# ACAT Science meeting - 4460

## Summary

This meeting was a three day workshop focused on the three ACAT subjects, Directed algebraic topology and concurrency (DAT and ATC) and homology computation (HC). There were two types of presentations: 4 PhD-students presented their work and 5 experienced researchers presented a subject and some open problems to be discussed among the participants. One of these lectures was aimed at a more general audience, hence reaching out to other researchers at Aalborg University. There were 14 participants and 6 of these were young people at PhD or PostDoc level.

## Scientific content and discussions

**Five experienced researchers presented a subject and some open problems:**

- S. Mimram: *Trace Spaces: Algorithmics and Applications.*
  **Abstract:** State-space reduction techniques, used primarily in model-checkers, all rely on the idea that some actions are independent, hence could be taken in any (respective) order while put in parallel, without changing the semantics. It is thus not necessary to consider all execution paths in the interleaving semantics of a concurrent program, but rather some equivalence classes. The purpose of this presentation is to describe a new algorithm to compute such equivalence classes, and a representative per class, which is based on ideas originating in algebraic topology. We introduce a geometric semantics of concurrent languages, where programs are interpreted as directed topological spaces, and study its properties in order to devise an algorithm for computing dihomotopy classes of execution paths. In particular, our algorithm is able to compute a control-flow graph for concurrent programs, possibly containing loops, which is "as reduced as possible"in the sense that it generates traces modulo equivalence. A preliminary implementation was achieved, showing promising results towards efficient methods to analyze concurrent programs.

- R.Mardare: *Stone duality in the context of Markov processes.*
  **Abstract:** The Stone representation theorem is one of the recognized landmarks of mathematics. It states that every (abstract) boolean algebra is isomorphic to a boolean algebra of sets. The basic fact of Stone duality is that an algebrization of propositional logic is dual to a certain kind of topological space. Jonsson and Tarski extended this to modal logic and showed a duality between boolean algebras with additional operators: essentially an algebraic form of the modalities, and certain topological spaces with additional operators. The research in this field was very productive in the last decades. The pioneering vision of Grothendieck showed the power of the dual viewpoint in the fruitful applications of duality

to algebraic geometry (schemes). Later the non-commutative geometry programme of Connes showed the possibility of working with dualized point of view. In this talk we will introduce the general concepts of the Stone duality theory and we will show some duality results that we have obtained, in collaboration with Kim Larsen and Prakash Panangaden. Our results refer to a certain duality that exists between the category of Markov processes and a special class of probabilistic (extensions of) Boolean algebras. Unlike other dualities, our representation theorem reflects properties of metric spaces.

- K. Ziemianski: *Path spaces on skeleta of tori*
  **Abstract:** A path on a torus $R^n/Z^n$ is directed, if and only if it lifts to a path on $R^n$ which is non-decreasing on all coordinates. The main goal of my talk is to present the calculation of the space of directed paths on the torus with a single point removed, or equivalently on the maximal non-trivial skeleton of the torus. The calculation uses a homotopy decomposition which is a version of construction due to Martin Raussen.

- E. Haucourt: *Directed Geometric Realisation, Fundamental Categories, their Representations and Applications to Concurrency.*
  **Abstract:** Directed Algebraic Topology (DAT) deals with "directed"spaces. The notion of direction can be formalized in several different ways. We provide two of them that are intuitive and actually equivalent. While simplicial sets are widely accepted as the "good"combinatorial framework for Algebraic Topology, Cubical Sets better fit to DAT. We define the directed geometric realization of a cubical set and the fundamental category of a directed space. Then we focus on those cubical sets which are products of 1-dimensional cubical sets. They are of specific interest for both following reasons: 1) they fundamental categories admit a concise description (Categories of Components) 2) they can be handled by computers (Areas) 3) they can be considered as the control flow structure of concurrent programs (PV language)

- L.Fajstrup: *Components and directed coverings*
  **Abstract:** Components of a directed space has been defined when there are no directed loops. One strategy for defining components in the presence of loops is to lift the problem to a (covering)space without loops and let the components of the space be the image of the components of the covering space. For a space with directed loops, there is a universal dicovering - the set of directed homotopy classes; with a certain topology. The universal dicovering is in general too trivial from a directed topological point of view and hence has too few components - much more than the directed loops is resolved in the universal dicovering. We define a relation on the universal dicovering, and we claim that the quotient is the covering needed to define components.

**Four PhD-students presented their work:**

- G.Jablonski: *Persistent homology of maps.*
  **Abstract:** Persistent homology is an important tool used to analyze the data from sampled topological spaces. In the talk I would like to present how one can use this method to examine the information about a map acting on the sampled space. I would like to present advances in the development of the algorithm and pay more attention to the implementation and the efficiency of the method. This is a joint work with Marian Mrozek and Herbert Edelsbrunner.

- M. Juda: *Computations for trace spaces.*
  **Abstract:** Trace Spaces are infinite dimensional, so it is difficult to use them as an input for computational topology algorithms. Fortunately the homotopy equivalence between Trace Spaces and finite prodsimplicial complexes gives a chance to compute topological invariants for them. Using the equivalence we are able to compute homology groups of Trace Spaces, but we only know a relationship between 0-dimensional homology and PV-programs. During the talk we would like to show examples of PV-programs and calculate homology for them. Our goal is to make a brainstorming session and discuss a possibility of application homological computations into concurrent program analysis.

- F.Weilandt: *Algorithms computing homological invariants for flows.*
  **Abstract:** The flow is a way of looking at the solutions of ordinary differential equations, assuming these are unique. The flow tells us how the states in the phase space develop. Fixing a time t, there is a function that sends each state to the new state after time t. This time-t-map can be analysed using methods from rigorous numerics that were developed by Marian Mrozek. The tool is the Conley index, a topological invariant assigning the homotopy type of a certain quotient space to a given subset of the phase space. It can also be expressed using relative homology groups. The talk will give an introduction to our ideas how the existing algorithms for discrete dynamical systems (i.e., they are described by iterations of a map) can be used to analyse the flow. This will be achieved by applying those algorithms to the time-t-map of the flow.

- P. Brendel: *Homology Computations via Acyclic Subspace.* **Abstract:** (Piotr Brendel, Paweł Dłotko, Marian Mrozek, Natalia Żelazna) Homology computations recently gain vivid attention in science. New methods, enabling fast and memory efficient computations are needed in order to process large simplicial complexes. We present the acyclic subspace reduction algorithm adapted to simplicial complexes. It provides fast and memory efficient preprocessing of the given data. A variant of the method for distributed computations is also presented.

After and between the presentations, there were discussions both among all participants and in smaller subgroups. Moreover, some presentations were extended to those inter-

ested. Participants learnt mathematical tools, new algorithmics available for calculation of homology and also problems in concurrency which may benefit from such tools.

## Assessment of the results and impact of the event on the future direction of the field

Bringing together researchers from different areas and in particular including young people in an open and collaborative athmosphere is bound to have impact. It is of course not easy to see already what the impact will be, but at least two connections seem to be clearly progressing: Areas of algebraic topology which have not been exploited in the applications were presented and it is expected that collaboration on applying such results to both algorithmics and also to theoretical problems in computer science will follow from the workshop. Moreover, another area of computer science, namely logic and in particular duality theorems, was presented. The connections to that should certainly be explored. If this is fruitful, it will be an outcome of the workshop.

## Programme

**Wednesday Sep. 5.**

- 9.30 -10.30 S. Mimram:*Trace Spaces: Algorithmics and Applications.*
  `"http://people.math.aau.dk/~fajstrup/KONFERENCER/mimram.pdf"`
  Slides.
- 11.00-12.00 R. Mardare: *Stone duality in the context of Markov processes.*
  `"http://people.math.aau.dk/~fajstrup/KONFERENCER/radu.pdf"`
  Slides.
- 13.45-14.15 G. Jablonski: *Persistent homology of maps.*
  `"http://people.math.aau.dk/~fajstrup/KONFERENCER/jablonski.pdf"`
  Slides.
- 14.30-15.00 M. Juda: *Computations for trace spaces.*
  `"http://people.math.aau.dk/~fajstrup/KONFERENCER/juda.pdf"`
  Slides.

**Thursday Sep. 6**

- 9.30-10.30 K. Ziemianski:*Path spaces on skeleta of tori.*
- 11.00-11.30 P. Brendel: *Homology Computations via Acyclic Subspace.*
  `"http://people.math.aau.dk/~fajstrup/KONFERENCER/brendel.pdf"`
  Slides.

- 11.45-12.15 F. Weilandt: *Algorithms computing homological invariants for flows.* `"http://people.math.aau.dk/~fajstrup/KONFERENCER/weilandt.pdf"` Slides.
- 14.00-15.00 E. Haucourt: *Directed Geometric Realisation, Fundamental Categories, their Representations and Applications to Concurrency*

**Friday Sep. 7**

- 9.30-10.30 L. Fajstrup: *Components and universal coverings*