"Recursion from Matrix Models to Quantum Algebraic Geometry," and "Topological Recursion and Quantum Algebraic Geometry."

(Master Class January 21-25 and International Conference January 28-February 1, Aarhus, Denmark)

ORGANIZED BY JØRGEN ANDERSEN (AU), ROBERT PENNER (AU/CALTECH), AND BRENDAN MCLELLAN (AU)

SUMMARY: The Centre for Quantum Geometry of Moduli Spaces (QGM) at Aarhus University in Aarhus, Denmark, hosted two successive scientific meetings in January-February of 2013. The first was a Master Class and was taught by Bertrand Eynard, Institute de Physique Théorique (IphT), and Nicolas Orantin, Instituto Superior Técnico, Lisboa. The Master Class featured recent work of Eynard and Orantin centering on matrix model techniques and quantum algebraic geometry. The second scientific meeting was an international conference on topological recursion and quantum algebraic geometry that was designed to complement the techniques learned in the Master Class. The conference featured speakers from numerous countries in Europe, as well as from the United States, Australia, and China.

SCIENTIFIC CONTENT: The Master Class from January 21 – 25 entitled, "Recursion from Matrix Models to Quantum Algebraic Geometry," was taught by Bertrand Eynard, Institute de Physique Théorique (IphT), and Nicolas Orantin, Instituto Superior Técnico, Lisboa, and was a detailed exposition of their seminal works from 2007 and 2008, which extend the scope of application of matrix model methods to an even greater class of theories, now called "Eynard-Orantin invariants." The Master Class spanned five days and covered the following material: Eynard started with some introductory examples of topological recursion, including Weil-Petersson volumes, Hurwitz numbers, Gromov-Witten invariants and crystal models. Orantin then continued with some constructions on Riemann surfaces, including algebraic curves, holomorphic forms, branchpoints, and Galois conjugates. He then covered spectral curves of genus one, including Seiberg Witten, local projective space, and some other non-hyperelliptic examples. The main definition of topological recursion was presented, including an introduction of the standard notation. Some examples of computations using topological recursion were presented using a convenient presentation involving three-valent graphs. Some important topics were then covered, including symmetry of the invariants, dilaton equations, form-cycle duality, symplectic invariance, and modular transformations. After these fundamental developments had been reviewed, Enyard proceeded to demonstrate the power of their approach. After showing how to write their invariants as intersection numbers, and reviewing the Laplace transform of spectral curves, he proceeded to demonstrate one of the main consequences of their approach, and showed how the important ELSV formula and the Marino-Vafa formula may be recovered within their formalism. He then proceeded to investigate some topics extending the previous material, and covered some issues regarding the non-perturbative part of their theory, background independence and modular invariance of their invariants. The Master Class ended with some discussion on integrability and the Hirota equations, demonstrating the versatility and vast applications that topological recursion has in general.

For the International Conference, a total of 16 senior researches from Europe, the United States, Australia and China presented talks on recent works of significance related to topological recursion and quantum algebraic geometry. Below are a selection of some highlights from these lectures:

- Bertrand Eynard started the conference off with a talk covering a general overview of the uses of topological recursion in mathematical physics. He later gave a more specialized talk on topological recursion for Gromov-Witten invariants, and presented a sketch of a proof of the BKMP conjecture. Nicolas Orantin, the other main pioneer of topological recursion, also gave a talk on applications involving the Givental group action.

- Stavros Garoufalidis (Georgia) gave two talks, the first on the quantum content of the gluing equations in topological quantum field theory, and the second talk presented his work on a proof that the coloured HOMFLY polynomial is q-holonomic.

- Jian Zhou gave a talk in the area of Quantum Algebraic Geometry which relates to work of Gukov and Sulkowski. Piotr Sulkowski presented some background on quantization and the super A-polynomial that served as a precursor to the talk of Sergei Gukov on periods and pre-potentials for (super) A-polynomial curves. This series of talks allowed conference participants to form a deeper appreciation for the super A-polynomial, the vast connections that it makes with other branches of physics and mathematics, and the application of topological recursion in this area.

- Paul Norbury and Motohico Mulase gave talks that demonstrate how the techniques of Eynard-Orantin have opened the door to new methods to study related counting problems in algebraic geometry.   This work is related to stationary Gromov-Witten invariants, which arise as the Eynard-Orantin invariants of a spectral curve. Melissa Liu presented related work on open-closed Gromov-Witten invariants of toric Calabi-Yau threefolds.

- Christian Reidys gave a talk based on joint work with Andersen, Chekhov, Penner, and Sulkowski, that exhibited the application of topological recursion methods in the diverse field of biology. His talk focused on RNA shapes over one and two backbones that has recently proven to benefit from the techniques of study of the Eynard-Orantin invariants.

ASSESSMENT OF RESULTS: The "Recursion from Matrix Models to Quantum Algebraic Geometry," Master Class was extremely well attended, with approximately 35 participants from across Europe, the United States, Australia and China. Not only was the class very well attended, but the content was also very well received, with the audience interacting throughout the entire lecture course. Also of note was the participation of a large number of PhD students who took the opportunity to actively engage with the senior researchers in both attending the course and the subsequent conference. This combined with the high quality mathematics that was presented by Eynard and Orantin, made the class an exciting event as it provided a hands on introduction to techniques used by practitioners in the field, as well as introducing young researchers to open problems and ideas for future applications.

The international conference was also very popular, with approximately 50 participants from across Europe, the United States, Australia and China. The highlight for many were lectures on the super A-polynomial by Sulkowski and Gukov, which provided a bridge from physics to new mathematics that uses the methods of Eynard and Orantin topological recursion to study new invariants of knots and three-manifolods. This provided a fitting accentuation of the power and diversity of the Eynard-Orantin invariants, and demonstrated the high level of mathematics discussed during both meetings. Numerous discussions, both formal and informal, occuring during both meetings will surely provide fertile ideas from which we can expect research results of significance in the coming months.

FINAL PROGRAMME:

Master Class: Recursion from Matrix Models to Quantum Algebraic Geometry

Monday 21 Jan

9.30-10.00

Coffee/tea outside the auditorium

10.00-10.45

Introduction: introductory examples of application of the TR: Weil-Petersson volumes, Hurwitz

numbers and others (Gromov-Witten, Kontsevich, maps, crystal models...) (Eynard)

11.15-12.00 As above

12.00-14.00 Lunch

14.00-14.45

Main definition: Some "reminders" of complex analysis on Riemann surfaces: algebraic curves,

holomorphic forms, 2nd kind differential. Branchpoints, Galois conjugate. Examples of spectral curves

of genus 1: Seiberg-Witten SU(2), local P^2,... Examples not hyperelliptical: (p,q)=(4,3) model Writing

the main definition of the Topological recursion, writing the F_g's. (Orantin)

14.45-15.15 Afternoon tea outside the auditorium

15.15-16.00 As above

18.00-

Wine and cheese

Tuesday 22 Jan

9.30-10.00

Coffee/tea and bread rolls outside the auditorium

10.00-10.45

Examples of computations with the Topological recursion: W_{03}, W_{11}, ... Writing as 3-valent

graphs (Orantin)

11.15-12.00 As above

12.00-14.00 Lunch

14.00-14.45

Properties: symmetry of W_{g,n}, no residue, dilaton equation, special geometry form-cycle duality,

symplectic invariance, modular transformations, Virasoro,... (Orantin)

14.45-15.15 Afternoon tea outside the auditorium

15.15-16.00 As above

Wednesday 23 Jan

9.30-10.00

Coffee/tea and bread rolls outside the auditorium

10.00-10.45

Properties: symmetry of $W_{g,n}$, no residue, dilaton equation, special geometry form-cycle duality,

symplectic invariance, modular transformations, Virasoro,... (Orantin)

11.15-12.00 As above

12.00-14.00 Lunch

14.00-14.45 As above

14.45-15.15 Afternoon tea outside the auditorium

15.15-16.00 As above

Thursday 24 Jan

9.30-10.00

Coffee/tea and bread rolls outside the auditorium

10.00-10.45

11.15-12.00

Writing the $W_{g,n}$'s and F_g's as intersection numbers, Laplace transform of spectral curves and

classes. Example of applications: Recovering ELSV formula, Marino-Vafa formula (Eynard)

As above

12.00-14.00 Lunch

14.00-14.45

Non-perturbative part, background independence, modular invariance (Eynard)

14.45-15.15 Afternoon tea outside the auditorium

15.15-16.00 As above

18.00-22.30 Special dinner

Friday 25 Jan

10.00-10.45 Coffee/tea and bread rolls outside the auditorium

10.00-10.45

Integrability, Hirota equations (Eynard)

11.15-12.00 As above

12.00-14.00 Lunch

Conference: Topological Recursion and Quantum Algebraic Geometry

Monday 28 January

Location: Natural History Museum, Wilhelm Meyers Allé 210, Building 1211

08.30-09.00: Coffee/tea

09.00-10.00: The use of topological recursion in mathematical physics (Bertrand Eynard)

10.00-10.30: Coffee break

10.30-11.30: The quantum content of the gluing equations (Stavros Garoufalidis)

11.30-13.30: Lunch break

13.30-14.30: Topological recursion and Givental group action (Nicolas Orantin)

14.30-15.00: Coffee break

15.00-16.00: Topological recursion and knot invariants: two early steps (Gaëtan Borot)

18.00:

Social networking dinner

Tuesday 29 January

Location: Natural History Museum, Wilhelm Meyers Allé 210, Building 1211

08.30-09.00: Coffee/tea

09.00-10.00: Eynard-Orantin recursions and quantum algebraic curves (Jian Zhou)

10.00-10.30: Coffee break

10.30-11.30: Quantization and super-A-polynomials (Piotr Sułkowski)

11.30-13.30: Lunch break

Time for discussion and tourism

Wednesday 30 January

Location: Natural History Museum, Wilhelm Meyers Allé 210, Building 1211

08.30-09.00: Coffee/tea

09.00-10.00: Orbifold Hurwitz numbers and Eynard-Orantin invariants (Paul Norbury)

10.00-10.30: Coffee break

10.30-11.30: On RNA-shapes over one and two backbones (Christian Reidys)

11.30-13.30: Lunch break

13.30-14.30: The colored HOMFLY polynomial is q-holonomic (Stavros Garoufalidis)

14.30-15.30: Coffee break

16.00-17.00: Nielsen lecture: Partitions, coverings, and modular forms (Don Zagier)

Thursday 31 January – Aud. D1 & D2

08.30-09.00: Coffee/tea

09.00-10.00: Partitions, Siegel-Veech constants, and quasimodular forms (Don Zagier) – Aud. D1

10.00-10.30: Coffee break

10.30-11.30: Top. Rec. for Gromov-Witten invariants, proof of the BKMP conjecture (Bertrand Eynard) – Aud. D1

11.30-14.00: Lunch break

14.00-15.00: Open-closed Gromov-Witten invariants of toric Calabi-Yau 3-orbifolds (Melisssa Liu) - Aud. D2

15.00-15.30: Coffee break

15.30-16.30: Ghosts on moduli of level curves (Alessandro Chiodo)

17.30: Walk from QGM

18.00: Dinner

Friday 1 February – Aud. G2

08.30-09.00: Coffee/tea

09.00-10.00: Periods and prepotential for (super) A-polynomial curves (Sergei Gukov)

10.00-10.30: Coffee break

10.30-11.30: Topological recursion for quantum Liouville theory (Leonid Chekhov)

11.30-13.30: Lunch break

13.30-14.30: Operadic structures in topological recursion (Brad Safnuk)

14.30-15.00: Coffee break

15.00-16.00: Quantum curves for geometric enumeration problems (Motohico Mulase)

PARTICIPANTS LIST:

Master Class: Recursion from Matrix Models to Quantum Algebraic Geometry

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Gupta, Subhojoy (AU)
Gukov, Sergei (Caltech)

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