ESF- Science Meeting - Scientific report

"Summer School in Probability 2012"
Villa Pallavicini, Bologna, Italy
3-7 September 2012

1. Scientific Summary

The *Summer School in Probability 2012* was directed to students and young researchers who wanted to learn and deepen their knowledge on Probability Theory and some of its most recent applications. Beside presenting and sharing the new developments of the field within the mathematical community, the aim of the school was to provide a stimulating environment where participants could learn new ideas and techniques, and have the opportunity to interact with some of the main experts of the field.

The focus of the school was on Interacting particle systems, that is a branch of Probability aimed to describe and explain macroscopic natural phenomena with models of many interacting particles following the same stochastic law. In particular it was devoted to the applications to Physics and Biology that were presented within the following three mini-courses:

(i) *Stochastic processes in population genetics*
by Jochen Blath, TU Berlin, Germany
(ii) *Localization and diffusivity for directed polymers*
by Hubert Lacoin, Université Paris Dauphine, France
(iii) *Topics on random walks: a short course*
by Dimitri Petritis, Université de Rennes, France

In addition to mini-courses, there were two tutorial lectures to the mini-course *Stochastic processes in population genetics*, held by Noemi Kurt (TU Berlin, Germany), where some key-exercises were discussed with participants, and a short communications session where young researchers and students had the opportunity to present their research topics and results.

2. Scientific Contents

The intent of the *Summer School in Probability 2012* was to present to students and young researchers some of the most recent applications of Probability Theory, especially to Physics and Biology. The general framework was that of *Interacting particle systems* while the particular focus of the school was on the following topics:

- population genetics;
- random polymers;
- random walks on graphs;

Each topic were presented by an international expert with a mini-course of four lectures of one hour and a half each. In what follows the content of each course is briefly described.

(1) The first course was on *Stochastic processes in population genetics* and was presented by Jochen Blath (TU Berlin, Germany).
In this course the audience was introduced to the application of modern stochastic processes in mathematical population genetics, where the ultimate goal is to understand the complex patterns of genetic variation that one observes in real

populations. Since many of the evolutionary processes leading to genetic variation are intrinsically probabilistic, it is not surprising that stochastic modeling becomes more and more important.
In fact, modeling 'probabilistic structures of evolution' is currently a booming and exciting field in applied probability, which brings together modern theory of stochastic processes (Lambda coalescents, generalized Fleming-Viot processes, diffusions, interacting particle systems, etc..) with concrete biological modeling that is ultimately data-driven and therefore also requires the derivation of methods for statistical inference.

(2) The second mini-course was on *Localization and diffusivity for directed polymers* and was presented by Hubert Lacoin (Université Paris Dauphine, France).
The main interest here was in providing a mathematical answer to the following physical problem: consider a polymer chain in a watery solution, which is inhomogeneous in the sense that it contains randomly placed impurities. The interactions with the impurities modify the shape of the polymer chain that can then be considered as a random object. The main question one would like to answer is how these impurities affect the global shape of the polymer chain.
Working in the framework of statistical mechanics, one can face this problem using different kind of simplifications. For example, one can model the polymer with N monomer by an N-steps path on the lattice, and place randomly the impurities (with deterministic or random hydrophilic strength) on the sites of the lattice. The distribution of the polymer shape is then given by a standard Gibbs-Boltzmann formalism.
The aim of this course was to give a mathematical overview of the known answers and conjectures about the above question. Schematically, these were distinguished in two cases:
   (i) When the space in which the polymer lives is high dimensional and the temperature is high-enough, the impurities do not affect the global shape of the polymer (essentially due to spatial averaging of the effect of the impurities).
   (ii) When the dimension is small or when the temperature is low, impurities drastically change the behavior of the polymer.

(3) The third mini-course was on *Topics on random walks* and was presented by Dimitri Petritis (Université de Rennes, France)
In this course the audience was introduced to the problem of determining the asymptotic properties of a random walk on a given graph. This is a classical and general problem with a huge amount of applications that continuously bring to the light new connections with other subjects and open new problems.
As the adapted random walks on graphs combine combinatorial, geometric, and probabilistic information, their asymptotic properties are strictly related to the graph properties. The study of random walks on undirected graphs jumping with uniform probability to the nearest neighbors of the current position is additionally greatly simplified by their weak reversibility property.
During the course some non trivial graphs, like the Penrose lattices, were introduced and reversible random walks on them were defined. The asymptotic properties of these random walks were then studied. The last part of the course was focused on random walks evolving on partially directed graphs and some unexpected properties, mainly due to the directedness, were established. Some developments towards C*-algebras and their applications in the study of random walks on constrained graphs were also presented.

3. Impact of the Event

In the last years Probability Theory has seen a great development mainly thanks to its successful application to various fields of science, starting from statistical physics and computer science, to economics, finance and biology. This interlacement of different research areas has generated a new surge of ideas that have brought to the light many stimulating connections with other branches of mathematics. This continuously renewing interest to the field will give rise to exciting new developments in our rigorous understanding of random phenomena. For these reasons, an optimal development of the research should concern also with the diffusion of the new ideas, particularly toward students and young researchers.

The *Summer School in Probability 2012* has acted along this way with the organization of three mini-courses, held by international experts of the field, on new and stimulating applications of Probability Theory to Physics and Biology.

Beside learning new ideas and techniques, the stimulating atmosphere of the school and the collective activities, such as tutorial lectures and short talk session, has allowed a positive interaction between the participants and gave them the opportunity to strengthen and consolidate relations with researchers from different groups and Universities.
We believe that this will be very fruitful for them, both at level of improvements on their scientific profile and for creating long time collaborations.

4. List of Participants

(including convenors and speakers)

- Dr. Edoardo Angeloni, Pubblica istruzione, Italy
- Dr. Lorenzo Baglioni, Università di Pisa, Italy
- Dr. Stefano Biagi, Università di Bologna, Italy
- Dr. Alessandra Bianchi, Università di Padova, Italy
- Professor Jochen Blath, Technische Universität Berlin, Germnay
- Mr. Eugenio Buzzoni, Università di Genova, Italy
- Professor Massimo Campanino, Università di Bologna, Italy
- Dr. Gioia Carinci, Università di Modena e Reggio Emilia, Italy
- Dr. Francesca Collet, Università di Bologna, Italy
- Professor Pierluigi Contucci, Università di Bologna, Italy
- Dr. Irene Crimaldi, Institute for Advanced Studies (IMT) of Lucca , Italy
- Dr. Giampaolo Cristadoro, Università di Bologna, Italy
- Dr. Giuseppe D'Onofrio, Università di Napoli, Italy
- Professor Mirko Degli Esposti, Università di Bologna, Italy
- Dr. Micaela Fedele, Courant Institute of New York, USA
- Dr. Michele Gianfelice, Università della Calabria , Italy
- Professor Cristian Giardinà, Università di Modena e Reggio Emilia, Italy
- Professor Claudio Giberti, Università di Modena e Reggio Emilia, Italy
- Dr. Noemi Kurt, Technische Universität Berlin, Germany
- Dr. Hubert Lacoin, Université Paris Dauphine, France
- Professor Marco Lenci, Università di Bologna, Italy
- Dr. Immacolata Oliva, Università di Bologna, Italy
- Professor Dimitri Petritis, Université de Rennes, France
- Dr. Louis PierreYves, Université de Poitiers, France
- Dr. Daniele Piras, Università di Roma Tre, Italy

- Mr. Enrico Properzi, Università di Bologna, Italy
- Dr. Elena Pulvirenti, Università di Roma Tre, Italy
- Dr. Daniele Regoli, Università di Padova, Italy
- Dr. Michele Salvi, Technische Universität Berlin, Germany
- Dr. Sergio Simonella, Università di Roma La Sapienza, Italy
- Dr. Mariarosaria Tupputi, Università di Bologna, Italy
- Professor Cecilia Vernia, Università di Modena e Reggio Emilia, Italy

5. Final Program

Mini-courses
Each mini-course was organized in four lectures of one hour and a half each, in the order presented in the timetable at the next page. It follows the detailed program.

(i) Stochastic processes in population genetics
by *Jochen Blath (Technische Universität Berlin)*

L1: Introduction to population genetics, the Wright-Fisher model, random genetic drift, genealogies, Kingman's coalescent, diffusion limits, role of space, the stepping stone model, duality for stochastic processes.
L2: Incorporating mutation, inference of mutation rates, Wright-Fisher diffusion with mutation. Incorporating selection, limiting diffusion, the ancestral selection graph. Cannings models, limiting genealogies, Λ-coalescents, Ξ-coalescents.
L3: Exceptional reproduction and Λ-coalescents, the Möhle-Sagitov limit theorem, inference of mutation rates under Λ-coalescents, the Moran model, the lookdown process, Fleming-Viot processes.
L4: Incorporating recombination and diploid. The ancestral recombination graph. Incorporating exceptional reproduction / selective sweeps. Optional: Spatial models, the symbiotic branching model.

The lectures for this mini-course were integrated with two tutorial lectures by *Noemi Kurt (Technische Universität Berlin)*.

(ii) Localization and diffusivity for directed polymers
by *Hubert Lacoin (Université Paris Dauphine)*

L1: Introduction of Directed Polymer in random environment: Physical motivation and some important questions;
L2: Diffusivity of the polymer trajectories at high temperature in high dimension.
L3: Localization phenomenon in the strong disorder regime.
L4: Superdiffusivity in dimension 1.

(iii) Topics on random walks
by *Dmitri Petritis (Université de Rennes)*

L1: Combinatorial, geometric, and probabilistic aspects of random walks: Monoids, groups, groups, and semi-groupoids; Cayley graphs; regular, constrained, and directed graphs; random walks adapted to graphs.
L2: Reversibility and its consequences: strong and weak reversibility; electric circuit analogy; isoperimetric inequalities; heat kernel estimates.
L3: Random walks on groupoids and semi-groupoids: simple random walk on Penrose tilings; simple random walk on partially directed graphs.

L4: C* -algebras and their groupoids: non-commutative shift spaces; random walks and quantum measurements.

Short talks session
The session was organized in five short talks of 20 minutes each. It follows the list of speakers of this session with the title of the related talk.

- *Lorenzo Baglioni (Università di Pisa)*: Hausdorff dimension of the level sets of the solution of a SPDE
- *Francesca Collet (Università di Bologna)*: The role of disorder in the dynamics of critical fluctuations of mean field models
- *Giuseppe D'Onofrio (Università di Napoli)*: Stochastic models in biomathematics: two examples
- *Micaela Fedele (New York University)*: Inverse problem for multi-species mean-field models
- *Michele Gianfelice (Università della Calabria)*: On the exact asymptotics of finite connection functions for the supercritical Bernoulli bond percolation on Z^d, $d \geq 3$.

SUMMER SCHOOL in PROBABILITY

Bologna - September 3-7, 2012

	Monday, September 3	Tuesday, September 4	Wednesday, September 5	Thursday, September 6	Friday, September 7
09:00 – 09:30	**Registration & Welcome**				
09:30 - 11:00	**Jochen Blath** first lecture on ***"Stochastic processes in population genetics"***	**Noemi Kurt** tutorial for the minicourse ***"Stochastic processes in population genetics"***	**Dimitri Petritis** third lecture on ***"Topics on random walks: a short course"***	**Noemi Kurt** tutorial for the minicourse ***"Stochastic processes in population genetics"***	**Hubert Lacoin** fourth lecture on ***"Localization and diffusivity for directed polymers"***
11:00 - 11:30	Break (coffee & tea)	Break (coffee & tea)	Break (coffee & tea)	Break (coffee & tea)	Break (coffee & tea)
11:30 - 13:00	**Hubert Lacoin** first lecture on ***"Localization and diffusivity for directed polymers"***	**Hubert Lacoin** second lecture on ***"Localization and diffusivity for directed polymers"***	**Hubert Lacoin** third lecture on ***"Localization and diffusivity for directed polymers"***	**Dimitri Petritis** fourth lecture on ***"Topics on random walks: a short course"***	**Jochen Blath** fourth lecture on ***"Stochastic processes in population genetics"***
13:00 - 15:00	Lunch at Villa Pallavicini	Lunch at Villa Pallavicini	Lunch at Villa Pallavicini	Lunch at Villa Pallavicini	Lunch at Villa Pallavicini
15:00 - 16:30	**Dimitri Petritis** first lecture on ***"Topics on random walks: a short course"***	**Dimitri Petritis** second lecture on ***"Topics on random walks: a short course"***	*There are no talks in the afternoon.*	**Jochen Blath** third lecture on ***"Stochastic processes in population genetics"***	*There are no talks in the afternoon.*
16:30 - 17:00	Break (coffee & tea)	Break (coffee & tea)		Break (coffee & tea)	
17:00 - 18:30	**Short talk session**	**Jochen Blath** second lecture on ***"Stochastic processes in population genetics"***			
20:00			Social Dinner		