Program

CAST Summer School and Conference in Budapest

July 9-20, 2012

The CAST Summer School and Conference was held in Budapest, at the Rényi Institute of Mathematics, in July 9-20, 2012. The event attracted 135 mathematicians, out of which 69 graduate students, many postdoctoral fellows and a number of senior research mathematicians.

In the two weeks of the event four minicourses (five lectures each) were delivered, with one discussion session for each, and 16 further research talks were given by top experts. The large number of participants created a very active, vibrant atmosphere on the conference. All the lectures have been recorded, and will be available soon both for the participants, and for those who could not participate this time. The organizers also plan to compile the lecture notes into a proceedings.

Below we give the schedule, the abstracts of the talks and a short description of the expenditure connected to the conference.

1 First week (July 9-13)

	Monday	Tuesday	Wednesday	Thursday	Friday
9:30-10:30	Lipshitz	Lipshitz	Lipshitz	Jen Hom	Lipshitz
11:00-12:00	Cieliebak	Cieliebak	Cieliebak	Cieliebak	Cieliebak
14:00-15:00	Lisca	Colin	Mrowka	Discussion 1	Discussion 2
15:30-16:30	Lekili	Niederkrüger	Ozsváth	Sivek	Bloom
chair:	Stipsicz	Lisca	Colin	Hutchings, Ozbagci	Kedra

2 Second week (July 16-20)

	Monday	Tuesday	Wednesday	Thursday	Friday
9:30-10:30	Hutchings	Hutchings	Hutchings	Hutchings	Ng
11:00-12:00	Hutchings	Ng	Ng	Ng	Ekholm
14:00-15:00	Ekholm	Bourgeois	Abouzaid	Discussion 3	Discussion 4
15:30-16:30	Smith	Rasmussen	Kutluhan	Juhász	Szabó
chair:	Ng	Ekholm	Smith	Rasmussen	Bourgeois

- Disussion 1 will be run by Kai Cieliebak, and will discuss problems related to his mini-course
- Disussion 2 will be run by Jen Hom, corresponding to problems and homeworks of the minicourse of Robert Lipshitz
- Disussion 3 will be run by Vinicius Gripp, corresponding to problems and homeworks of the minicourse of Michael Hutchings
- Discussion 4 will be run by Chris Cornwell, corresponding to problems and homeworks of the minicourse of Lenny Ng

3 Mini-courses

• Robert Lipshitz: Bordered Heegaard Floer homology

Overall goals: Bordered Floer homology is an extension of Ozsvath-Szabó's Heegaard Floer homology to 3-manifolds with boundary, enjoying good properties with respect to gluings. The first few lectures will introduce the key features of bordered Heegaard Floer homology: its formal structure, a precise definition of the invariants of surfaces, a sketch of the definitions of the 3-manifold invariants, and some hints at the analysis

underlying the theory. In the last lectures we will talk about bordered Heegaard Floer homology as a computational tool, both in theory and practice. Most of the results we will talk about are joint work with Peter Ozsvath and Dylan Thurston; the remaining ones are due to subsets of {Denis Auroux, Jen Hom, Adam Levine, Ina Petkova, Rumen Zarev}.

Lecture-by-lecture plan:

Lecture 1: Overview of bordered Floer homology. Arc diagrams for surfaces. The algebra associated to an arc diagram.

Lecture 2: Bordered Heegaard diagrams. Holomorphic curves and Reeb chords. The "type D" module associated to a bordered 3-manifold. Example: the surgery exact triangle.

Lecture 3: Analysis underlying the invariants and the pairing theorem.

Lecture 4: Computing with bordered Floer homology I: knot complements.

Lecture 5: Computing with bordered Floer homology II: factoring mapping classes.

• Kai Cieliebak: Stein structures: existence and flexibility

Abstract: In 1990 Eliashberg gave a topological characterization of those smooth manifolds of real dimension greater than four that admit the structure of a Stein complex manifold. The goal of these lectures is to explain the proof of Eliashberg's theorem and more recent results on the topology of Stein structures such as a Stein version of the h-cobordism theorem, a uniqueness theorem for subcritical Stein structures, and a remarkable class of "flexible" Stein structures that also satisfy uniqueness. Prerequisites are basic notions of symplectic and differential topology, in particlar Morse theory. The relevant complex geometry will be explained in the lectures.

Lecture-by-lecture plan:

Lecture 1: ***J*-convexity.** J-onvex functions and hypersurfaces, definitions and examples of Stein manifolds, topology of Stein manifolds, continuous J-convex functions, Richberg's smoothing theorem.

Lecture 2: ***J*-convex surroundings.** J-convex functions near totaly real submanifolds, from families of hypersurfaces to J-convex functions, J-convex model functions, Stein neighbourhoods.

Lecture 3: **Existence of Stein structures.** h-pinciples for totally real embeddings and isotropic immersions, Whitney's theory of half-dimensional immersions, stabilization of Legendrian submanifolds, proof of Eliashberg's existence theorem, failure in dimension four.

Lecture 4: **Morse-Smale theory for J-convex functions.** h-cobordism theorem, moving critical levels, moving attaching spheres, creation and cancellation of critical points, Stein h-cobordism theorem.

Lecture 5: **Flexibility of Stein structures.** Uniqueness of subcritical Stein structures, exotic Stein structures, Murphy's h-principle for loose Legendrian knots, uniqueness of flexible Stein structures, applications to symplectomorphisms and pseudo-isotopies.

• Michael Hutchings: Embedded contact homology, cobordism maps, and applications

Abstract: The first part of this course will explain the definition of embedded contact homology, review some necessary background, and compute some basic examples. The second part of the course will discuss the recent construction of cobordism maps on ECH using Seiberg-Witten theory and present some applications to symplectic embedding problems and refinements of the Weinstein conjecture.

• Lenny Ng: Knot contact homology and applications

Abstract: Knot contact homology is a Floer-theoretic knot invariant derived from counting holomorphic curves in the cotangent bundle of $\mathbb{R}^3$ with Lagrangian boundary condition on the conormal bundle to the knot. Among other things, this can be used to produce a three-variable polynomial that detects the unknot and conjecturally contains many known knot invariants; a different part of the package yields an effective invariant of transverse knots in $\mathbb{R}^3$. This minicourse is an introduction to knot contact homology as well as connections to other knot invariants, transverse knot theory, and physics. It includes joint work with Tobias Ekholm, John Etnyre, and Michael Sullivan, and will culminate in a lecture by Tobias Ekholm on some recent developments linking this subject to string theory (joint between the two of us, Mina Aganagic, Robbert Dijkgraaf, and Cumrun Vafa).

Lecture-by-lecture plan:

Lecture 1: **Legendrian contact homology.** Holomorphic curves, differential graded algebras, and augmentations, with special emphasis on Legendrian knots in $\mathbb{R}^3$.

Lecture 2: **The conormal construction and Legendrian contact homology in five dimensions.** Braids and the algebraic formulation of knot contact homology.

Lecture 3: **The cord algebra.** The augmentation polynomial and its relation to the A-polynomial and the HOMFLY polynomial.

Lecture 4: **Transverse homology, the filtered version associated to transverse knots.** Effectiveness and comparison to the transverse invariant in knot Floer homology.

Lecture 5 (given by Tobias Ekholm): **Augmentations and open Gromov-Witten invariants in the resolved conifold.**

4 Titles and abstracts

- *Paolo Lisca: Stein fillable contact structures versus positive open books*

  Abstract: A 2–dimensional open book (S, h) determines a closed, oriented 3–manifold $Y_{(S,h)}$ and a contact structure $\xi_{(S,h)}$ on $Y_{(S,h)}$. The contact structure $\xi_{(S,h)}$ is Stein fillable if h is *positive*, i.e. it can be written as a product of right–handed Dehn twists. Work of Wendl implies that when S has genus zero the converse holds, that is

  $$\xi_{(S,h)} \text{ Stein fillable} \quad \Longrightarrow \quad h \text{ positive.} \qquad (*)$$

  On the other hand, Wand and Baker/Etnyre/Van Horn–Morris constructed counterexamples to (**??**) with S of genus two. After recalling the necessary background material, I will describe a proof of (**??**) assuming that S is a one–holed torus and $Y_{(S,h)}$ is a Heegaard Floer L–space.

- *Yankı Lekili: Symplectic topology of rational blowdowns*

  Abstract: We study some finite quotients of the A_n Milnor fibre which coincide with the Stein surfaces that appear in Fintushel and Stern's rational blowdown construction. We show that these Stein surfaces have no exact Lagrangian submanifolds by using the already available and deep understanding of the Fukaya category of the A_n Milnor fibre coming from homological mirror symmetry. On the contrary, we find Floer theoretically essential monotone Lagrangian tori, finitely covered by the monotone tori that we studied in the A_n Milnor fibre. We conclude that these Stein surfaces have non-vanishing symplectic cohomology. This is joint work with Maydanskiy.

- *Vincent Colin: HF=ECH, from hat to plus*

  Abstract: I will describe a cobordism which allows to define a chain map from the plus version of Heegaard Floer homology to Embedded Contact Homology. It commutes with the U-maps defined on both sides and induces an isomorphism at the homology level. This is joint work with Paolo Ghiggini and Ko Honda.

- *Klaus Neiderkrüger: Fillability of contact structures in higher dimension*

  Abstract:

- *Tom Mrowka: Foams, webs and instantons*

  Abstract:

- *Peter Ozsváth: Bordered Floer homology and the spectral sequence of a double branched cover*

  Abstract:

- *Steven Sivek: A contact invariant in sutured monopole homology*

  Abstract: Kronheimer and Mrowka recently used monopole Floer homology to define an invariant of sutured manifolds, following work of Juh?sz in Heegaard Floer homology. Contact 3-manifolds with boundary are natural examples of such manifolds. In this talk, I will construct an invariant of a contact structure as an element of the associated sutured monopole homology group. I will discuss several interesting properties of this invariant, including gluing maps which are analogous to the Heegaard Floer sutured gluing maps of Honda, Kazez, and Mati?, and applications to Legendrian knots. This is joint work with John Baldwin.

- *Jonathan Bloom: A bordered monopole Floer theory*

  Abstract: I will report on work-in-progress to develop a bordered monopole Floer theory. We associate an algebra to a surface, a module to 3-manifold with boundary, and a map of modules to a 4-manifold with corners (all in the A-infinity sense). These structures satisfy the natural gluing theorems inherent in a 4-dimensional TQFT with corners, and are closely related to Khovanov's invariant of tangles and Szabó's geometric spectral sequence. This is joint work with John Baldwin.

- *Tobias Ekholm: Exact Lagrangian immersions with a single double point*

  Abstract: We show that if a closed orientable $2k$-manifold K, $k > 2$, with Euler characteristic not equal to -2 admits an exact Lagrangian immersion into complex $2k$-space with one transverse double point and no other self intersections, then K is diffeomorphic to the sphere. The proof combines Floer homological arguments with a detailed study of moduli spaces of holomorphic disks with Lagrangian boundary conditions determined by K. We will also discuss related results in the odd dimensional case. The talk reports on joint work with Ivan Smith.

- *Ivan Smith: Symplectic Khovanov Cohomology I*

- *Mohammed Abouzaid: Symplectic Khovanov Cohomology II*

  Abstract: Khovanov homology is an invariant of oriented links in the 3-sphere, defined either via elementary combinatorics of a diagrammatic calculus, or via representation theory and homological algebra. Symplectic Khovanov cohomology is a Lagrangian Floer-theoretic invariant of oriented links in the 3-sphere, which is conjecturally isomorphic to its combinatorial sibling. We will outline a proof of that equivalence in characteristic zero. The proof involves a formality theorem for certain Fukaya categories and bimodules which underlie the definition of the geometric invariant. The first lecture will introduce relevant background, and the second will give the main formality argument.

- *Frederic Bourgeois: On the geography of Legendrian submanifolds*

Abstract: Important classes of Legendrian submanifolds can be described via generating families. This approach can be used to define Legendrian invariants. I will describe the construction of Legendrian submanifolds which have all possible values for this invariant. This is joint work with Josh Sabloff and Lisa Traynor. I will also explain that the generating family approach is also related to the use of several augmentations in Legendrian contact homology. This is joint work with Baptiste Chantraine.

- *Jake Rasmussen: Khovanov homology of torus knots*

  Abstract: I'll describe some conjectures relating the Khovanov homology of torus knots to objects in geometry (Hilbert schemes of points on a plane curve) and algebra (Cherednik algebras), and discuss their relation to the theory of colored Khovanov homology.

- *Cagatay Kutluhan: $HF = HM$*

  Abstract: This talk will be about joint work with Yi-Jen Lee and Cliffor H. Taubes on the equivalence of Heegaard Floer and Seiberg–Witten Floer homologies of closed 3-manifolds. Our proof of this equivalence uses certain twisted versions of Hutchings' embedded contact homology for stable Hamiltonian strtuctures as a mediator.

- *András Juhász: Naturality of Heegaard Floer homology*

  Abstract: Previously, the Heegaard Floer invariants of 3-manifolds and links have only been defined up to isomorphism. This suffices for many applications, but to be able to talk about diffeomorphism or cobordism maps on such groups one needs a functorial construction. We will provide the missing ingredient for functoriality, and define the action of the based mapping class group. Our approach relies on the bifurcation analysis of 2-parameter families of gradient vector fields. This is joint work with Dylan Thurston and Peter Ozsváth.

- *Zoltán Szabó: Knot Floer homology and bordered algebras*

  Abstract:

5 Financial report

The event was mainly sponsored by the CAST network (50,000 Euros), which support was complemented by the *Lendület* program of the Hungarian Academy of Sciences through the *Alacsony dimenziós topológia* grant (PI: András Stipsicz) and by the Rényi Institute.The total budget of the conference was 70,362 Euros, in the following distribution:

- Travel support: 27,631 Euros. This amount covered full travel support for all the European participants and some of the lecturers, and partial travel support for 19 participants from overseas. For local travel within Budapest 154 weekly passes were handed out for those who needed to use public transportation in order to get to the Rényi Institute. The Lendület program covered 16,562 Euros of the above sum, while the rest of 11,069 Euros were financed by CAST.

- Housing support: 32,505 Euros. All students were hosted by a youth hostel; for postdoctoral participants apartments were rented near the Institute, while senior participants were staying in Hotel Gellért. The Lendület program covered 2,377 Euros, the Rényi Institute contributed 1,423 Euros, while 30,128 Euros of the CAST support was used for housing expenses.

- Students and postdoctoral participants received breakfast, and all participants received meal coupons for the days of the lectures in the nearby dining hall of the Eötvös University. The total cost of this was 10,226 Euros (covered by the CAST support).

6 Participants

Convenor:

- Professor Andras Stipsicz Budapest, (HU) Convenor

Speakers:

- Dr. Mohammed Abouzaid New York, NY, (US) Speaker
- Dr. Jonathan Bloom Cambridge, MA, (US) Speaker
- Professor Frederic Bourgeois Bruxelles, (BE) Speaker
- Professor Kai Cieliebak Munich, (DE) Speaker
- Professor Vincent Colin Nantes, (FR) Speaker
- Dr. Christofer Cornwell Durham, NC, (US) Speaker
- Professor Tobias Ekholm Uppsala, (SE) Speaker
- Mr. Vinicius Gripp Berkeley, CA, (US) Speaker
- Dr. Jennifer Hom New York, NY, (US) Speaker
- Professor Michael Hutchings Berkeley, CA, (US) Speaker
- Dr. Andras Juhasz Cambridge, (UK) Speaker
- Dr. Cagatay Kutluhan Cambridge, MA, (US) Speaker
- Dr. Yanki Lekili Cambridge, (UK) Speaker
- Dr. Robert Lipshitz New York, NY, (US) Speaker
- Professor Paolo Lisca Pisa, (IT) Speaker
- Professor Tomasz Mrowka Cambridge, (US) Speaker
- Dr. Lenny Ng Durham, NC, (US) Speaker
- Dr. Klaus Niederkruger Toulouse Cedex 9, (FR) Speaker
- Professor Peter Ozsvath Princeton, (US) Speaker
- Professor Jacob Rasmussen Cambridge, (UK) Speaker
- Dr. Steven Sivek Cambridge, MA, (US) Speaker
- Dr. Ivan Smith Cambridge, (UK) Speaker
- Professor Zoltan Szabo Princeton, (US) Speaker

Participants:

- Mr. Fatih Celik Istanbul, (TR) Participant
- Mr. Paolo Aceto Firenze, (IT) Participant
- Mr. Russel Avdek Los Angeles, CA, (US) Participant
- Miss Emel Aydin Bornova, (TR) Participant
- Mr. Stefan Behrens Bonn, (DE) Participant
- Dr. Ahmet Beyaz Ankara, (TR) Participant
- Mr. Karatug Bircan Istanbul, (TR) Participant
- Mr. Jozsef Bodnar Budapest, (HU) Participant
- Mr. Matthew Borman Chicago, IL, (US) Participant
- Dr. Maciej Borodzik Warsaw, (PL) Participant
- Dr. Till Brnnle Bruxelles, (BE) Participant
- Dr. Daniele Celoria Pisa, (IT) Participant
- Mr. Mustafa Cengiz Istanbul, (TR) Participant
- Dr. Hyunjoo Cho Rochester, (US) Participant
- Ms. Corrin Clarkson New York, NY, (US) Participant
- Dr. Moshe Cohen Ramat Gan, (IL) Participant
- Mr. Sylvan Courte Lyon Cedex 07, (FR) Participant
- Dr. Peter Csorba Budapest, (HU) Participant
- Mr. Georgios Dimitroglou Uppsala, (SE) Participant
- Mr. Sean Droms Charlottesville, VA, (US) Participant
- Mr. Sinan Eden Lisbon, (PT) Participant
- Mrs. Ilknur Egilmez Istanbul, (TR) Participant
- Mr. Albin Eriksson stman Uppsala, (SE) Participant
- Mr. Dmitry Faifman Tel-Aviv, (IL) Participant
- Dr. David Farris Bangalore, (IN) Participant
- Dr. Ana Garcia Lecuona University Park, PA, (US) Participant
- Dr. Allison Gilmore Los Angeles, CA, (US) Participant

- Dr. Marco Golla Cambridge, (UK) Participant
- Dr. Roman Golovko Montreal, (CA) Participant
- Mr. Forrest Gordon Baton Rouge, LA, (US) Participant
- Mr. Jean Gutt Bruxelles, (BE) Participant
- Mr. Thomas Guyard Nantes, (FR) Participant
- Mr. Adam Gyenge Budapest, (HU) Participant
- Ms. Ina Halacheva Toronto, ON, (CA) Participant
- Mr. Jonathan Hales Stony Brook, (US) Participant
- Mr. Jonathan Hanselman New York, NY, (US) Participant
- Mr. Luis Haug Zurich, (CH) Participant
- Ms. Krsiten Hendricks New York, NY, (US) Participant
- Mr. Ramon Horvath Uppsala, (SE) Participant
- Mr. Marton Horvath Budapest, (HU) Participant
- Ms. Diana Hubbard Chestnut Hill, MA, (US) Participant
- Dr. Emoke Imre Budapest, (HU) Participant
- Mr. Dominic Jnichen Koln, (DE) Participant
- Miss Hioyin Jung Seoul, (KR) Participant
- Mr. Boldizsar Kalmar Budapest, (HU) Participant
- Ms. Cecilia Karlsson Uppsala, (SE) Participant
- Dr. Keiko Kawamuro Iowa City, (US) Participant
- Dr. Mary Kearny Baton Rouge, LA, (US) Participant
- Dr. Jaroslaw Kedra Aberdeen, (UK) Participant
- Dr. Michael Khanevsky Princeton, (US) Participant
- Professor Adam Knapp New York, NY, (US) Participant
- Dr. Thomas Kragh Cambridge, (US) Participant
- Mr. David Krcatovich East Lansing, MI, (US) Participant
- Mr. Myeonggi Kwon Seoul, (KR) Participant
- Mr. Peter Lambert-Cole Baton Rouge, LA, (US) Participant

- Mr. Kyle Larson Austin, TX, (US) Participant
- Mr. Tamas Laszlo Budapest, (HU) Participant
- Miss Juhyun Lee Seoul, (KR) Participant
- Dr. Adam Levine Waltham, MA, (US) Participant
- Mr. Francesco Lin Pisa, (IT) Participant
- Dr. Andrew Lobb Durham, (UK) Participant
- Ms. Emilia Lundberg Uppsala, (SE) Participant
- Mr. Jean-Mathieu Magot Saint Martin d'Hres, (FR) Participant
- Mr. Andy Manion Princeton, (US) Participant
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- Professor Ferit Ozturk Istanbul, (TR) Participant
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- Mr. Simon Schatz Strasbourg Cedex, (FR) Participant
- Mr. Cotton Seed Princeton, NJ, (US) Participant
- Miss Merve Seyhun Istanbul, (TR) Participant
- Professor Dongsoo Shin Daejeon, (KR) Participant
- Mr. Pavel Silveira Diaz Trieste, (IT) Participant
- Ms. Agnieszka Skrkowska Warsaw, (PL) Participant
- Mr. Romero Solha Barcelon, (ES) Participant
- Mr. Gilberto Spano Nantes, (FR) Participant
- Ms. Laura Starkston Austin, TX, (US) Participant
- Dr. Endre Szabo Budapest, (HU) Participant
- Professor Andras Szucs Budapest, (HU) Participant
- Dr. Tamas Terpai Geneva, (CH) Participant
- Mr. Dmitry Tonkonog Moscow, (RU) Participant
- Mr. Igor Uljarevic Zurich, (CH) Participant
- Mr. Ramon Vera Durham, (UK) Participant
- Dr. Vera Vertesi Cambridge, MA, (US) Participant
- Dr. Stephan Wiesendorf Koln, (DE) Participant
- Mr. Andrew Williams Los Angeles, CA, (US) Participant
- Dr. Toru Yoshiyasu Tokyo, (JP) Participant
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