

FINAL REPORT FOR THE CONFERENCE: “TRENDS IN SET THEORY”

1. SUMMARY

The conference took place in the days 8-11 of July, 2012 at the Banach Center of Mathematical Institute of the Polish Academy of Sciences in Warsaw, Poland and was focused on the interactions between set theory and other parts of mathematics, from Banach spaces and C^* -algebras to topological groups, sets of the reals, general topological spaces, and others. This included both the advances concerning appropriate set-theoretic tools from forcing theory, Ramsey theory, descriptive set theory or other branches of set theory as well as concrete applications of these methods in the mathematical practice.

The conference had the official status of a satellite event to the 6th European Congress of Mathematicians held in Kraków, Poland in the days 1-7 of July 2012. Three leading set-theorists A. Kechris, S. Shelah, S. Todorcevic were among the main speakers of the Congress in Kraków and agreed to come to our conference. This attracted many other mathematicians both to our conference and the Congress. We financed the stay of 18 invited speakers which gave longer presentations and 18 younger mathematicians (graduate students and postdocs). We had total of 44 presentations and one problem session. Besides the contribution from European Science Foundation research networking programme INFTY - New frontiers of infinity: mathematical, philosophical and computational prospects we obtained financial help from the Mathematical Institute of the Polish Academy of Sciences and from the University of Wrocław.

The following mathematicians were our invited speakers A. Aviles (Murcia), J. Brendle (Kobe), M. Dzamonja (UEA Norwich), M. Elekes (Renyi Institute), S. Geschke (Hausdorff Center), M. Hrusak (UNAM Morelia), A. Kechris (Caltech), W. Kubis (AVCR and UJK Kielce), J. Lopez-Abad (ICMAT Madrid), S. Shelah (Jerusalem and Rutgers), S. Solecki (Urbana-Champaign), L. Nguyen Van The; (Aix-Marseille), S. Thomas (Rutgers), S. Todorcevic (CNRS and Toronto), T. Tsankov; (Paris 7), B. Velckovic (Paris 7), P. Zakrzewski (Warszawa), J. Zapletal (AVCR and Gainesville).

2. DESCRIPTION OF THE SCIENTIFIC CONTENT OF AND DISCUSSION AT THE EVENT

2.1. Set Theory of the Real Line. The real line is the place where the problems of modern set-theoretic nature were first detected at the end of 19th century. The real lines continues to provide inspiration for a combinatorial and set-theoretic descriptive insight. The research concerning the Lebesgue measure the Baire category, special subsets of the reals continues to be an active part of set theory and interacts with newer research directions.

Many of the talks at our conference concerned the real line and the classical set-theoretic problems in a pure form or in unexpected contexts. One could include here

talks of M. Balcerzak, S. Głąb, W. Wohofsky, P. Zakrzewski, M. Elekes. Nowadays many results involve forcing as it turned out the previous decades that many basic statements concerning the basic concepts related to the real line are undecidable on the basis of the usual axioms of set theory. So, increasingly, these results interact with the theory of forcing.

2.2. Descriptive Set Theory in Polish Groups. Many participants represented this very important trend in set-theoretic research conducted in the recent years. A. Kechris and S. Solecki who participated in the conference are classics of the subject and interacted with many other participants interested in this direction of set-theoretic applications. Also T. Tsankov, D. Bartosova or L. Nguyen Van The presented ingenious ideas involving descriptive set-theory in the context of Polish topologies which met an enthusiastic reception. Polish topologies appear at a very fundamental level in many structures in functional analysis, ergodic theory, dynamical systems and others. Applying there the tools of descriptive set theory allows us to progress, among others, in the classification of the structures and relations among them.

2.3. Combinatorial Set Theory in Banach Spaces. One of the most recent trends in applications of set-theory concerns the theory of Banach spaces. We had the pleasure to interact with major researchers of this line S. Todorcevic, J. Lopez-Abad and A. Aviles. Their talks concerned their recent results on biorthogonal systems, relations between gaps and projections and the Banach-Sacks property.

A special role in the theory of Banach spaces is played by the classical spaces of continuous functions on a compact Hausdorff space with the supremum norm denoted $C(K)$ in what follows. All Banach spaces are isometrically subspaces of the spaces of the form $C(K)$, but more importantly, quite often, the natural examples and counterexamples are of the form $C(K)$, especially on the nonseparable level. Of course, also spaces like $C([0, 1])$, $c_0 \sim C([0, \omega])$ or $C([0, \alpha])$ play fundamental role in the general theory (\sim will denote an isomorphism between Banach spaces and \equiv will denote an isometry). The above connection forms a foundation of another basic link, besides the weak* topology of the dual ball, between set-theoretic topological structure (of a nonmetrizable K) and the Banach space $C(K)$. This was seen in the presentation of J. Meri, T. Kania, M. Cuth, W. Kubiś.

Quite a natural classical Banach space ℓ_∞/c_0 can be interpreted as induced by even more natural combinatorial structure of all subsets of \mathbb{N} modulo finite sets. Indeed, the Stone space of the Boolean algebra $\wp(\mathbb{N})/Fin$ is canonically homeomorphic to $\mathbb{N}^* = \beta\mathbb{N} \setminus \mathbb{N}$ and $C(\mathbb{N}^*)$ is canonically isometric to ℓ_∞/c_0 . Alternatively one can see the finite linear combinations of subsets of \mathbb{N} modulo finite sets as a dense subset of ℓ_∞/c_0 .

The structure of the Boolean algebra $\wp(\mathbb{N})/Fin$ or the compact space \mathbb{N}^* is very sensitive to the choice of additional combinatorial axioms (see [?]). The existence of points in \mathbb{N}^* of a given character, the possibilities of extending bounded continuous functions on $\mathbb{N}^* \setminus \{x\}$ to \mathbb{N}^* for $x \in \mathbb{N}^*$, the class of continuous images of \mathbb{N}^* or its autohomeomorphisms, all have basic properties which change drastically when we change the allowed infinitary combinatorics (e.g. assume CH or Martin's axiom and the negation of CH, Proper forcing axiom, or work in a few canonical models of set-theoretic universe). However, it is unclear to what extent this dependence is transferred to the structure of Banach space ℓ_∞/c_0 which is not only interesting

as a classical, natural object but plays structural roles, for example, is a universal space for Banach spaces of density $\leq 2^\omega$ under the assumption of CH. The talks of M. Dzamonja and M. Krupski concerned these problems particularly.

2.4. Set-theoretic Topology. Some of the participants of the conference decided to devote their presentations to set theoretic topology. This included D. Soukup, A. Farhat, D. Guerrero-Sanchez, M. Hrusak, E. Tachtsis, Błaszczyk and Banach. Set-theoretic topology was a leading field of applications of set-theory in the 70ties or the 80ties of the previous century. Many of the participants which today focus on applications in other fields of mathematics made their contributions to set-theoretic topology several decades ago. This makes the set-theoretical topological arguments in a sense paradigmatic for other applications. That is why the above talks attracted attention of everyone.

2.5. Algebra an Set Theory. S. Thomas had a very inspiring talk on set theoretic methods in unitary group representation. It was done from the descriptive set theory point of view. A. Bartoszewicz talked about other applications of set theory in algebra.

2.6. Developing Set-theoretic Tools. While doing applications of set theory we should not forget about developing new set-theoretic tools. New applications require new combinatorial methods. So, it is not surprising that several of the presentations included mainly what one could call pure set theory. The participants which presented them talked about very canonical tools which were tested in applications many times. They included ultrafilters of subsets of \mathbb{N} , Ramsey theory, the structure $\wp(\mathbb{N})/Fin$ and the constructible universe. Example of such talks were talks of J. Brendle, T. Weinert, D. Chodounsky, U. Khomsky, Z. Vidnyanszky, J. Flaskova.

2.7. Problem session. Of course such a big conference is one big problem session. However most discussions take place in small groups or just between two mathematicians. During the problem session we had a chance to see in public some of the open problems of interest to the participants. They included:

Problem 2.1 (T. Banach). *An ultrafilter u is called discrete if and only if for every $f : \omega \rightarrow \mathbb{R}$ there is an element $a \in u$ such that $f[a]$ is discrete.*

$A \subseteq \mathbb{Z}$ is called asymptotically discrete in \mathbb{Z} if $A = \{a_n : n \in \mathbb{N}\}$ in an increasing order and

$$\lim_{n \rightarrow \infty} (a_{n+1} - a_n) = \infty.$$

An ultrafilter u is called asymptotically discrete if and only if for every injective $f : \omega \rightarrow \mathbb{Z}$ there is an element $a \in u$ such that $f[a]$ is asymptotically discrete in \mathbb{Z} . Are there asymptotically discrete ultrafilters in ZFC?

Problem 2.2 (S. Shelah). *A MAD family $\mathcal{A} \subseteq [\omega]^{\aleph_2}$ is saturated if for every $A \subseteq \omega$ which is not included in a finite union of elements of \mathcal{A} , we have that A intersect 2^{\aleph_0} members of \mathcal{A} . Is there such a family in ZFC?*

Problem 2.3 (I. Juhasz). *Is there a ZFC example of a noncompact space X such that every infinite discrete subset of X has a complete accumulation point?*

Problem 2.4 (I. Juhasz). *Let P be a property of topological spaces. A space X is called P bounded if for every $Y \subseteq X$ with property P the closure of Y is compact. Is there a Tichonov, Lindelöf bounded but not c.c.c. bounded space?*

Problem 2.5 (A. Farhat). *Let X be as in A. Farhat's talk. Can X be constructed from Aronszajn tree and be hereditarily Lindelöf?*

Problem 2.6 (P. Koszmider). *Is it consistent that in every nonmetrizable compact space there is a nonmetrizable compact totally disconnected space?*

3. ASSESSMENT OF THE RESULTS AND IMPACT OF THE EVENT ON THE FUTURE DIRECTION OF THE FIELD

The idea of the conference as focused on applications of set theory in diverse mathematical disciplines was quite original and created a new environment for interactions and the exchange of ideas. We had two leading combinatorial set-theorists Saharon Shelah and Stevo Todorovic and a leading Descriptive set-theorist Alexander Kechris. Internationally leading mathematicians also included S. Solecki. S. Thomas On the other hand half of the participants were young mathematicians and the remaining participants were somewhere in between. This created quite a dynamic interaction which must inspire many young participants. We also hope that the lesson from this conference that can be learned by the young mathematicians is that applying set theory in other fields of mathematics is at least as exciting as creating new set theory. Of course many discussions from the conference will result in publications and new scientific results. Because of this diverse gathering which on the other hand spoke the same language of set theory we expect a deep impact of the event.

4. FINAL PROGRAMME OF THE MEETING

- Saturday, 7th of July 2012
 - Check-in at the hotels, guest rooms.
 - 7 - 9 pm: Pick up of the conference materials, at the tables in front of restaurant Lanse around the corner from the MDM hotel. We offer a free welcome drink (beer, wine, water, juice).
- Sunday, 8th of July 2012
 - 8.45-9.00 pick-up of the conference materials
 - Morning session
 - * 9.00-9.45 S. Shelah; Null ideal (random forcing) for inaccessible exist?
 - * 9.45-10.00 break
 - * 10.00-10.30 J. Zapletal; The free set property for calibrated ideals
 - * 10.30-11.00 break
 - * J. Brendle; Recent results on splitting and disjointness
 - * 11.30-11.45 break
 - * 11.45-12.00 T. Weinert; Idiosyncromatic Poetry
 - * 12.00-12.05 break
 - * 12.05-12.20 W. Wohofsky; Variants of the Borel Conjecture and Sacks dense ideals
 - * 12.20-12.25 break

- * 12.25-12.40 M. Doucha; Borel equivalence relations and Laver forcing
- * 12.40-12.45 break
- * 12.45-13.00 D. Chodounsky; Grigorieff forcing and automorphisms of $\wp(\omega)/Fin$
- Lunch break
 - * Lunch
- Afternoon session
 - * 16.00-16.45 Piano Concert of Chopin’s music at the Royal Lazienki Gardens.
 - * 16.45-17.05 walking back from the concert,
 - * 16.45-17.15 coffee at the conference venue
 - * 17.15-17.45 M. Hrusak; Countable Frechet groups
 - * 17.45-18.15 break
 - * 18.15-18.30 Y. Khomskii; Definable Hausdorff Gaps
 - * 18.30-18.40 break
 - * 18.40-18.55 Z. Vidnyanszky; Transfinite constructions in $V = L$
- Monday, 9th of July 2012
 - Morning session
 - * 9.00-9.45 A. Aviles; Multiple gaps
 - * 9.45-10.00 break
 - * 10.00-10.30 J. Lopez Abad; On the Banach-Saks property and convex hulls
 - * 10.30-11.00 break
 - * 11.00-11.30 W. Kubis; Recent developments in the Fraisse-Jonsson theory
 - * 11.30-11.45 break
 - * 11.45-12.00 M. Cuth; Separable reduction theorems by the method of elementary submodels
 - * 12.00-12.05 break
 - * 12.05-12.20 E. Tachtsis; On a Topological Choice Principle by Murray Bell
 - * 12.20-12.25 break
 - * 12.25-12.40 D. Soukup; Constructing Lindelof, non D -spaces
 - * 12.40-12.45 break
 - * 12.45-13.00 A. Farhat; The basis problem for compacta satisfying high separation axioms
 - Lunch break
 - * 13.00-15.00 Lunch
 - Afternoon session
 - * 16.00-16.45 S. Thomas; A Descriptive View of Unitary Group Representations
 - * 16.45-17.15 break
 - * 17.15-17.45 B. Velickovic; Towards a structure theory of Maharam algebras
 - * 17.45-18.00 break
 - * 18.00-18.15 P. Luecke; Sigma-1-1-definability at uncountable regular cardinals

- * 18.15-18.20 break
- * 18.20-18.35 A. Bartoszewicz; Set theoretical methods in algebraic constructions
- * 18.35-18.40 break
- * 18.45-19.00 E. Wajch; An axiomatic theory of classes for physics
- Tuesday, 10th of July 2012
 - Morning session
 - * 9.00-9.45 A. Kechris; Amenability, unique ergodicity and random orderings
 - * 9.45-10.00 break
 - * 10.00-10.30 M. Dzamonja; Embeddings of spaces of the form $C(K)$
 - * 10.30-11.00 break
 - * 11.00-11.30 T. Tsankov; The minimal flows of S_∞
 - * 11.30-11.45 break
 - * 11.45-12.00 D. Bartosova; Universal minimal flow in the language of filters
 - * 12.00-12.05 break
 - * 12.05-12.20 V. Gregoriades; Turning Borel sets into Clopen sets effectively
 - * 12.20-12.25 break
 - * 12.25-12.40 P. Schlicht; Separating the bounding and dominating numbers for classes of uncountable structures
 - * 12.40-12.45 break
 - * 12.45-13.00 A. Kwela; Ranks of F-limits of filter sequences
 - Lunch break
 - * 13.00-15.00 Lunch
 - Afternoon session
 - * 16.00-16.45 S. Todorcevic; A set-theoretic analysis of the quotient problem and the biorthogonal system problem
 - * 16:45-17.00 break
 - * 17.00-17.30 A short problem session;
 - * 17.30-17.45 break
 - * 17.45-18.00 M. Krupski; Universality properties of ℓ_∞/c_0
 - * 18.00-18.05 break
 - * 18.05-18.20 J. Meri; Spaces with few operators and the lack of complex structure
 - * 18.20-18.25 break
 - * 18.25-18.40 T. Kania; Operator ideal of weakly compactly generated operators
 - Evening
 - * 19.45-23.00 The Conference Barbeque
- Wednesday, 11th of July 2012
 - Morning session
 - * 9.00-9.45 S. Solecki; Spatial models of Boolean actions.
 - * 9.45-10.00 break
 - * 10.00-10.30 S. Geschke; Definable graphs of low complexity
 - * 10.30-11.00 break

- * 11.00-11.30 P. Zakrzewski; On Borel sets belonging to every invariant ccc sigma-ideal on $2^{\mathbb{N}}$
- * 11.30-11.45 break
- * 11.45-12.00 M. Balcerzak; Selected results on measure-category products of ideals
- * 12.00-12.05 break
- * 12.05-12.20 S. Glab; Covering properties of ideals
- * 12.20-12.25 break
- * 12.25-12.40 J. Flaskova; Rapid ultrafilters and summable ideals
- * 12.40-12.45 break
- * 12.45-13.00 D. Guerrero Sanchez; Are Eberlein-Grothendieck scattered spaces sigma-discrete?
- Lunch break
 - * 13.00-15.00 Lunch
- Afternoon session
 - * 16.00-16.45 L. Nguyen Van The; Expanding Fraisse classes into Ramsey classes
 - * 16.45-17.15 break
 - * 17.15-17.45 M. Elekes; Set theory and Hausdorff measures
 - * 17.45-18.00 break
 - * 18.00-18.15 T. Banach; The character of points in the Higson corona of a metric space
 - * 18.15-18.20 break
 - * 18.20-18.35 A. Blaszczyk; Topologies defined on trees.
- Thursday, 12th of July 2012
 - Check-out