

**"The Ehrenpreis Conjecture" and
"Progress in low-dimensional topology: Teichmüller theory and 3-manifold groups"
(Master Class 6-10 August and International Conference 11-14 August, Aarhus, Denmark)**

Organized by Jørgen Andersen, Douglas LaFountain, Athanase Papadopoulos, and Robert Penner

Summary: The Centre for Quantum Geometry of Moduli Spaces (QGM) at Aarhus University in Aarhus, Denmark, hosted two successive scientific meetings in August of 2012, specifically a Master Class from 6-10 August entitled "The Ehrenpreis conjecture", taught by Jeremy Kahn of Brown University and Vladimir Marković of the California Institute of Technology, as well as a subsequent International Conference from 11-14 August entitled "Progress in low-dimensional topology: Teichmüller theory and 3-manifold groups". The main purpose of the master class was to provide an opportunity for both senior mathematicians and beginning researchers to intensively study recent seminal work of Kahn and Marković that has led to their resolution of several outstanding problems in Teichmüller theory and 3-manifold topology, and the class focused in particular on the new combinatorial/analytic techniques which they have developed to do so. The conference following then provided a forum whereby many of the same scientists attending the master class announced and explained new applications stemming from the work of Kahn- Marković , as well as conveyed new progress on related problems in Teichmüller theory and 3-manifold groups. This conference featured speakers from numerous countries in Europe, as well as from the United States, Japan, and China.

Scientific content: "The Ehrenpreis Conjecture" Master Class from 6-10 August was taught by Jeremy Kahn and Vladimir Marković, and was a detailed exposition of their seminal works from 2009 (in which they proved the Surface Subgroup Conjecture for hyperbolic 3-manifolds by constructing π_1 -injective immersed surfaces) and from 2011 (in which they proved the Ehrenpreis Conjecture demonstrating that any two hyperbolic Riemann surfaces are virtually close in the Teichmüller metric). The master class spanned five days and traced out the following pedagogical arc: Marković began the course by giving a brief though insightful introduction to hyperbolic geometry in two and three dimensions, focusing specifically on certain useful features of right-angled hexagons as well as complex Fenchel-Nielsen coordinates useful for studying hyperbolic surfaces in hyperbolic 3-space. He then built on this to demonstrate how with the proper choice of a particular type of right-angled hexagon, one may glue together two such hexagons to obtain a "good" pair of pants with cuff-lengths all within a given ε of some hyperbolic length R . Moreover, by then gluing together good pairs of pants with a twist along cuffs within the same ε of the number 1, one obtains what Kahn-Marković call a "good pants decomposition" for the resulting hyperbolic surface. Building on this introduction to the needed hyperbolic geometry, Kahn then stated both the Surface Subgroup Theorem and the Ehrenpreis Conjecture as the goals of the course, along with a strategy of proof that would rely on the proof of a Unifying Theorem, namely that for large enough R there exist "model" surfaces S_R admitting (ε, R) -good pants decompositions such that for all closed hyperbolic 2- or 3-manifolds M there is a finite cover of S_R for which a nearly isometric immersion in M exists.

Kahn and Marković then proceeded to share lecturing duties, first addressing the proof of this Unifying Theorem for the case of hyperbolic 3-manifolds M . The general idea here was to find good pairs of pants immersed along closed geodesics in M , and such that around each such geodesic the

associated pants were equidistributed so as to be able to be paired and glued to form immersed surfaces. This equidistribution relied on a detailed analysis of the exponential mixing of the frame flow in M in the context of the complex Fenchel-Nielsen coordinates for the good pants in M . A similar approach was then used to prove the Unifying Theorem for the case of hyperbolic 2-manifolds, but in this case, because the normal bundle to closed geodesics had one less degree-of-freedom as in the 3-manifold case, more work was needed to obtain equidistribution of good pairs of pants around closed geodesics. Thus, Kahn and Marković described their "good pants homology" for surfaces whose development led to the required results.

Within this development of the tools and statements needed to prove the Surface Subgroup Theorem and the Ehrenpreis Conjecture, Kahn also took two lectures to briefly describe the recent application of the Surface Subgroup Conjecture in the work of Agol (based on work of Wise) to prove the Virtual Haken Conjecture.

For the International Conference, a total of 12 senior researchers from Europe, the United States, Japan and China presented talks on recent works of significance in the fields of Teichmüller theory and 3-manifold groups. Below are a selection of some of the highlights from these lectures:

- Ian Agol presented two lectures on his proof of the Virtual Haken Conjecture, following the work of Wise and Kahn-Markovic. In the first, he gave an overview of the history of both the problem and previous partial solutions, including the result that the Surface Subgroup Theorem plus Locally Extended Residual Finiteness of hyperbolic groups yields Virtually Haken. In the second lecture, he then described his proof of the Virtual Haken Conjecture which depends on the result that cubulated hyperbolic groups are virtually special.
- David Gabai presented an effective proof of the generalized Waldhausen conjecture. Specifically, in joint work with Toby Colding, he built on Tao Li's result that a closed non-Haken 3-manifold M has only finite many irreducible Heegaard splittings by describing an algorithm with which to actually compute this upper bound when M is hyperbolic.
- Scott Wolpert presented work which generalizes formulas for Fenchel-Nielsen twists, geodesic-lengths and the Weil-Petersson metric to the setting of punctured surfaces triangulated by ideal geodesics. Specifically, he reviewed Bonahon's embedding of Teichmüller space for compact surfaces into the space of transverse cocycles on a maximal geodesic lamination, and further reviewed Penner's lambda-length embedding of decorated Teichmüller space for punctured surfaces to the positive Euclidean orthant. For punctured surfaces triangulated by ideal geodesics, he then considered weighted sums of geodesics with weights summing to zero at each cusp, and analyzed such configurations by "doubling across cusps" and "opening nodes" to obtain compact surfaces with FN twists. He thus showed that the dual of a Thurston shear is the total length, that the WP symplectic pairing is given by weight summation by parts at punctures, and describe the formula for the WP metric pairing.
- Christian Zickert spoke on Thurston's gluing equations for $\mathrm{PGL}(n, \mathbb{C})$. More specifically, Thurston's gluing equations are polynomial equations used to explicitly compute hyperbolic structures or, more generally, representations in $\mathrm{PGL}(2, \mathbb{C})$. This is done via so called shape coordinates. Zickert generalized the shape coordinates to obtain a parametrization of representations in $\mathrm{PGL}(n, \mathbb{C})$, and gave applications to quantum topology, in particular discussing an intriguing duality between the shape coordinates and the Ptolemy coordinates of Garoufalidis-Thurston-Zickert. The shape coordinates and Ptolemy coordinates can be viewed as 3-dimensional analogs of the X and A coordinates on higher Teichmüller spaces due to Fock and Goncharov.

Assessment of results: The "Ehrenpreis Conjecture" Master Class was extremely well-attended, with nearly 40 participants from across Europe, the United States, Japan, and China. Not only was the class very well-attended, but the content was also very well-received, with the audience interacting actively throughout the lecture course. Of note was the participation of a large number of PhD students who took the opportunity to actively engage with the senior researchers both teaching and attending the course. This, combined with the high quality of the mathematics being presented by Kahn and Marković, made the class quite an exciting event, as it marked the resolution of a number of significant outstanding conjectures, as well as serving as a springboard for new developments in the field

The International Conference was also very popular, with over 50 participants from across Europe, the US, Japan, China, and Bangladesh. The highlight for many were Agol's lectures on his recent resolution of the Virtual Haken Conjecture, building directly on the work of Kahn-Marković as taught in the Master Class. This provided a fitting capstone to the week's events, and demonstrates the high level of mathematics discussed during both meetings. Numerous discussions, both formal and informal, occurring during both meetings will surely provide fertile ideas from which we can expect research results of significant in the coming months.

Final programme:

Master class: The Ehrenpreis conjecture

Monday 6 August

09.45-10.00: Coffee/tea by Aud. E

10.00-10.45: An Introduction to Hyperbolic Geometry 1/2 by Vladimir Markovic (Aud. E)

11.15-12.00: An Introduction to Hyperbolic Geometry 2/2 (Aud. E)

12.00-14.00: Lunch break

14.00-14.45: The Surface Subgroup Theorem and the Ehrenpreis Conjecture 1/2 by Jeremy Kahn (Aud. E)

14.45-15.15: Coffee break

15.15-16.00: The Surface Subgroup Theorem and the Ehrenpreis Conjecture 2/2 (Aud. E)

18.00: Wine and cheese / Social networking in the Department Staff Lounge

Tuesday 7 August

09.45-10.00: Coffee/tea by Aud. E

10.00-10.45: The Gluing of Pants and the Transport of Measure 1/2 by Vladimir Markovic (Aud. E)

11.15-12.00: The Gluing of Pants and the Transport of Measure 2/2 (Aud. E)

12.00-14.00: Lunch break

14.00-14.45: Sufficient Conditions for Quasiconformal Conjugacy 1/2 by Vladimir Markovic (Aud. E)

14.45-15.15: Coffee break

15.15-16.00: Sufficient Conditions for Quasiconformal Conjugacy 2/2 (Aud. E)

Wednesday 8 August

09.45-10.00: Coffee/tea by Aud. E

10.00-10.45: The Frame Flow is Exponentially Mixing 1/2 by Jeremy Kahn (Aud. E)

11.15-12.00: The Frame Flow is Exponentially Mixing 2/2 (Aud. E)

12.00-14.00: Lunch break

14.00-14.45: Mixing and an Equidistributed Measure on Good Pants 1/2 by Vladimir Markovic (Aud. E)

14.45-15.15: Coffee break

15.15-16.00: Mixing and an Equidistributed Measure on Good Pants 2/2 (Aud. E)

Thursday 9 August

09.45-10.00: Coffee/tea by Aud. E

10.00-10.45: Mixing, Counting, and Equidistribution of Good Pants 1/2 by Jeremy Kahn (Aud. E)

11.15-12.00: Mixing, Counting, and Equidistribution of Good Pants 2/2 (Aud. E)

12.00-14.00: Lunch break

14.00-14.45: An Introduction to Good Pants Homology 1/2 by Vladimir Markovic (Aud. E)

14.45-15.15: Coffee break

15.15-16.00: An Introduction to Good Pants Homology 2/2 (Aud. E)

18.00: Special dinner at restaurant in town

Friday 10 August

09.45-10.00: Coffee/tea by Aud. E

10.00-10.45: Good Pants Homology for the Brave and Foolhardy 1/2 by Jeremy Kahn (Aud. E)

11.15-12.00: Good Pants Homology for the Brave and Foolhardy 2/2 (Aud. E)

12.00-14.00: Lunch break

14.00-14.45: Good Pants Homology, Randomization, and the Ehrenpreis conjecture 1/2 by Jeremy Kahn (Aud. E)

14.45-15.15: Coffee break

15.15-16.00: Good Pants Homology, Randomization, and the Ehrenpreis conjecture 2/2 (Aud. E)

Conference: Progress in Low-dimensional Topology: Teichmüller theory and 3-manifold groups

Saturday 11 August

09.45-10.00: Coffee/tea by Aud. D1

10.00-11.00: On some non-Euclidean trigonometric formulae and applications by Athanase Papadopoulos

11.30-12.30: The degree of a CP1-structure by Bertrand Deroin

12.30-14.00: Lunch break

14.00-15.00: Extremal length geometry of Teichmüller space by Hideki Miyachi

15.00-15.30: Coffee break

15.30-16.30: Variation of extremal length on Teichmüller space by Weixu Su

18.00: Wine and cheese / Social networking in "Vandrehallen"

Sunday 12 August – Free Day

Monday 13 August

09.45-10.00: Coffee/tea by Aud. D1

10.00-11.00: An effective proof of the generalized Waldhausen conjecture by David Gabai

11.30-12.30: An extension of the Weil-Petersson metric to the Hitchin Component by Martin Bridgeman

12.30-14.00: Lunch break

14.00-15.00: The virtual Haken conjecture by Ian Agol

15.00-15.30: Coffee break

15.30-16.30: Thurston's gluing equations for $PGL(n, \mathbb{C})$ by Christian Zickert

18.00: Special dinner at restaurant in town

Tuesday 14 August

09.45-10.00: Coffee/tea by Aud. D1

10.00-11.00: Gromov's systolic characterisation of Jacobians and application by Norbert A'Campo

11.30-12.30: Linear slices of the quasifuchsian space of punctured tori by Yohei Komori

12.30-14.00: Lunch break

14.00-15.00: Earthquakes in the length spectrum Teichmüller spaces of infinite genus surfaces by Dragomir Saric

15.00-15.30: Coffee break

15.30-16.30: Products of twists, geodesic-lengths and Thurston shears by Scott Wolpert

List of Participants: Masterclass "The Ehrenpreis conjecture" 6-10 August 2012

1. Kahn, Jeremy (Brown University) - SPEAKER
2. Markovic, Vladimir (CalTech) - SPEAKER
3. Andersen, Jørgen Ellegaard (AU)
4. Bourque, Maxime Fortier (Brown University)
5. Browder, William (Princeton)
6. De, Amit (AU)
7. Deroin, Bertrand (Université Sud-Paris)
8. Egsgaard, Jens Kristian (AU)
9. Fernandez, Mario Garcia (AU)
10. Gekhtman, Ilya (University of Chicago)
11. Gothen, Peter (University of Porto)
12. Gupta, Subhojoy (AU)
13. Herbig, Hans-Christian (AU)
14. Himpel, Benjamin (AU)
15. Hurdato, Sebastian (UC Berkeley)
16. Iezzi, Francesca (University of Warwick)
17. Kanstrup, Tina (AU)
18. Komori, Yohei (Waseda University)
19. LaFountain, Douglas (AU)
20. Lin, Hsueh-Yung (École Normale Supérieure de Lyon)
21. Martens, Johan (AU)
22. Marzioni, Simone (University of Bologna)
23. Masai, Hidetoshi (Tokyo Institute of Technology)
24. Masulli, Paolo (AU)
25. McLellan, Brendan (AU)
26. Miyachi, Hideki (Osaka University)

List of Participants: Masterclass "The Ehrenpreis conjecture" 6-10 August 2012

27. Neofytidis, Christoforos (University of Munich)
28. Pandit, Suhas (Abdus Salam International Center for Theoretical Physics, Trieste, Italy)
29. Papadopoulos, Athanase (IRMA)
30. Penner, Bob (AU/Caltech)
31. Poulsen, Niccolo Skovgård (AU)
32. Shiga, Hiroshige (Tokyo Institute of Technology)
33. Shinomiya, Yoshihiko (Tokyo Institute of Technology)
34. Sikander, Shehryar (AU)
35. Su Weixu (Fudan Uni, Shanghai/Uni of Strasbourg)
36. Swann, Andrew (AU)
37. Tadokoro, Yuuki (Kisarazu National College of Technology/QGM)
38. Yuasa, Wataru (Tokyo Institute of Technology)
39. Zograf, Peter (Steklov Mathematical Institute)

List of participants

1. Andersen, Jørgen Ellegaard (AU) - ORGANIZER
2. LaFountain, Douglas (AU) - ORGANIZER
3. Papadopoulos, Athanese (IRMA) - ORGANIZER
4. Penner, Bob (AU/CalTech) - ORGANIZER
5. A'Campo, Norbert (University of Basel) - SPEAKER
6. Agol, Ian (UC Berkeley) - SPEAKER
7. Bridgeman, Martin (Boston University) - SPEAKER
8. Derooin, Bertrand (Université Paris-Sud) - SPEAKER
9. Gabai, Dave (Princeton University) - SPEAKER
10. Kahn, Jeremy (Brown University) - SPEAKER
11. Komori, Yohei (Waseda University) - SPEAKER
12. Miyachi, Hideki (Osaka University) - SPEAKER
13. Markovic, Vladimir (CalTech) - SPEAKER
14. Saric, Dragomir (Queens College, City University of New York) - SPEAKER
15. Su, Weixu (Fudan University, Shanghai /IRMA) - SPEAKER
16. Wolpert, Scott (University of Maryland) - SPEAKER
17. Andersen, Henning Haahr (AU)
18. Browder, William (Princeton)
19. De, Amit (AU)
20. Dobrovolska, Galyna (University of Chicago)
21. Egsgaard, Jens Kristian (AU)
22. Fernandez, Mario Garcia (AU)
23. Frøyshov, Kim (AU)
24. Gekhtman, Ilya (University of Chicago)

Conference 11-14 August 2012: "Progress in low-dimensional topology..."

25. Gothen, Peter (University of Porto)
26. Gupta, Subhojoy (AU)
27. Herbig, Hans-Christian (AU)
28. Himpel, Benjamin (AU)
29. Hurtado, Sebastian (UC Berkeley)
30. Iezzi, Francesca (University of Warwick)
31. Ito, Kentaro (Nagoya University)
32. Kanstrup, Tina (AU)
33. Kim, Joonhyung (Konkuk University)
34. Lin, Hsueh-Yung (École Normale Supérieure de Lyon)
35. Masai, Hidetoshi (Tokyo Institute of Technology)
36. Masulli, Paolo (AU)
37. McLellan, Brendan (AU)
38. Meaze, A.K.M Moinul (University of Chittagong, Bangladesh)
39. Neofytidis, Christoforos (University of Munich)
40. Obitsu, Kunio (Kagoshima University)
41. Pandit, Suhas (Abdus Salam International Center for Theoretical Physics, Italy)
42. Poulsen, Niccolo Skovgård (AU)
43. Santoso, Jenny (University of Stuttgart)
44. Shiga, Hiroshige (Tokyo Institute of Technology)
45. Shinomiya, Yoshihiko (Tokyo Institute of Technology)
46. Sikander, Shehryar (AU)
47. Swann, Andrew (AU)
48. Tadokoro, Yuuki (Kisarazu National College of Technology/QGM)
49. Tang, Robert (University of Warwick)
50. Webb, Richard (University of Warwick)

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51. Yuasa, Wataru (Tokyo Institute of Technology)
52. Zickert, Christian (University of Maryland)
53. Zograf, Peter (Steklov Institute of Mathematics)