Science Meeting — Scientific Report

Proposal Title: Modern trends in topological quantum field theory

Application Reference N°: 4856

1) Summary

The Science Meeting *Modern trends in topological quantum field theory* was a two-month programme held at the Erwin-Schrödinger-Institute (ESI) in Vienna, Austria. Characteristic for such meetings at the ESI is a strong focus on informal discussions which initiate or continue scientific collaborations.

On the more formal side, the programme comprised the following activities:

- Two one-week workshops, each with about 20 talks of 50 minutes.
- Two master classes, each consisting of three 90-minute lectures.
- A one-week meeting of the junior research network 'String Geometry' (sponsored by the German Science Foundation).
- Eight individual talks outside the workshop periods.

Altogether these activities were attended by 70 participants. Of these, 55 gave a talk. 50 of the participants came from countries with organisations contributing to the ITGP budget. The total duration of stay amounted to about 116 person-weeks.

Major topics of the workshops were: extended topological field theories; higher categorical structures needed for topological field theories; cobordism categories with additional structure; differential cohomology theories; non-semisimple tensor categories, and link and three-manifold invariants derived from them; logarithmic conformal field theories.

In addition, connections were made with various other problems, such as: equivariantization of categories; open string field theory; defects in topological field theories; statistical mechanics for systems with boundaries; module categories over fusion categories.

Among the scientific developments that became manifest during the programme, the following two may be accentuated: First, monoidal categories are now fully recognized as the lingua franca for communication between different fields, with higher categories gaining increasing importance. And second, non-semisimple tensor categories are coming to the forefront of research, with converging interests in (logarithmic) conformal field theory, in representation theory (of Hopf algebras, Kac-Moody algebras and vertex algebras), and in the construction of new renormalized link and three-manifold invariants as well as of (partial) topological quantum field theories.

2) Scientific content of and discussions at the event

The idea of the meeting was to promote interaction between mathematicians and mathematical physicists of different background working on or using topological field theories and their various extensions. On the mathematical side, the meeting involved mathematicians working in topology, geometry and representation theory. From the perspective of physics, the programme included participants with a background in string theory and in statistical mechanics.

The two workshop periods concentrated on two different types of problems:

Workshop 1: Extended Topological field theories, higher structures and differential cohomology theories.

This workshop also dealt with infinity categories, topological conformal field theory and its application to Gromov-Witten invariants, and the quantization of classical topological field theories using higher structures.

Workshop 2: Topological and conformal field theories with non-semisimple monoidal categories.

Different algebraic structures, in particular Hopf algebras and vertex algebras, lead to representation categories that are monoidal categories. Recent developments presented during workshop 2 have revealed that non-semisimple categories arising from Hopf algebras and from vertex algebras have a much more closely related structure than specialists in both fields had realized before. Moreover, they are surprisingly similar to non-semisimple monoidal categories used for the construction of renormalized quantum invariants of links and three-manifolds.

Let us describe some important results presented during the programme, grouping thematically related contributions.

Explicit constructions of topological field theories.

Several talks were devoted to explicit constructions of topological field theories: combinatorial methods continue to provide a rather explicit computational access to topological field theories. This was demonstrated by Dror Bar-Natan (Toronto) for the combinatorics of perturbative BF-theories in three dimensions. Azat Gainutdinov (Hamburg) used quantized versions of Schur-Weyl duality to investigate state sum models of spin type. Rinat Kashev (Geneva) presented a construction of a topological field theory with infinite-dimensional state spaces using the combinatorial framework of triangulated manifolds.

Kazui Habiro (Kyoto) discussed recent developments on Kirby calculus; Thang Le (Atlanta) discussed the role of the Habiro ring on three-manifolds; finally, Qingtao Chen (Trieste) explained congruent skein relations with a motivation to study the LMOV conjectures.

Cobordism categories with additional structure.

Variants of topological field theories can be constructed on cobordism categories which involve manifolds with additional structure. Ingo Runkel (Hamburg) presented a combinatorial construction of spin topological field theories in two dimensions; a related construction of a state-sum model for surfaces with spin structures was presented by John Barrett (Nottingham). Alexis Virelizier (Lille) explained his construction (with Turaev) of three-dimensional homotopy TFTs in which manifolds with covers enter. Chris Schommer-Pries (Bonn) reported how finite tensor categories give rise to fully local partially defined thee-dimensional topological field theories, generalizing the case of (spherical) fusion categories for which the TFTs are fully defined. Additional structure on cobordism categories (e.g. an orientation) then translate into additional structure on the categories.

Mapping class groups.

The vector spaces assigned to surfaces by a three-dimensional topological field theory carry a representation of the relevant mapping class group. Jørgen Andersen (Aarhus) explained how mapping class group representations obtained via geometric quantization generalize from Reshetikhin-Turaev models based on the compact group $\mathrm{SU}(n)$ to the complex group $\mathrm{SL}(n, \mathbb{C})$. Joost Slingerland (Maynooth) discussed local representations of the loop braid group which governs configurations of rings in three-dimensional space. Such representations arise in discrete gauge theories, e.g. those relevant for the toric code. Finally, Gregor Masbaum (Paris) used methods from integral topological field theories to investigate abstract group theoretical properties of mapping class groups.

Classical field theory.

Classical topological field theory remains an active field with many open questions. Conversely, topological field theories shed a light on the mathematical structure of classical field theories, whereby they e.g. provide one possible mathematical perspective on partial differential equations). Thomas Strobl (Lyon) discussed the Dirac sigma model as a joint generalization of Poisson sigma models and gauged WZW models. Here Lie algebroid structures arise. Homotopy Frobenius structures appear in the generalization of AKSZ TFTs presented by Theo Johnson-Freyd (Berkeley), who also discussed aspects of the quantization of these models. Operadic methods also played an important role in the relations between the Grothendieck-Teichmüller group and polyvector fields that was explained by Sergei Merkulov (Stockholm). Generalized Chern-Simons invariants constructed from L_∞-algebras were also central in the contribution by Andrey Lazarev (Lancaster).

Quantization.

There are several procedures of quantization at various levels of mathematical and conceptual rigour. In a semiclassical approach to quantum field theory, determinants of Laplacians figure prominently. Gluing relations in topological field theories raise the question about gluing formulas for determinants. This was the topic of the contribution by Boris Vertman (Bonn). A much more direct approach is possible for topological field theories based on finite groups, so-called Dijkgraaf-Witten theories; Jeffrey Morton (Hamburg) presented a general construction of such extended TFTs, based on suitable linearization functors. A rather advanced general scheme for formalizing quantization via pull-push in twisted generalized cohomology was presented by Joost Nuiten and Urs Schreiber (Nijmegen).

Differential cohomology and gerbes.

Topological field theories in three dimensions are closely related to conformal field theories in two dimensions. The latter typical have topological terms in their actions, which are naturally understood by differential cohomology theories and their geometric realizations, gerbes and higher versions of gerbes.

Thomas Schick (Göttingen) discussed geometric models for higher twisted K-theory, including a proposal for the notion of a homotopy bundle gerbe. Alan Carey (Canberra) presented a different geometric approach, based on Baum-Douglas geometric cycles. Mahmoud Zeinalian (Brookville) constructed a model of differential K-theory and its S^1-iteration maps that is based on a geometrically defined spectrum. Ulrike Tillmann (Oxford) introduced commutative K-theory by imposing the condition that transition functions of bundles commute, obtaining in this way new generalized cohomology theories.
Ulrich Bunke and Thomas Nikolaus (Regensburg) presented their result that differential cohomology can be presented by a sheaf of spectra on manifolds, allowing for a discussion of twisted differential cohomology for an arbitrary multiplicative differential cohomology theory.
Christian Becker (Potsdam) introduced two different notions of relative differential cohomology; he presented long exact sequences and fibre integration for both of them.
Applications in the realm of two-dimensional conformal field theories were discussed in two talks: Martin Schnabl (Prague) explained a new perspective on boundary states in such theories using Witten's open string field theory. Thomas Nikolaus (Regensburg) used recent tools from homotopy theory to get a refined understanding of topological T-duality, including a topological view on T-folds as generalized string compactifications.

Fusion categories and their module categories.
Fusion categories are the simplest possible input for the construction of a fully extended topological field theories in three dimensions. Much of their structure theory still remains to be unraveled. Scott Morrison (Canberra) gave an overview over existing classification results. Sonia Natale (Córdoba, Argentina) showed several classification results on the subclass of weakly group-theoretical braided fusion categories. Equivariant versions of fusion categories are relevant for equivariant topological field theories. Sebastian Burciu (Bucharest) presented new results on the Grothendieck groups of equivariant fusion categories.
Module categories over fusion categories appear in particular in the description of surface defects of three-dimensional topological field theories. Gregor Schaumann (Bonn) presented a categorified version of $*$-modules for module categories that is closely linked to Frobenius structures on algebras of internal homs.

Defects in topological field theories.
Defects of various codimension play an increasing role in the theory and in applications of quantum field theories. Surface defects in three-dimensional topological field theories appear in the construction of quantum codes, in state sum models of condensed matter systems and in a new perspective of the TFT construction of RCFT correlators. Catherine Meusburger (Erlangen) presented a careful discussion of dualities at various categorical levels for such defects. Alessandro Valentino discussed surface defects and boundary conditions of three-dimensional topological field theories of Reshetikhin-Turaev type; in particular he explained an obstruction that takes values in the Witt group of modular tensor categories.
Defects are also of interest in dimensions different from three. Nils Carqueville (Stony Brook/Vienna) explained how to employ defects in two-dimensional field theories to produce equivalences of matrix factorizations. Kevin Walker (Santa Barbara) showed in par-

ticular that three-dimensional defects in four-dimensional topological field theories (capturing e.g. the anomaly of three-dimensional topological field theories of Reshetikhin-Turaev type) can be used to understand the modularization procedure of Bruguières and Müger in a geometric manner.

Non-semisimple categories, logarithmic conformal field theory.
In their original formulation, combinatorial constructions of invariants of three-manifolds, like those of Turaev-Viro or Reshetikhin-Turaev, strongly rely on *semisimple* categories with suitable additional structure. Insight both from mathematics and from physics (as explained for instance in the contribution by Jesper Jacobsen (Paris) on geometrical critical phenomena) have, however, led to look for suitable categories that are no longer semisimple, but still allow to construct at least substantial aspects of a a three-dimensional topological field theory.
Such categories have been much better understood by representation theorists recently: Drazen Adamovic (Zagreb) explained new results on the construction of representations of certain C_2-cofinite vertex algebras appearing in logarithmic conformal field theories. Simon Wood (Canberra) showed that Virasoro minimal models admit C_2-cofinite logarithmic extensions. These theories can also be realized using kernels of screening operators. Alexei Semikhatov (Moscow) discussed how properties of screening charges are naturally captured by Nichols algebras which can be used to understand the representation categories of such vertex algebras explicitly. David Ridout (Canberra) explained the necessity to consider simultaneously different parabolic Borel algebras to get consistent non-semisimple representation theories associated with WZW-theories at fractional level.
In a whole series of lectures, Christian Blanchet (Paris), François Costantino (Strasbourg), Nathan Geer (Utah State) and Bertrand Patureau (Vannes) presented a class of non-semisimple categories constructed as representation categories of a finite-dimensional quantum group that allow one to construct both invariants of at least classes of decorated three-manifolds and representations of mapping class groups of surfaces, and thus two central pieces of data of a three-dimensional TFT. Closely related invariants, generalized Kashaev invariants for knots in three-manifolds, were discussed by Jun Murakami (Tokyo). Finally, Christoph Schweigert (Hamburg) explained how to construct, for certain types of non-semisimple categories, invariants of mapping class groups that are candidates for bulk correlation functions of logarithmic conformal field theories.

3) Results and impact of the event on future directions

We note a few scientific developments that have become manifest during the programme:

- Monoidal categories (possibly with additional structures, e.g. a braiding) have become a powerful lingua franca, allowing for efficient communication across borders between different fields.
- Higher categories have gained increasing importance. Bicategories are by now freely used in talks, three-categorical structures are found in concrete problems. ∞-categories start to pervade the field and strengthen its links to modern trends in algebraic topology.
- Both problems arising in physics, in particular in the study of critical systems, and representation theoretic developments have put three-dimensional TFT-like structures based on non-semisimple tensor categories to the forefront. Much progress has been made concerning these structures on manifolds of codimension 1 and 2. The structures in three dimensions require additional insight. We expect further progress in this field in the coming years.

We also would like to point out some of the new connections that have been established during the programme:

- Developments in the representation theory of Hopf algebras and in the representation theory of infinite-dimensional algebras (Kac-Moody algebras and vertex algebras) have lead to rather similar classes of non-semisimple monoidal categories.
  Prior to this workshop, this convergence had not been realized by the respective communities (including the organizers). These new developments have lead to a major excitement among the participants of the second workshop. In fact, the free afternoon was spontaneously devoted to an additional afternoon session to which almost all participants contributed.
- These non-semisimple categories have the promise to lead to structures rather close to the one of a complete extended three-dimensional topological field theory. It is exciting to speculate that these structures will allow one to extend the TFT approach to RCFT correlators to larger classes of conformal field theories.

Many participants started discussions in smaller groups. We briefly list some of the discussions that were started and of which we are aware:

- Jesper Jacobsen and Martin Schnabl realized that an equation derived from factorization constraints plays a central role both in open string field theory and in statistical mechanics for systems with boundaries.
- Jürgen Fuchs, Christoph Schweigert and Martin Schnabl realized structural similarities between the TFT approach to RCFT correlators and open string field theory. We hope that this will provide a more precise handle on string field theory as well as insights into possible extensions of the validity of the TFT approach beyond rational theories.
- Sebastian Burciu and Jeffrey Morton discussed the equivariantization of categories with a group action, and also some generalizations of Mackey functors and groupoidification.

- Jürgen Fuchs, Christoph Schweigert and Alessandro Valentino on the one hand and Gregor Schaumann on the other hand realized that it is most beneficial to join their efforts in understanding topological surface defects in three-dimensional topological field theories of Turaev-Viro type.

4 a) Program of the meeting

The meeting was composed of several closely interrelated components.

First of all, it is characteristic for meetings at the Erwin-Schrödinger-Institute that many informal discussions are used to initiate or to continue scientific collaborations. Some of their aspects have been mentioned in the previous sections.

Apart from this general activity, the meeting involved the following components.

- The core parts were two workshops of one week each. Each workshop consisted of about 20 talks of 50 minutes.
  The two workshops had a different thematic focus:
  - The first workshop concentrated on extended topological field theories, higher categorical structures needed for topological field theories, and differential cohomology theories.
  - The central topics of the second workshop were logarithmic non-semisimple tensor categories and conformal field theories, including their representation theoretic origins.

  Both workshops also had some talks devoted to classical topological field theories and their quantization.

- Another important component were two master classes, each of which consisted of three lectures of 90 minutes:
  - Azat Gainutdinov (Hamburg) lectured on *Tensor categories in logarithmic conformal field theories*. He explained how tensor categories, and in particular non-semisimple ones, arise in two-dimensional conformal field theories. In particular, he described how quantum groups at roots of unity and associated non-semisimple tensor categories are related to representation categories of vertex algebras, thereby explaining the relevance of these structures for the study of logarithmic conformal field theories.
  - Gregor Masbaum (Paris 7) gave lectures on *Integral TQFT and applications to quantum representations of mapping class groups*. He showed how the combinatorial construction of the SO(3)-TQFT with quantum parameter equal to a root of unity based on skein theory (due to Blanchet-Habegger-Masbaum-Vogel) admits, for roots of unity of prime order, an integral refinement: In every genus, the vector spaces of conformal blocks contains a natural mapping class group invariant lattice of full rank defined over a ring of algebraic integers. The presence of this lattice allows for new insights into the structure of mapping class groups.

During the week of the master class, Nils Carqueville, the new Simons junior professor at the University of Vienna and a participant of the programme (including both workshops), gave classes within a regular course and seminar series on topological field theory at the University of Vienna. The participants of the master class attended these classes as well.

- A different component, which was unforeseen in the original planning of the programme, was a meeting of the junior research network "String Geometry", sponsored by the German Science Foundation DFG. The network focusses on higher categorical aspects of geometry. Since many of the members of this network were participants of the first workshop, it was decided that the semestrial meeting of the network should take place in Vienna (and thus for the first time outside Germany). This component involved 9 talks exhibiting recent progress by the participants as well as intensive collaborative research.
- In addition, there were eight individual 60-minute talks by participants outside the workshop periods.

Finally it can be mentioned that the ITGP steering committee had its last meeting at the Schrödinger Institute during the period of the programme.

More detailed information about the schedule, including abstracts and, when applicable, slides of the talks, is available at the programme's web site
`http://www.ingvet.kau.se/juerfuch/conf/esi14` .

4 b) List of speakers and participants

As a consequence of the format of the event, the majority of the participants also were speakers. Those who did not give a talk are marked by an asterisk in the list below.

- Drazen Adamovic (Zagreb)
- Jørgen Andersen (Aarhus)
- Dror Bar-Natan (Toronto)
- John Barrett (Nottingham)
- Christian Becker (Potsdam)
- Mette Bjerre⋆ (Aarhus)
- Christian Blanchet (Paris)
- Igor Buchberger⋆ (Karlstad)
- Ulrich Bunke (Regensburg)
- Sebastian Burciu (Bucharest)
- Alan Carey (Canberra)
- Nils Carqueville (Stony Brook / Vienna)
- Qingtao Chen (Trieste)
- Hongyi Chu⋆ (Osnabrück)
- Ben Cooper⋆ (Zürich)
- Francesco Costantino (Strasbourg)
- Jens Kristian Egsgaard⋆ (Aarhus)
- Jürgen Fuchs⋆ (Karlstad)
- Azat Gainutdinov (Hamburg)
- Nathan Geer (Utah State)
- Kazuo Habiro (Kyoto)
- Yonatan Harpaz⋆ (Nijmegen)
- Bas Janssens (Erlangen)
- Jesper Jacobsen (Paris)
- Theo Johnson-Freyd (Berkeley)
- Rinat Kashaev (Genève)
- Ludmil Katzarkov⋆ (Vienna)
- Andrey Lazarev (Lancaster)
- Thang Le (Atlanta)
- Simone Marzioni⋆ (Aarhus)
- Gregor Masbaum (Paris)
- Joanna Meinel⋆ (Bonn)
- Sergei Merkulov (Stockholm)

- Catherine Meusburger (Erlangen)
- Scott Morrison (Canberra)
- Jeffrey Morton (Hamburg)
- Jun Murakami (Tokyo)
- Sonia Natale (Córdoba)
- Anton Nikitenko* (Klosterneuburg)
- Thomas Nikolaus (Regensburg)
- Joost Nuiten (Nijmegen)
- Pranav Pandit (Vienna)
- Bertrand Patureau (Vannes)
- Ulrich Pennig (Münster)
- Nicolai Reshetikhin* (Berkeley)
- David Ridout (Canberra)
- Ingo Runkel (Hamburg)
- Ramanujan Santharoubane* (Paris)
- Gregor Schaumann (Bonn)
- Claudia Scheimbauer* (Zürich)
- Thomas Schick (Göttingen)
- Martin Schnabl (Prague)
- Christopher Schommer-Pries (Bonn)
- Urs Schreiber (Utrecht)
- Christoph Schweigert (Hamburg)
- Alexey Semikhatov (Moscow)
- Joost Slingerland (Maynooth)
- Markus Spitzweck* (Osnabrück)
- Thomas Strobl (Lyon)
- Ulrike Tillmann (Oxford)
- Alessandro Valentino (Hamburg)
- Boris Vertman (Bonn)
- Alexis Virelizier (Lille)
- Christian Voigt (Glasgow)
- Michael Völkl (Regensburg)
- Konrad Waldorf (Greifswald)
- Kevin Walker (Santa Barbara)
- Christoph Wockel (Hamburg)
- Simon Wood (Canberra)
- Mahmoud Zeinalian (Brookville)