

# Workshops and Master Classes on Geometry and Quantization of Moduli Spaces

## SCIENTIFIC REPORT FOR THE EUROPEAN SCIENCE FOUNDATION RESEARCH NETWORKING PROGRAMME ITGP — INTERACTIONS OF LOW-DIMENSIONAL TOPOLOGY AND GEOMETRY

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The following activities were organized within the Research Programme *Geometry and Quantization of Moduli Spaces*, which took place at Centre de Recerca Matemàtica (CRM), Barcelona (March–June 2012):

- *Master Class on Topological Quantum Field Theories*  
CRM, 19, 20, 23 April, 2012
- *Workshop on Topological Quantum Field Theories*  
Institut de Matemàtiques, Universitat de Barcelona, 24, 25, 26, 27 April, 2012
- *Master Class on Representations of Surface Groups*  
CRM, 9, 10, 11 May, 2012
- *Workshop on Representations of Surface Groups*  
CRM, 14, 15, 16, 17 May, 2012

The Research Programme *Geometry and Quantization of Moduli Spaces* was centred on the geometry of moduli spaces, mostly (but not exclusively) associated to compact Riemann surfaces. The topics of the above workshops, Topological Quantum Field Theories (TQFT) and Representations of Surface Groups, are experiencing extraordinary growth within this broad Research Programme. Each workshop was preceded by a master class delivered by leading experts, designed to equip young researchers with general background and specific training on the area of the workshop. The four activities brought together leading experts along with postdoctoral researchers and advanced graduate students working in these areas.

### 1. ORGANIZERS

Organizers of the Workshop and Master Class on Topological Quantum Field Theories:

- Joergen Ellegaard Andersen (QGM, Aarhus, Denmark)
- Gregor Masbaum (Paris VI, France)

Organizers of the Workshop and Master Class on Representations of Surface Groups:

- Oscar García-Prada (ICMAT, CSIC, Madrid, Spain)
- Steven Bradlow (University of Illinois at Urbana–Champaign, USA)

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## 2. SCIENTIFIC PROGRAMME

**Master Class on Topological Quantum Field Theories**

This master class was organized in three series of lectures running simultaneously. They were devoted to the isomorphism between the Combinatorial and the Conformal Field Theory Construction of TQFT, recently proved by J. Andersen and K. Ueno. They were delivered by leading experts on the following topics:

- Jørgen Ellegaard Andersen (QGM, Aarhus): *The Isomorphism between the Combinatorial and the Conformal Field Theory Construction of TQFT*
- Gregor Masbaum (IMJ, Université de Paris 6): *Skein Theory and TQFT*
- Christian Pauly (Univ. Nice, France): *Conformal Blocks and TQFT*

The lectures were be very well structured, clear and inspiring.

**Workshop on Topological Quantum Field Theories**

This workshop focused on new developments in Topological Quantum Field Theories (TQFT), with emphasis on the following topics: integral TQFT and applications to the mapping class group, the geometric approach to TQFT and the Andersen–Ueno isomorphism, its application to moduli space and geometric group theory, the volume conjecture and relations to hyperbolic geometry of 3-manifolds, the AJ-conjecture and semi-classical asymptotics of TQFT, Quantum Teichmuller theory and TQFT’s for non-compact gauge groups.

The list of speakers and the titles of their lectures to the workshop were as follows:

- Jørgen Ellegaard Andersen (QGM, Aarhus): *The geometric construction of the Reshetikhin–Turaev TQFT*
- François Costantino (Strasbourg, France): *On spin networks and geometry*
- Benjamin Himpel (QGM, Aarhus): *The asymptotic expansion of the Witten–Reshetikhin–Turaev invariants*
- Søren Fuglede Jørgensen (QGM, Aarhus): *Quantum invariants of torus bundles and their asymptotics*
- Rinat Kashaev (Univ. Geneve): *The pentagon equation and mapping class group representations*
- Thang T. Q. Le (Georgia Institute of Technology, Atlanta, USA): *Nahm sums, stability and the colored Jones polynomial*
- Gregor Masbaum (IMJ, Univ. Paris 6): *All finite groups are involved in the mapping class group*
- Brendan McLellan (QGM, Aarhus, Denmark): *Non-Abelian Localization For Chern–Simons Theory*
- Bertrand Patureau-Mirand (Univ. Bretagne-Sud, Vannes, France): *Quantum invariants of 3-manifolds via link surgery presentations and non-semi-simple categories*
- Christian Pauly (Univ. Nice, France): *On the monodromy of the Hitchin connection*
- Ramanujan Santharoubane (IMJ, Univ. Paris 6): *Limits of the quantum  $SO(3)$  representations for the one-holed torus*

## Master Class on Representations of Surface Groups

This master class was organized in four series of lectures running simultaneously, covering many aspects of the subject of the Workshop on Representations of Surface Groups and delivered by experts on the corresponding topics:

- Olivier Guichard (Orsay, France) : *Dynamics and geometry of Fuchsian groups and Kleinian groups and generalisations*
- Tony Pantev (Univ. Pennsylvania, USA): *Higgs bundles and non-abelian Hodge theory*
- Florent Schaffhauser (Univ. Los Andes, Colombia): *Fundamental groups of Klein surfaces and their representation varieties*
- Domingo Toledo (Univ. Utah, USA): *Harmonic maps and topology of varieties*

The lectures were be very well structured, clear and inspiring.

## Workshop on Representations of Surface Groups

One theme of this workshop was the role of Higgs bundles, with special emphasis on Higgs bundles over non-orientable surfaces, the Hitchin fibration, and Higgs bundles on higher dimensional varieties.

The list of speakers and the titles of their lectures to the workshop were as follows:

- Philip Boalch (École Normale Supérieure, France): *Irregular connections, Dynkin diagrams, and fission*
- Peter Gothen (Univ. Porto, Portugal): *The singular fibre of the Hitchin map*
- Jacques Hurtubise (McGill Univ., Montreal, Canada): *Flat bundles and Grassmann framings*
- Alessandra Iozzi (ETH Zurich, Switzerland): *Weakly maximal and causal representations of surface groups*
- Vincent Koziarz (Univ. Bordeaux 1, France): *The Toledo invariant on smooth varieties of general type*
- Sean Lawton (Univ. Texas, USA): *The topology of parabolic character varieties of free groups*
- Melissa Liu (Columbia Univ., USA): *The Yang–Mills equations over Klein surfaces*
- Alessia Mandini (Univ. Técnica de Lisboa, Portugal): *Polygon spaces and an involution on the moduli space of parabolic higgs bundles*
- André Gama Oliveira (Univ. Trás-os-Montes e Alto Douro, Portugal): *Quadratic pairs and representations of surface groups*
- S. Ramanan (Chennai, India): *Involutions of the Higgs moduli space*
- Laura Schaposnik (Univ. Oxford, UK): *Spectral data for  $G$ -Higgs bundles*
- Botong Wang (Purdue Univ., USA): *Moduli space of stable parabolic Higgs bundles*
- Martin Wijnholt (Univ. München, Germany): *Higgs bundles in high energy physics*
- Kang Zuo (Univ. Mainz, Germany): *On the characteristic  $p$  nonabelian Hodge theory in the geometric case*

## 3. LIST OF PARTICIPANTS

**Participants of the master class and workshop on Topological Quantum Field Theories**

1. Amit De — QGM, Aarhus
2. Andratx Bellmunt — Univ. Barcelona
3. André Gama Oliveira — Centre of Mathematics of UTAD
4. Andrés Jaramillo Puentes — Univ. los Andes
5. Benjamin Himpel — Aarhus Univ.
6. Burglind Juhl-Jöricke
7. Carles Casacuberta — Univ. Barcelona
8. Christian Pauly — Univ. Nice
9. Cristina Martinez — Univ. Autònoma de Barcelona
10. Darío Sánchez — Univ. Salamanca
11. David Fernández Álvarez — IC-MAT/CSIC Madrid
12. Domingo Toledo — Univ. of Utah
13. Gregor Masbaum — Institut de Mathématiques de Jussieu
14. Hans-Christian Herbig — QGM, Aarhus
15. Hiraku Nozawa — Univ. Santiago de Compostela
16. Ignasi Mundet — Univ. Barcelona
17. Jens Kristian Egsgaard — Aarhus Univ.
18. Jens-Jakob Kratmann Nissen
19. Joachim Kock — UAB
20. Joanna Meinel — Max-Planck-Institut für Mathematik
21. Jørgen Ellegaard Andersen — Aarhus Univ.
22. Luis Alvarez-Cónsul — IC-MAT/CSIC Madrid
23. Maria Immaculada Gálvez Carrillo — Univ. Politècnica de Catalunya
24. Oscar García-Prada — CSIC
25. Paolo Masulli — Aarhus Univ.
26. Ramanujan Santharoubane — Institut de Mathematiques de Jussieu, Paris
27. Ramsès Fernández — Univ. Barcelona
28. Renaud Detcherry — Ecole Polytechnique
29. Reza Rezazadegan — Univ. Nantes
30. Richard A. Wentworth — Univ. Maryland
31. Shehryar Sikander — QGM, Aarhus
32. Søren Fuglede Jørgensen — Aarhus Univ.
33. Thomas Baier — Univ. Porto
34. Victoria Hoskins — Oxford Univ.
35. Vivek Mallick — CRM
36. William Goldman — Univ. Maryland
37. François Costantino — Strasbourg
38. Rinat Kashaev — Univ. Geneve
39. Thang T. Q. Le — Georgia Institute of Technology, Atlanta, USA
40. Brendan McLellan — QGM, Aarhus
41. Bertrand Patureau-Mirand — Univ. Bretagne-Sud, Vannes, France

**Participants of the master class and workshop on Surface Group Representations**

1. Rachid Ali Radi — CERIST and USTHB of Algiers
2. Luis Alvarez-Cónsul — IC-MAT/CSIC Madrid
3. Caleb Ashley — Howard Univ., Washington. DC
4. Thomas Baier — Univ. Porto

5. Andratx Bellmunt — Univ. Barcelona
6. Oren Ben-Bassat — Univ. Haifa
7. Philip Boalch — ENS-DMA-CNRS
8. Steven Bradlow — Univ. Illinois at Urbana-Champaign
9. Luis Angel Calvo Pascual — ICMAT
10. David Fernández — ICMAT/CSIC Madrid
11. Carlos Florentino — Instituto Superior Técnico
12. Vladimir Fock — Institut de Recherche Mathématique Avancée (IRMA)
13. Emilio Franco Gómez — ICMAT
14. Oscar García-Prada — CSIC
15. Peter Gothen — Univ. Porto
16. Victoria Hoskins — Oxford Univ.
17. Jacques Hurtubise — McGill Univ.
18. Alessandra Iozzi — ETH Zürich
19. Andres Jaramillo Puentes — Univ. los Andes
20. Burglind Juhl-Jöricke — Weizmann Institute of Science
21. Joachim Kock — Univ. Autònoma de Barcelona
22. Vincent Koziarz — Université Bordeaux I
23. Sean Lawton — Univ. Texas-Pan American
24. Melissa Liu — Columbia Univ.
25. Alessia Mandini — Instituto Superior Técnico
26. Cristina Martinez — Univ. Autònoma de Barcelona
27. Gregor Masbaum — Institut de Mathématiques de Jussieu
28. Pere Menal — UAB
29. Ignasi Mundet — Univ. Barcelona
30. Peter Newstead — Univ. Liverpool
31. Maria-Helena Noronha — California State Univ. Northridge
32. Hiraku Nozawa — Univ. Santiago de Compostela
33. André Oliveira — Centre of Mathematics of UTAD
34. Tony Pantev — Univ. Pennsylvania
35. Ana Peón — ICMAT Madrid
36. Sundararaman Ramanan — Chennai Mathematical Institute
37. Roberto Rubio Núñez — Univ. Oxford
38. Darío Sánchez — Univ. Salamanca
39. Florent Schaffhauser — Univ. Los Andes
40. Laura Schaposnik — Oxford Univ.
41. Zhe Sun — Univ. Paris-Sud 11
42. Domingo Toledo — Univ. Utah
43. Botong Wang — Purdue Univ.
44. Richard A. Wentworth — Univ. Maryland
45. Martijn Wijnholt — Ludwig-Maximilians-Univ. München
46. Kang Zuo — Johannes Gutenberg Univ. Mainz

#### 4. SCHEDULES

The following pages include the schedules of the Master Class on Topological Quantum Field Theories, the Workshop on Topological Quantum Field Theories, the Master Class on Representations of Surface Groups, and the Workshop on Representations of Surface Groups.

# Master Class and Workshop on Topological Quantum Field Theories

an ITGP midterm activity

Centre de Recerca Matemàtica, Bellaterra (Barcelona), 19–27 April, 2012

Integrated in the Research Programme  
Geometry and Quantization of Moduli Spaces  
CRM, Barcelona, March–June 2012

## Schedule of the Master Class

	Thursday 19 April	Friday 20 April		Monday 23 April
10:30-11:30	Andersen I	Masbaum II	<b>Weekend</b>	Andersen II
12:00-13:00	Pauly I	Pauly III		Masbaum IV
13:00-15:00	lunch	lunch		lunch
15:00-16:00	Masbaum I	Masbaum III		Andersen III
16:30-17:30	Pauly II	Pauly IV		Andersen IV

The lectures of the Master Class will take place in the Auditorium of the CRM, at the Campus of the Universitat Autònoma de Barcelona (see <http://www.crm.cat> for location and travel information).

## Schedule of the Workshop

	Tuesday 24 April	Wednesday 25 April	Thursday 26 April	Friday 27 April
10:30-11:30	Costantino	Kashaev	Pauly	Jørgensen
11:30-12:00	coffee	coffee	coffee	coffee
12:00-13:00	Santharoubane	Patureau-Mirand	Masbaum	Andersen
13:00-15:00	lunch	lunch	lunch	
15:00-16:00	Le	Himpel	POSTER	
16:30-17:30	Andersen V (MC)	McLellan	SESSION	

The lectures of the Workshop will take place in Room B7 of the Faculty of Mathematics of the Universitat de Barcelona (see <http://www.mat.ub.edu/en/> for location).

**REPRESENTATIONS OF SURFACE GROUPS**  
**SCHEDULE OF THE MASTER CLASS**

<b>Wednesday, May 9</b>	
10:15 – 10:30	REGISTRATION
10:30 – 11:30	Tony Pantev <i>Higgs bundles and non-abelian Hodge theory</i>
11:30 – 12:00	COFFEE BREAK
12:00 – 13:00	Domingo Toledo <i>Harmonic maps and topology of varieties</i>
13:00 – 15:00	LUNCH
15:00 – 16:00	Florent Schffhauser <i>Fundamental groups of Klein surfaces and their representation varieties</i>
16:00 – 16:30	BREAK
16:30 – 17:30	Olivier Guichard <i>Dynamics and geometry of Fuchsian groups and Kleinian groups and generalisations</i>
<b>Thursday, May 10</b>	
10:30 – 11:30	Domingo Toledo <i>Harmonic maps and topology of varieties</i>
11:30 – 12:00	COFFEE BREAK
12:00 – 13:00	Tony Pantev <i>Higgs bundles and non-abelian Hodge theory</i>
13:00 – 15:00	LUNCH
15:00 – 16:00	Olivier Guichard <i>Dynamics and geometry of Fuchsian groups and Kleinian groups and generalisations</i>
16:00 – 16:30	BREAK
16:30 – 17:30	Florent Schffhauser <i>Fundamental groups of Klein surfaces and their representation varieties</i>
<b>Friday, May 11</b>	
10:30 – 11:30	Tony Pantev <i>Higgs bundles and non-abelian Hodge theory</i>
11:30 – 12:00	COFFEE BREAK
12:00 – 13:00	Domingo Toledo <i>Harmonic maps and topology of varieties</i>
13:00 – 15:00	LUNCH
15:00 – 16:00	Florent Schffhauser <i>Fundamental groups of Klein surfaces and their representation varieties</i>
16:00 – 16:30	BREAK
16:30 – 17:30	Olivier Guichard <i>Dynamics and geometry of Fuchsian groups and Kleinian groups and generalisations</i>

**REPRESENTATIONS OF SURFACE GROUPS**  
**SCHEDULE OF THE WORKSHOP**

<b>Monday, May 14</b>	
10:30 – 11:30	Sundararaman Ramanan <i>Involutions of the Higgs moduli space</i>
11:30 – 12:00	COFFEE BREAK
12:00 – 13:00	Alessandra Iozzi <i>Weakly maximal and causal representations of surface groups</i>
13:00 – 15:00	LUNCH
15:00 – 16:00	Peter Gothen <i>The singular fibre of the Hitchin map</i>
16:00 – 16:30	BREAK
16:30 – 17:30	André Gama Oliveira <i>Quadratic pairs and representations of surface groups</i>
<b>Tuesday, May 15</b>	
10:30 – 11:30	Laura Schaposnik <i>Spectral data for <math>\mathbb{P}^1</math>-Higgs bundles</i>
11:30 – 12:00	COFFEE BREAK
12:00 – 13:00	Kang Zuo <i>On the characteristic <math>\mathbb{P}^1</math> nonabelian Hodge theory in the geometric case</i>
13:00 – 15:00	LUNCH
15:00 – 16:00	Vincent Koziarz <i>The Toledo invariant on smooth varieties of general type</i>
16:00 – 16:30	BREAK
16:30 – 17:30	Botong Wang <i>Moduli space of stable parabolic Higgs bundles</i>
<b>Wednesday, May 16</b>	
10:30 – 11:30	Martin Wijnholt <i>Higgs bundles in high energy physics</i>
11:30 – 12:00	COFFEE BREAK
12:00 – 13:00	Philip Boalch <i>Irregular connections, Dynkin diagrams, and fission</i>
13:00 – 15:00	LUNCH
15:00 – 16:00	Sean Lawton <i>The topology of parabolic character varieties of free groups</i>
16:00 – 16:30	BREAK
16:30 – 17:30	Alessia Mandini <i>Polygon spaces and an involution on the moduli space of parabolic higgs bundles</i>
<b>Thursday, May 17</b>	
10:30 – 11:30	Melissa Liu <i>The Yang-Mills equations over Klein surfaces</i>
11:30 – 12:00	COFFEE BREAK
12:00 – 13:00	Jacques Hurtubise <i>Flat bundles and Grassmann framings</i>