GEOMETRY, INTEGRABILITY, QUANTIZATION

SISSA, Trieste, 9-12 July 2012

FINAL REPORT

Summary

The purpose of the meeting was to give a survey of the state of the art about different aspect of integrability in mathematics and physics, both at the classical and quantum level, with a special attention to the geometric contents of the theory. The main topics touched upon were the recent developments in Poisson geometry and quantum groups, with their applications to integrable systems and Painlevé transcendants, and their relations with noncommutative geometry; moduli spaces of decorated sheaves and quiver representations, with their applications in geometry and physics (instantont counting, Alday-Gaiotto-Tachikawa relations); the geometry of nonlinear integrable hierarchies and the associated geometric structures; some aspects of the theory of Lie and Courant algebroids.

There were 60 registered participants, 7 of which affiliated to SISSA, the remaining ones coming from various European universities. There have been moreover several unregistered SISSA participants, so that the total number of participants was about 75.

The organizers believe that the workshop was a success, both for the interested and level of the talks, and for the general atmosphere, which allowed for many fruitful interactions among the participants.

Description of the scientific content and discussion at the event

The main objectives of the workshop, as indicated in the previous Summary of contents, were covered by the following topics. We shall give here a description of the contents of the various talks.

In his presentation, A. Alekseev (Geneva) told about solutions of modern generalizations of the classical Horn problem and its relations to planar networks. He described his joint work with A. Szenes and M. Podkopaieva, where they introduced a combinatorial problem defined in terms of planar graphs. Their approach fits well with some problems in linear algebra, such as the Gelfand-Zeitlin scheme, and has strong relations with cluster algebras and tropical geometry.

V. Retakh (Rutgers) described his recent achievements in the construction of a new class of noncommutative integrable equations that are analogs of Painlevé II transcendants. He told about his recent results with V. Rubtsov about the relations of this noncommutative Painlevé II with the quasi-determinants of Gelfand-Retakh and the nonabelian Toda chains.

Linear degenerate PDEs and quadratic complexes were considered by E. Ferapontov (Loughborough). Such complexes are specified by a single quadratic relation in Plücker coordinates in $\mathbb{P}^3$. He showed that any linear degenerated PDE can be obtained in this way, and classified the degenerate wave equations.

C. Bartocci (Genova) described a construction of the moduli space of framed sheaves on Hirzebruch surfaces by means of monads.

A new approach in noncommutative geometry was the topic of the talk by D. Gurevich (Valenciennes). This approach is based on a series of joint works with P. Saponov and P. Pyatov, where they developed a version of differential calculus based on some Hecke type solutions of the quantum Yang-Baxter equation and the corresponding quantum groups and reflection equation algebras.

I. Reider (Angers) talked about his construction of nonabelian analogs of Jacobians arising from his studies of moduli spaces of sheaves on projective surfaces. An interesting version of the 2-dimensional geometric Langlands correspondence appears as a result of this construction.

O. Lisovyy (Tours) spoke about an analog of the AGT correspondence which he discovered with N. Iorgov and A. Gamayun. This relates generating series representing the τ function of Painlevé VI with 4-point correlators of primary fields in 2-dimensional conformal field theory.

F. Sala (Edinburgh) talked about the relation between instantons on toric ALE spaces and framed sheaves on a root stack having the ALE space as its coarse moduli space. This serves to provide an interpretation to the instanton counting computations done by Bruzzo, Poghosyan and Tanzini.

The talk by I. Roulstone (Guildford) was devoted to a surprising application of various geometries, such as symplectic, contact, and generalized complex structures, to some PDEs arising from theoretical meteorology. This was based on his joint works with B. Banos, J. Gibbon, and V. Rubtsov, where they studied the 2D Euler equation and the 3D Navier-Stokes system, and defined some geometric criteria for stability and instability phenomena.

The talk by G. Ortenzi (Milano) was about the vortex filament dynamics. He studied an elliptic system of quasi-linear PDEs describing the motion of vortex filament with or without axial flow. The case of a gradient catastrophe was considered.

Symplectic field theory and new integrable hierarchies were the subject of the talk by P. Rossi (Zurich). He showed some topological recursion relations for the Hamiltonian symplectic field theory of symplectic mapping tori. An analog of the Nijenhuis operator and the related descendant relations were discussed.

A covariant formalism in classical field theory was the topic of the contribution by F. Hélein (Paris). He provided a formulation of dynamical problems using a multisymplectic formalism and argued that this is most likely the appropriate way for understanding the Hamiltonian structure of relativistic field theory.

P. Tortella (São Carlos, Brazil) talked about Nijenhuis operators and Λ-connections. This is a recent and interesting development of his studies on the structure and classification of moduli spaces of Λ-modules.

Hypersymplectic and generalized complex structures on algebroids were the object of study of P. Antunes (Coimbra). He showed some interesting compatibility conditions of these structures with Poisson and Dirac geometries related to Lie and Courant algebroids.

C. Roger (Lyon) told about matrix valued integrable hierarchies, Hamiltonian structures, and an analog of the Miura map playing the role of a Hamiltonian morphism between these matrix hierarchies.

M. Bertola (Concordia, Montreal) discussed the solution of the Painlevé II equation within the Ablowitz-Segur family and showed that they are all free in some region of the complex plane.

In his talk M. Cafasso (Angers) spoke about the Riemann-Hilbert problem related to a computation of Its-Slavnov-Korepin correlation function, Fred-

holm determinants and their integral representations. It was shown that the kernels of these integral operators satisfy a Painlevé II matrix which corresponds to a noncommutative Painlevé II equation proposed by Retakh and Roubtsov.

G. Falqui (Milano) concluded the workshop with his talk based on a series of papers, in collaboration with A. Chervov, V. Rubstov and A. Sylantiev, devoted to a class of matrices with noncommutative entries subject to some quadratic relations. These matrices appeared in works of Yu. Manin in the 80s and are known indeed as Manin matrices. Some rich properties of Manin and q-Manin matrices were discussed, as well as their relations to integrable systems and combinatorial problems.

U. Bruzzo (SISSA), B. Enriquez (Strasbourg), Y. Kosmann-Schwarzbach (Paris), J. Krasil'shchik (Moscow) and V. Lychagin (Tromsø) described their collaborations with V. Rubtsov.

Assessment of the results and impact of the event on the future directions of the field

It is the opinion of the organizers that the workshop fulfilled its scopes in a satisfactory way. The talks were interesting, and the structure of the workshop has left space for personal discussion. The workshop has allowed many of the participants to get in touch with the most recent advances in the fields covered in the conference. Moreover, the workshop served the scope of allowing for the interaction between researchers working in various areas of mathematical physics and geometry, using the ideas of quantum and classical integrability as a common denominator for the specialists in the different subjects.

Many among the talks in the workshop related to directions of investigation that are actively studied and definitely will have a remarkable impact on the present research in the area.

A nice feature of the workshop was the presence of many young researchers, most of them from SISSA and from Russia and Ukraine. The exposure to so many modern concepts and ideas will definitely be beneficial to their training. They also enjoyed a week-long stay in the stimulating environment provided by the synergy between the participants in the workshop and the existing staff and student of SISSA.

Geometry, Integrability, Quantization

a workshop dedicated to Vladimir Rubtsov on the occasion of his 60th birthday

SISSA, Trieste — July 9 to 12, 2012

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SISSA

GNSAGA — INDAM

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Istituto Nazionale di Fisica Nucleare

Scientific committee: U. Bruzzo, B. Enriquez, Y. Kosmann-Schwarzbach, J. Krasil'shchik, V. Lychagin, A. Morozov

Programme

Monday, July 9th

10:00-11:00 V. Retakh, Noncommutative integrable systems and quasideterminants

11:30-12:30 A. Alekseev, The Horn problem and planar networks

14:30-15:30 E. Ferapontov, Quadratic line complexes, conformal structures in $\mathbb{P}^3$ and linearly degenerate PDEs

15:45-16:45 C. Bartocci, Monadic description of framed sheaves on Hirzebruch surfaces

17:00-18:00 D. Gurevich, A new approach in Noncommutative Geometry

Tuesday, July 10th

9:30-10:30 I. Reider, Correspondences, nonabelian Jacobian and geometric Langlands program for complex projective surfaces

10:45-11:45 O. Lisovyy, Algebraic solutions of Painleve VI

12:00-13:00 F. Sala, Sheaves and instantons on ALE spaces

15:00-16:00 I. Roulstone, A geometric description of Navier-Stokes flows

16:30-17:30 G. Ortenzi, Flutter of a vortex filament and gradient catastrophe

20:00 Dinner

Wednesday, July 11th

9:30-10:30 P. Rossi, Symplectic geometry from symplectic topology

10:45-11:45 Volodya's work

12:00-13:00 F. Hélein, Covariant functionals on the covariant phase space

15:00-16:00 P. Tortella, Nijenhuis structures and Λ-connections

16:30-17:30 P. Antunes, Hypersymplectic structures on Courant algebroids

Thursday, July 12th

9:30-10:30 C. Roger, Matrix-valued Miura transform

10:45-11:45 Voloyda's work

12:00-13:00 M. Bertola, The Cauchy matrix model and the Meijer-G random point field

15:00-16:00 M. Cafasso, Fredholm determinants, Riemann-Hilbert problems and noncommutative Painlevé equations

16:30-17:30 G. Falqui, Manin matrices and applications to integrable systems

U. Bruzzo, B. Enriquez, Y. Kosmann-Schwarzbach, J. Krasil'shchik, V. Lychagin will talk about their collaborations with Vladimir Rubtsov.

Other participants: N. Amburg, G. Aminov, S. Artamonov, L. Dabrowski, V. Dolotin, P. Dunin-Barkovsky, D. Galakhov, T. Grava, G. Helminck, H. Kanno, N. Konovenko, E. Krylov, G. Landi, G. Marelli, A. Marshakov, K. Maruyoshi, A. Mironov, S. Mironov, Al. Morozov, And. Morozov, Ant. Morozov, M. Pedroni, O. Ptitisyna, V. Poberezhny, I. Polyubin, F. Popov, P. Pushkar, C. Rava, B. Runov, V. Shadura, A. Sleptsov, A. Stern, E. Suslova, A. Tanzini, D. Vasiliev, F. Yagi