

Workshop  
*Recent Advances in Topological Quantum Field Theory*  
Lisbon, September 10th – 14th, 2012

November 14, 2012

## Summary

This is the scientific report for the workshop *Recent Advances in Topological Quantum Field Theory*, which was held at the University of Lisbon from September 10 to September 14, 2012. The organizers were

- João Faria Martins, CMA-FCT/UNL, Lisbon
- Atle Hahn, GFM-UL, Lisbon
- Aleksandar Mikovic, GFM-UL/Universidade Lusófona, Lisbon
- Roger Picken, CAMGSD-IST, Lisbon

The workshop brought together 26 researchers from Canada, Denmark, France, Germany, Italy, Portugal, Singapore, the United Kingdom, and the United States of America working in Geometric Topology, Stochastic Analysis and Mathematical Physics, with a particular interest in Topological Quantum Field Theory. There were a total of 14 talks (between 45 and 55 minutes each) and five short courses (45 + 45 minutes) by leading international experts.

We left ample time for questions, problem sessions and informal discussion.

The workshop was an excellent opportunity for local researchers with an interest in Topological Quantum Field Theory to connect with each other and internationally.

For additional information on the workshop (including the pdf-files of some of the talks) see the workshop website:

*<http://gfm.cii.fc.ul.pt/tqft2012>*

## Scientific content: Conference Program & Abstracts

	Monday	Tuesday	Wednesday	Thursday	Friday
8:30–9:00	<i>Registration</i>				
9:00–9:10	<i>Opening</i>				
9:10–9:55	Crane	Crane	Carter	Carter	Tavares
10:00–10:45	Charles	Charles	Gohla	Stosic	Adams
10:45–11:15	<i>Coffee Break</i>	<i>Coffee Break</i>	<i>Coffee Break</i>	<i>Coffee Break</i>	<i>Coffee Break</i>
11:15–12:00	Mackaay	McLellan	Sengupta	Gukov	de Haro
12:00–14:00	<i>Lunch Break</i>	<i>Lunch Break</i>		<i>Lunch Break</i>	
14:00–14:45	Mackaay	Gukov		Zampini	
14:50–15:35	Faria Martins	Thompson	<i>Excursion</i>	Fine	
15:35–16:05	<i>Coffee Break</i>	<i>Coffee Break</i>		<i>Coffee Break</i>	
16:05–17:00	Wise	Castelo Ferreira		Christensen	
20:00			<i>Conference Dinner</i>		

### David Adams: Simplicial approach to knot and link invariants via discretized TQFTs

I will review how abelian BF gauge theory (a particular TQFT) can be discretized using a simplicial cell decomposition of the 3-dimensional spacetime manifold and its dual cell decomposition. This leads to a simplicial version of the Gauss linking number formula for linked knots. Then I will discuss simplicial discretization of nonabelian BF theory where the goal is to find simplicial versions of integral formulae for the coefficients of the Alexander-Conway polynomial (a knot invariant). Finally, prospects for extending the simplicial approach to other knot and link invariants will be discussed.

### Scott Carter: Knotted foams and G-families of quandles

A 2-foam is a compact topological space that is modeled on the dual to the tetrahedron,  $Y^2$ . Every point in the foam has a neighborhood that is homeomorphic to a neighborhood of a point in  $Y^2$ . In  $Y^2$  there is a single vertex at which three edges and six faces are incident. Along each edge, three faces intersect. A foam can be knotted. The local pictures for crossings correspond to chains in a homology theory that is associated to a  $G$ -family of quandles. In this talk, I will outline the homology theory, and demonstrate some non-trivial knottings.

### Pedro Castelo Ferreira: String D-branes description from 2+1D topological field theory

The string world sheet D-branes vertex operators are derived from the orbifolding of 2+1D topological massive gauge theories coupled to a dynamical scalar field. It is shown that the boundary conformal field theory states corresponding to D-branes are described by the vacuum states of the bulk theory and the brane tension is set by the bulk mass scales. [hep-th/0308101]

### Laurent Charles: Topological quantum field theory and semi-classical limit

I will present recent results obtained in collaboration with J. Marche, on the semi-classical limit of knot states and the Witten asymptotic conjecture. I will address the following points

- Witten asymptotic expansion of the partition function

- Definition of Reshetikhin-Turaev invariants from skein relations
- $SU(2)$ -character manifold of knot exteriors
- Verlinde basis and theta functions
- $q$ -difference relations and Toeplitz operators
- semi-classical limit of knot states.

### **Dan Christensen: Computation of traces in the representation theory of the symmetric and unitary groups**

I will review the classification of representations of the symmetric and unitary groups, and how they are related to each other. In particular, I will describe the Young projection operators whose images give the irreducible representations. Then I will describe how computations of traces of maps of symmetric group representations can be used to compute traces of maps of  $U(n)$  representations for all  $n$  at once and will explain how this can be expressed in the language of traced monoidal categories. Finally, I will give new formulas which use the Young projection operators to construct a family of \*orthogonal\* projections which are convenient for such computations. If there is time, I will explain why this is potentially useful for computations in lattice gauge theory.

### **Louis Crane: From TQFT to quantum gravity to the UFT**

I will outline how the categorical construction techniques from TQFT could be extended to construct a unified field theory

### **Sebastian de Haro: Holographic Renormalization and the Holographic Cotton Tensor**

After introducing the technique of holographic renormalization, I will focus on the case of self-dual four-dimensional geometries which are asymptotically locally AdS (anti-de Sitter) and discuss some of their holographic properties. I will show that the holographic stress energy tensor encoded in these geometries is the Cotton tensor of the conformal boundary geometry.

### **Joao Faria Martins: Categorifying the Knizhnik-Zamolodchikov connection**

In the context of higher gauge theory, we construct a flat and fake flat 2-connection in the configuration space of  $n$  particles in the complex plane, which categorifies the Knizhnik-Zamolodchikov connection. We define the differential crossed module of horizontal 2-chord diagrams, categorifying the Lie algebra of horizontal chord diagrams in a set of  $n$  parallel copies of the interval, therefore yielding a categorification of the 4-term relation. We discuss the representation theory of differential crossed modules in chain-complexes of vector spaces, which makes it possible to formulate the notion of an infinitesimal 2-R matrix in a differential crossed module. We present several open problems.

The talk is based on joint work with Lucio Cirio

### **Dana Fine: A rigorous path integral for supersymmetric quantum mechanics**

We construct an approximate path integral for the propagator at time  $t$  in imaginary-time supersymmetric quantum mechanics on a Riemannian manifold. The approximate path integral takes the form of a product, depending on a partition of  $[0,t]$ , of approximate heat kernels. The fine-partition limit exists, thereby providing a rigorous definition of the path integral, and agrees with the heat kernel for the Laplace-Beltrami operator on forms. We extract the small- $t$  behavior of the limit, which, upon restricting the paths to loops, agrees with the steepest-descent approximation to the heuristic path integral. This yields a path integral proof of the Gauss-Bonnet-Chern theorem.

## **Bjoern Gohla: Toward Power Objects of Gray-Categories**

We shall report on an ongoing project to define an internal hom for Gray-categories, which can serve to describe tri-connections on manifolds.

## **Sergei Gukov: Super-A-polynomial**

The generalized volume conjecture states that “color dependence” of the colored Jones polynomial is governed by an algebraic variety, the zero locus of the A-polynomial (for knots) or, more generally, by character variety (for links or higher-rank quantum group invariants). This relation, based on  $SL(2, \mathbb{C})$  Chern-Simons theory, explains known facts and predicts many new ones.

In particular, since the colored Jones polynomial can be categorified to a doubly-graded homology theory, one may wonder whether the generalized (or quantum) volume conjecture admits a natural categorification. In these lectures, I will argue that the answer to this question is “yes” and introduce a two-parameter deformation of the A-polynomial that describes the “color behavior” of the HOMFLY homology, much like the ordinary A-polynomial does it for the colored Jones polynomial. This deformation, called the super-A-polynomial, is strong enough to distinguish mutants, and its most interesting properties include relation to knot contact homology and knot Floer homology.

These lectures are based on a joint work with Hiroyuki Fuji, Marko Stosic, and Piotr Sulkowski.

## **Marco Mackaay: $sl_3$ web algebras and categorical Howe duality**

Kuperberg gave a diagrammatic presentation of the representation theory of quantum  $sl_3$ , using spiders or webs. In order to categorify the quantum  $sl_3$  knot invariants, Khovanov introduced foams, which are 2d cobordisms with a particular type of singularity.

In my talk, which is on joint work with Weiwei Pan and Daniel Tubbenhauer, I will introduce a new algebra  $K(S)$ , called the web algebra. This algebra is defined using webs and foams and is the  $sl_3$  analogue of Khovanov’s arc algebra. In this definition,  $S$  is a sign sequence of length  $n$ .

I will first show that  $K(S)$  is a finite-dimensional graded Frobenius algebra.

After that, I will explain skew Howe duality. In this particular instance, it implies that  $W(S)$ , the space of webs with boundary determined by  $S$ , is isomorphic to a certain weight space of  $V$ , the irreducible quantum  $sl_n$  module of highest weight  $(3^k)$ , with  $n=3k$ .

Finally, I will explain the main result of my work with Pan and Tubbenhauer, which is the categorification of this particular instance of Howe duality. It gives a precise relation between  $K(S)$  and the cyclotomic KLR-algebra with highest weight  $(3^k)$ . In particular, this implies that  $K(S)$  categorifies  $W(S)$ .

## **Brendan McLellan: Perturbative Chern-Simons Theory Revisited**

In 2005 Chris Beasley and Edward Witten completely reformulated Chern-Simons theory using contact structures on closed three manifolds. This formulation allowed them to apply the method of non-abelian localization to compute the bare partition function for a special class of contact structures on some closed Seifert three manifolds. This talk will investigate some perturbative aspects of Beasley and Witten’s work with the aim of understanding their reformulation beyond the special contact structures they consider.

## **Ambar Sengupta: Categorical Parallel-transport**

There has been considerable interest and activity in the interface between geometry and category theory. In this talk we discuss the notions of categorical bundles and categorical connections on such bundles. We explore several examples, including a class of examples involving bundles of ‘decorated’ paths over spaces of paths.

Much of this talk reports on ongoing joint work with S. Chatterjee and A. Lahiri.

### **Marko Stosic: Structure of colored HOMFLY homology of knots**

In this talk, we give a list of structural properties of colored HOMFLY homology that categorifies colored HOMFLY polynomial. The main ingredients are the “colored differentials” that relate homological invariants of knots colored by different representations. The differentials are predicted by the physics insights that include BPS states counting and Landau-Ginzburg theories, and give a very rigid structure on colored HOMFLY homology theories.

### **Sara Tavares: Observables in two-dimensional BF theory**

Yang-Mills theory in two-dimensions is remarkably close to a topological theory. In this talk we will readdress the connection with BF theories highlighting the TQFT interpretation of the spinfoam approach to quantization. We will also introduce the mathematical model that allows us to treat surfaces with inbuilt topological defects and how we expect them to relate to operators in the quantum field theory.

### **George Thompson: Casson type invariants from physics**

I evaluate the path integral representation of the Euler characteristic of the space of flat  $SU(n)$  connections on the Lens spaces  $L(p,1)$ .

### **Derek Wise: Extended TQFT in a bimodule 2-category**

Extended topological quantum field theory generalizes topological field theory by going one codimension further, allowing not only manifolds boundary, but manifolds with corners. I will describe work in progress with Jeffrey Morton towards defining an extended topological quantum field theory based on gauge theory for a compact Lie group. In our framework: 1) each compact  $(n-2)$ -manifold is assigned a von Neumann algebra, 2) each  $(n-1)$ -dimensional cobordism is assigned a bimodule of von Neumann algebras, and 3) each  $n$ -dimensional cobordism between cobordisms is assigned a bimodule homomorphism.

### **Alessandro Zampini: Gauge theory on quantum Hopf bundle**

The idea of this talk is to describe how the  $U(1)$  Hopf fibrations over the two dimensional quantum sphere depends on the differential calculi on the various spaces, how it is possible to define a Hodge duality in order to define Laplacians, how these Laplacians are coupled to gauge connections.

## Annex: Full list of participants (including organizers)

Lastname	Firstname	Affiliation	E-mail
Adams	David	Nanyang Technological Univ., Singapore	dhadams@ntu.edu.sg
Cardoso	Gabriel	IST, Lisbon, Portugal	gcardoso@math.ist.utl.pt
Carter	Scott	University of South Alabama, USA	carter@southalabama.edu
Castelo Ferreira	Pedro	GFM-UL, Lisbon, Portugal	pedro.castelo.ferreira@gmail.com
Charles	Laurent	Université Pierre et Marie Curie, France	charles@math.jussieu.fr
Christensen	Dan	University of Western Ontario, Canada	jdc@uwo.ca
Cirafici	Michele	IST, Lisbon, Portugal	cirafici@math.ist.utl.pt
Crane	Louis	Kansas State University, USA	crane@math.ksu.edu
Dawe Martins	Rachel Anne	IST, Lisbon, Portugal	rmartins@math.ist.utl.pt
de Haro	Sebastian	University of Amsterdam, The Netherlands	s.deharo@auc.nl
Esteves	João	IST, Lisbon, Portugal	mesteves@math.ist.utl.pt
Faria Martins	João	CMA-FCT/UNL, Lisbon, Portugal	jn.martins@fct.unl.pt
Fine	Dana	UMass Dartmouth, USA	dfine@umassd.edu
Gukov	Sergei	Caltech, USA	gukov@theory.caltech.edu
Hahn	Atle	GFM-UL, Lisbon, Portugal	atle.hahn@gmx.de
Mackaay	Marco	University of the Algarve, Portugal	mmackaay@ualg.pt
McLellan	Brendan	QGM, Aarhus, Denmark	mclellan@qgm.au.dk
Mikovic	Aleksandar	Universidade Lusófona, Lisbon, Portugal	p1644@ulusofona.pt
Osorio	Alvaro	IST, Lisbon, Portugal	aveliz@gmail.com
Picken	Roger	CAMGSD-IST, Lisbon, Portugal	rpicken@math.ist.utl.pt
Sengupta	Ambar	Louisiana State University, USA	ambarnsg@yahoo.com
Stosic	Marko	IST, Lisbon, Portugal	mstosic@math.ist.utl.pt
Tavares	Sara	University of Nottingham, UK	sara.orianagmail.com
Thompson	George	ICTP, Trieste, Italy	thompson@ictp.it
Wise	Derek	University of Erlangen, Germany	derek.wise@gravity.fau.de
Zampini	Alessandro	L.M.U. Muenchen, Germany	azampini@gmail.com