iulg/trm`

4. Lecture Notes from my previous classes on this topic (see below), and also the notes from Dirk Pattinson's Introduction to Coalgebra class at NASSLLI 2003.