

## Report on ESF funded Science Meeting 4439

### Summary

- Title: Workshops on *Forcing and Large Cardinals*, September 23–27, and *Descriptive Set Theory*, September 30 – October 4, 2013.  
Part of the ESI Programme *Forcing, Large Cardinals and Descriptive Set Theory*, Vienna, September 9 – October 18, 2013.
- Programme website: <http://www.logic.univie.ac.at/2013/ESI/>
- Venue and main organizer: The Erwin Schrödinger Institute of the University of Vienna (ESI), Vienna, <http://www.esi.ac.at/>
- Programme committee: S. Friedman (Universität Wien), M. Goldstern (Technische Universität Wien), A. Kechris (California Institute of Technology), J. Kellner (Universität Wien), H. Woodin (University of California at Berkeley).

### Scientific Content and Activities

#### Background

Forcing was invented by Paul Cohen in the 1960's to demonstrate that a failure of Cantor's continuum hypothesis is consistent with the usual system of axioms ZFC for set theory. Since then this technique has been used to establish the consistency with ZFC of an immense range of other statements, not only from abstract set theory but also from "mainstream" areas of mathematics. Sometimes more than the consistency of ZFC is required for such a result; one very often assumes (and often needs to assume) the consistency of ZFC together with a *strong axiom of infinity* or *large cardinal axiom*. Together with the methods of inner model theory, it is often the case that when a statement is unprovable in ZFC then its consistency with the ZFC axioms is not only derivable from but in fact equivalent to the consistency of ZFC together with an appropriate large cardinal axiom.

The work on independence results in set theory through the use of large cardinal axioms and forcing can even be applied to statements about definable sets of real numbers, the central topic of descriptive set theory. However as descriptive set theory has developed it has become apparent that many of its aspects are immune to the independence results that apply to other areas of set theory and these aspects lead to profound connections with areas of mathematics such as ergodic theory and functional analysis. In recent years the emphasis in descriptive set theory has therefore shifted to results which are provable in ZFC, and the study of independence has been replaced by a study of *unclassifiability*, whereby it is shown that classification problems in mathematics do *not* admit a reasonable set of invariants.

#### Activities

**The Forcing and Large Cardinals Workshop** This was a very relaxed and interesting workshop featuring 19 lectures on a wide range of aspects of forcing and large cardinal set theory (a 20th lecture was cancelled due to Arthur Apter's regrettably late notice of his inability to attend). Some highlights: Chodounsky told us of the current state of the still-unsolved Katowice problem (can the powerset of  $\omega$  mod finite and the powerset of  $\omega_1$  mod finite be isomorphic?). Holy showed us how to get a  $\Sigma_1$  wellorder of  $H(\kappa^+)$  with  $2^\kappa$  large when  $\kappa = \kappa^{<\kappa}$ . Borodulin-Nadzieja discussed his work with (our local researcher) Farkas and Plebanek on the geometry of analytic  $P$ -ideals. Cummings, Sinapova and Unger each presented cutting-edge results in singular cardinal combinatorics. Sakai and Viale discussed strong forcing axioms. On the more applied side, we heard talks from Koszmider and Dzamonja related to Banach spaces and from Kojman and Rinot on

large graphs. And Mildenberger presented her subtle work on iterated forcing needed to analyze near-coherence classes of ultrafilters.

**The Descriptive Set Theory Workshop** This remarkable workshop presented 30 talks and evidenced the vitality and rapid growth of this area of set theory. In addition to the senior researchers Bartoszyński, Dodos, Gao, Jackson, Kanovei, Rosendal, Solecki and Törnquist, the workshop featured an astounding group of new descriptive set theory PhD's who are already obtaining deep and striking results at the start of their careers. Some highlights: Bartosova exposed her striking work on unique amenability, Kwiatkowska revealed surprising properties of the pseudo-arc and both Aaron Hill and Su Gao discussed rank one measure-preserving transformations. Ground-breaking results were presented by Asger Törnquist who solved long-standing open questions concerning almost disjoint families, by Andrew Marks, who recently obtained the best results on Martin's conjecture regarding Turing-invariant functions, by Marcin Sabok, who established the unclassifiability of separable  $C^*$ -algebras and by Slawek Solecki, who recently obtained a far-reaching generalisation of earlier work on the Ramsey theory of trees.

## Outcomes and achievements

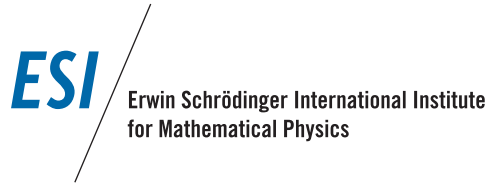
Below are some of the work created as a result of the programme, or important results obtained and specific collaborations that took place; this information was provided by the participants.

- Borodulin-Nadziejka, Piotr: I worked with Barnabas Farkas on our preprint "Representations of analytic  $P$ -ideals in Banach spaces and in Polish groups" (20 pages).
- Elekes, Marton: Thanks to discussions with various participants (Solecki, Dodos, Debs ...) we made significant progress, which will result in a paper (M. Elekes, V. Kiss, Z. Vidnyan-szky) "Ranks on the Baire class  $\alpha$  functions", approx. 25 pages. The main step forward since my talk is that we have found very nicely-behaved ranks on the Baire  $\alpha$  classes and have an essentially optimal generalization of the classical results of Louveau and Kechris about ranks on the Baire class 1 functions.
- Farah, Ilijas: I mostly talked to Magidor about our paper in preparation "Omitting types in logic of metric structures is hard." At present the paper has 19 pages.
- Holy, Peter: We worked out the essential parts of the following: "Locally Lightface  $\Sigma_1$ -definable Well-Orders Of  $H(\kappa^+)$ ", Peter Holy and Philipp Luecke (25 pages). We obtain the following: Assume  $V = L$ . For any regular uncountable cardinal  $\kappa$  that is not the successor of a singular cardinal, there is a forcing extension in which  $2^\kappa$  has any reasonably prescribed value  $\lambda$ , cofinalities up to  $\lambda$  agree between the forcing extension and  $L$  and there is a  $\Sigma_1$ -definable wellorder of  $H(\kappa^+)$  using only  $\kappa$  as a parameter. I also had discussions about generalized almost disjoint coding with David Schrittesser and Philipp Luecke and with Sy Friedman about future work in the Outer Model Programme.
- Ikegami, Daisuke: I am preparing a preprint entitled "Inner models from logics". I also discussed with Ralf Schindler the Necessity Maximality Principle for local statements due to Hamkins and Ralf observed that  $ZFC +$  "NMP for local statements" is equiconsistent with  $ZFC +$  "There exist a proper class of Woodin cardinals" (the implication from large cardinals to BNMP was already known by Hamkins).
- Kanovei, Vladimir: The following is a preprint prepared as a result of the ESI 2013 Programme: "On countable cofinality of definable chains in Borel partial orders", 6 pages. We prove that in some cases definable chains of Borel partial orderings are necessarily countably cofinal.
- Leiderman, Arkady: I had fruitful conversations with Lyubomyr Zdomskyy about my joint work with S.S. Gabrielyan and J. Kakol, "The strong Pytkeev property for topological

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groups and topological vector spaces". It helped me to finalize the presentation of our results and to submit the paper for publication in *Monatshefte für Mathematik*.

- Rosendal, Christian: I continued working on my paper "Large scale geometry of metrisable groups" (50 pages +).
- Schrittester, David: My discussions with Sy Friedman and Ralf Schindler resulted in our joint preprint "Coding over core models", 17 pp. I also met with Asger Törnquist to discuss the lifting problem of the automorphism group of a measure space and with Yurii Khomskii to discuss problems in the separation of ideals in the projective hierarchy.
- Selivanov, Victor: I met with several participants (including Matthias Schroeder, Luca Motto Ros, Andre Nies and Alessandro Andretta) to discuss different issues from computability theory and descriptive set theory. A concrete project resulted from discussions with Schroeder and we are now working on a joint paper "More hierarchies of  $qcp_0$ -spaces".
- Sinapova, Dima: I started a project with Spencer Unger. We are analyzing the pcf structure in the model of our paper "Combinatorics at  $\aleph_\omega$ ". Right now we have a first draft preprint, titled "Scales in Combinatorics at  $\aleph_\omega$ ", 6 pages. Our conjecture is that our model will show the consistency of not SCH and no very good scale.
- Tsaban, Boaz: I worked on the preprint "Selective covering properties of product spaces, II:  $\gamma$  spaces" (Arnold W. Miller, Boaz Tsaban, Lyubomyr Zdomskyy), 24 pp. I also collaborated with Zdomskyy to characterize exactly when Hurewicz spaces remain Hurewicz in an extension of the universe by Cohen forcing, thus solving a problem of Scheepers and Tall.



DVR 0065528

**Workshop on  
“Forcing and Large Cardinals”**

**organized by**

**Sy-David Friedman, Martin Goldstern, Alexander Kechris,  
Jakob Kellner and W. Hugh Woodin**

**September 23 - 27, 2013**

• **Monday, September 23, 2013**

09:00 – 09:30 **Opening & Registration**

09:30 – 10:20 **David Chodounsky**

*A report on the Katowice problem*

10:30 – 11:00 *coffee break*

11:00 – 11:50 **Peter Holy**

*Locally  $\Sigma_1$ -definable Wellorders of  $H(\kappa^+)$*

12:00 – 15:30 *lunch break*

15:30 – 16:00 **Piotr Borodulin-Nadzieja**

*Geometry of analytic  $P$ -ideals*

16:15 – 16:45 **Dima Sinapova**

*Very good scales, squares, and SCH*

• **Tuesday, September 24, 2013**

09:30 – 10:20 **Heike Mildenberger**

*Finitely Many Near-Coherence Classes of Ultrafilters*

10:30 – 11:00 *coffee break*

11:00 – 11:50 **Spencer Unger**

*Aronszajn trees and the successors of a singular cardinal*

12:00 – 14:00 *lunch break*

14:00 – 14:30 **James Cummings**

*Forcing and the combinatorics of successors of singulars*

14:30 – 15:30 *coffee break*

15:30 – 16:00 **Piotr Koszmidar**

*The Banach space  $C(N^*)$  in the Cohen model*

16:15 – 16:45 **Hiroshi Sakai**

*Separation of  $MA^+$  ( $\sigma$ -closed) from reflection principles*

- **Wednesday, September 25, 2013**

09:30 – 10:20 **Ralf Schindler**

*Does  $\Pi_1^1$  determinacy yield  $0^\#$ ?*

10:30 – 11:00 *coffee break*

11:00 – 11:50 **Matteo Viale**

*Category forcings,  $MM^{+++}$  and the generic absoluteness of the theory of the Chang model  $L(\text{Ord}^{\omega_1})$*

12:00 *lunch break/social program*

- **Thursday, September 26, 2013**

09:30 – 10:20 **Philipp Lücke**

*Continuous Images of Closed Sets in Generalized Baire Spaces*

10:30 – 11:00 *coffee break*

11:00 – 11:50 **Assaf Rinot**

*Hedetniemi's conjecture for uncountable graphs*

12:00 – 14:00 *lunch break*

14:00 – 14:30 **Mohammad Golshani**

*Adding a lot of Cohen reals by adding a few.*

14:30 – 15:30 *coffee break*

15:30 – 16:00 **Menachem Kojman**

*Perfect graph coloring*

16:15 – 16:45 **Grigor Sargsyan**

*On the strength of the unique branch hypothesis (UBH)*

- **Friday, September 27, 2013**

09:00 – 09:50 **Daisuke Ikegami**

*Inner models from logics and the generic multiverse*

10:00 – 10:40 *coffee break*

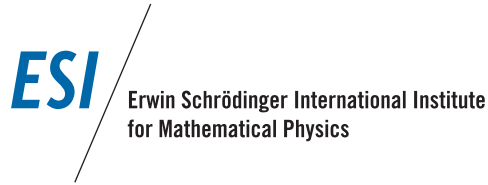
10:40 – 11:10 **Mirna Dzamonja**

*Constructions from one Cohen real*

11:30 – 12:00 **Diego Mejia**

*Rothberger gaps in  $F_\sigma$  ideals*

**All lectures take place in the ESI Boltzmann Lecture Hall**



DVR 0065528

**Workshop on  
“Descriptive Set Theory”**

**organized by**

**Sy-David Friedman, Martin Goldstern, Alexander Kechris,  
Jakob Kellner and W. Hugh Woodin**

**September 30 - October 4, 2013**

• **Monday, September 30, 2013**

09:00 **Opening & Registration**

09:30 – 10:20 **Hiroshi Ando**

*Ultraproducts, QWEP von Neumann algebras, and the Effros-Marechal topology.*

10:20 – 11:10 *coffee break*

11:10 – 12:00 **Dana Bartošová**

*Near ultrafilters, groups of automorphisms and unique amenability.*

12:00 – 14:00 *lunch break*

14:00 – 14:50 **Kostas Beros**

*TBA*

15:05 – 15:35 **Marton Elekes**

*Ranks on Baire class  $\alpha$  functions.*

15:35 – 16:15 *coffee break*

16:15 – 16:45 **Udayan Darji**

*TBA*

17:00 – 17:30 **Miodrag Sokic**

*Semilattices.*

• **Tuesday, October 1, 2013**

09:00 – 09:50 **Vassilis Gregoriades**

*Classes of Polish spaces under effective Borel isomorphism.*

09:50 – 10:20 *coffee break*

10:20 – 11:10 **Aleksandra Kwiatkowska**

*Projective Fraïssé limits and the pseudo-arc.*

11:25 – 11:55 **Samuel Ziegler**

*Weak equivalence and invariant random subgroups.*

11:55 – 13:30 *lunch break*

13:30 *Departure for Social Programme (Rust) from Sensengasse 2a*

- **Wednesday, October 2, 2013**

09:00 – 09:50 **Asger Törnquist**  
*Diagonalizing almost disjoint families.*

09:50 – 10:20 *coffee break*

10:20 – 11:10 **Aaron Hill**  
*TBA*

11:25 – 11:55 **Su Gao**  
*TBA*

11:55 – 14:00 *lunch break*

14:00 – 14:50 **Andrew Marks**  
*TBA*

15:05 – 15:35 **Steve Jackson**  
*TBA*

15:35 – 16:15 *coffee break*

16:15 – 16:45 **Alberto Marcone**  
*The complexity of isometric embeddability between ultrametric Polish spaces with fixed set of distances.*

17:00 – 17:30 **Pandelis Dodos**  
*Some recent results in Ramsey Theory.*

- **Thursday, October 3, 2013**

09:00 – 09:50 **Robin Tucker-Drob**  
*A von-Neumann algebra free proof of solid ergodicity for Bernoulli shifts.*

09:50 – 10:20 *coffee break*

10:20 – 11:10 **Jay Williams**  
*Cone measures, biembeddability, and isomorphism of Kazhdan groups.*

11:25 – 11:55 **Vladimir Kanovei**  
*On the countable cofinality of definable chains in Borel partial orders.*

11:55 – 14:00 *lunch break*

14:00 – 14:50 **Christian Rosendal**  
*Large scale geometry of metrisable groups.*

15:05 – 15:35 **Maciej Malicki**  
*Non-locally compact Polish groups and essentially countable orbit equivalence relations.*

15:35 – 16:15 *coffee break*

16:15 – 16:45 **Scott Schneider**  
*Simultaneous reducibility of pairs of Borel equivalence relations.*

17:00 – 17:30 **Brandon Seward**  
*Locally nilpotent groups and hyperfinite equivalence relations.*

- **Friday, October 4, 2013**

09:00 – 09:50 **Marcin Sabok**  
*Completeness of the isomorphism problem for separable  $C^*$ -algebras.*

09:50 – 10:20 *coffee break*

10:20 – 11:10 **François Le Maître**  
*Topological generators for full groups.*

11:25 – 11:55 **Kostyantyn Slutskyy**

*Automatic continuity for homomorphisms into free products.*

11:55 – 13:45 *lunch break*

13:45 – 14:35 **Todor Tsankov**

*Weakly almost periodic functions, model-theoretic stability, and minimality of topological groups.*

14:50 – 15:20 **Phillip Wesolek**

*Conjugacy class conditions in totally disconnected locally compact Polish groups.*

15:20 – 16:00 *coffee break*

16:00 – 16:50 **Slawomir Solecki**

*Dual Ramsey theorem for trees.*

17:05 – 17:35 **Tomek Bartoszynski**

*TBA*

**All lectures take place in the ESI Boltzmann Lecture Hall**



## Annex: list of participants

(KGRC is the Kurt Gödel Research Center at the University of Vienna; DMG is the Institute of Discrete Mathematics and Geometry at the University of Technology, Vienna. The speakers are listed in the preceding programs.)

- Hiroshi **Ando**, IHES, France
- Alessandro **Andretta**, University of Torino, Italy
- Carolin **Antos-Kuby**, KGRC
- Giorgio **Audrito**, University of Torino, Italy
- Joan **Bagaria**, University of Barcelona, Spain
- Dana **Bartosova**, University of Toronto, Canada
- Tomek **Bartoszynski**, National Science Foundation, USA
- Kostas **Beros**, University of Wisconsin, USA
- Piotr **Borodulin-Nadzieja**, University of Wroclaw, Poland
- Riccardo **Camerlo**, Politecnico di Torino, Italy
- David **Chodounsky**, Academy of Sciences of the Czech Republic
- John **Clemens**,
- James **Cummings**, Carnegie-Mellon University, USA
- Udayan **Darji**, University of Louisville, Kentucky, USA
- Gabriel **Debs**, University of Le Havre and Institut Mathematique de Jussieu, France
- Vincenzo **Dimonte**, DMG
- Pandelis **Dodos**, University of Athens, Greece
- Gregor **Dolinar**, University of Ljubljana, Slovenia
- Ohad **Drucker**, The Hebrew University, Jerusalem, Israel
- Mirna **Dzamonja**, University of East Anglia, Norwich, UK
- Katsuya **Eda**,
- Marton **Elekes**, Renyi Institut, Budapest, Hungary
- Ilijas **Farah**, York University, Toronto, Canada
- Barnabas **Farkas**, Budapest University of Technology and Economics, Hungary
- Arthur **Fischer**, KGRC
- Vera **Fischer**, KGRC
- Laura **Fontanella**, KGRC
- Sy-David **Friedman**, KGRC
- Sakae **Fuchino**, Kobe University, Japan

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- Su **Gao**, University of North Texas, USA
  - Micha **Gavrilovich**, St.Petersburg
  - Martin **Goldstern**, DMG
  - Mohammad **Golshani**, IPM, Teheran, Iran
  - Martin **Grebik**, Charles University in Prague, Czech Republic
  - Vassilis **Gregoriades**, Universität Darmstadt, Germany
  - Aaron **Hill**, University of North Texas, USA
  - Stefan **Hoffelner**, KGRC
  - Peter **Holy**, University of Bristol, UK
  - Radek **Honzik**, KGRC
  - Daisuke **Ikegami**, University of California, Berkeley, USA
  - Steve **Jackson**, University of North Texas, USA
  - Istvan **Juhasz**, Renyi Institut, Budapest, Hungary
  - Adriane **Kaïchouh**, Université Lyon 1, France
  - Vladimir **Kanovei**, Kharkevich Institute, Moscow, Russia
  - Asaf **Karagila**, The Hebrew University, Jerusalem, Israel
  - Ahmad **Karimi**, Tarbiat Modares University, Tehran, Iran
  - Jakob **Kellner**, KGRC
  - Juliette **Kennedy**, University of Helsinki, Finland
  - Yurii **Khomski**, KGRC
  - Menachem **Kojman**, Ben Gurion University, Beer-Sheva, Israel
  - Piotr **Koszmider**, Polish Academy, Warsaw
  - Vadim **Kulikov**, KGRC
  - Aleksandra **Kwiatkowska**, University of California, Los Angeles, USA
  - François **Le Maître**, ENS de Lyon, France
  - Arkady **Leiderman** , Ben Gurion University, Israel
  - Stephane **Leroux**, TU Darmstadt, Germany
  - Philipp **Lücke**, Universität Bonn, Germany
  - Martino **Lupini**, York University, Toronto, Canada
  - Menachem **Magidor**, The Hebrew University, Jerusalem, Israel
  - Maciej **Malicki**, Polish Academy, Warsaw, Poland
  - Alberto **Marcone**, University of Udine, Italy
  - Andrew **Marks**, Caltech, USA

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- Andrea **Medini**, KGRC
  - Diego **Mejia**, Kobe University, Japan
  - Heike **Mildenberger**, Universität Freiburg, Germany
  - Diana Carolina **Montoya Amaya**, KGRC
  - Assaf **Rinot**, Bar-Ilan University, Israel
  - Christian **Rosendal**, University of Illinois at Chicago, USA
  - Marcin **Sabok**, Polish Academy, Warsaw, Poland
  - Hiroshi **Sakai**, Kobe University, Japan
  - Grigor **Sargsyan**, Rutgers University, USA
  - Ralf **Schindler**, Universität Münster, Germany
  - Scott **Schneider**, University of Michigan, USA
  - David **Schrittesser**, Universität Münster, Germany
  - Matthias **Schröder**, KGRC
  - Victor **Selivanov**, A.P. Ershov Institute of Informatics Systems, Siberian Branch Russian Academy of Sciences
  - Brandon **Seward**, University of Michigan, Ann Arbor, USA
  - Dima **Sinapova**, University of Illinois at Chicago, USA
  - Kostyantyn **Slutskyy**, University of Copenhagen, Denmark
  - Miodrag **Sokic**, Caltech, USA
  - Slawomir **Solecki**, University of Illinois, Urbana-Champaign, USA
  - Lajos **Soukup**, Renyi Institut, Budapest, Hungary
  - Anda-Ramona **Tanasie**, KGRC
  - Peter **Telec**,
  - Fabio **Tonti**, KGRC
  - Asger **Törnquist**, University of Copenhagen, Denmark
  - Victor **Torres Perez**, KGRC
  - Boaz **Tsaban**, Bar-Ilan University, Israel
  - Todor **Tsankov**, University of Paris 7, France
  - Robin **Tucker-Drob**, Caltech, USA
  - Spencer **Unger**, Carnegie-Mellon University, USA
  - Jouko **Väänänen**, University of Helsinki, Finland
  - Boban **Velickovic**, University of Paris 7, France
  - Jonathan **Verner**, Charles University in Prague, Czech Republic

- Matteo **Viale**, University of Torino, Italy
- Zoltán **Vidnyánszky**, Eötvös Loránd University, Hungary
- Philip **Welch**, University of Bristol, UK
- Phillip **Wesolek**, University of Illinois, Chicago, USA
- Jay **Williams**, Caltech, USA
- Wolfgang **Wohofsky**, DMG
- Tin Lok **Wong**, KGRC
- Lyubomyr **Zdomskyy**, KGRC
- Yizheng **Zhu**, Universität Münster, Germany
- Samuel **Ziegler**, University of Illinois at Chicago, USA