Scientific report for meeting funded by ITGP

"Masterclass and conference on Pressure metric and Higgs bundles"
Application reference number: 4919

October 21, 2013

1 Brief summary

This is a report on the following scientific meetings held at the Centre for Quantum Geometry of Moduli Spaces (QGM) at Aarhus University:

- A Masterclass on the "Pressure metric" by Martin Bridgeman (Boston), Dick Canary (Michigan), and Andres Sambarino (Paris-Sud 11) from August 12-16.

- Three hours of lectures on Higgs bundles by Oscar Garcia-Prada (Madrid) on August 17.

- An international conference from August 19-22 on pressure metric and Higgs bundles.

We shall briefly describe the scientific content of the proceedings of each of these three events in §2, and present an assessment in §3. The appendix provides the programme and other details of the meeting.

To summarize, this conference reported on some recent developments in the fields of moduli spaces and representation varieties. The Masterclass was an excellent exposition of an exciting new result, the existence of a mapping-class-group invariant Riemannian metric on the Hitchin components of moduli spaces. After a crash-course on the related subject of Higgs bundles, the subsequent conference included many illuminating talks by numerous experts. A wide variety of topics including topological, dynamical and analytical aspects of moduli spaces were addressed. Videos of the talks are available at `http://qgm.au.dk/video/`.

It was well-attended, with 57 participants from European, US and Asian academic institutions, including 20 speakers, and 15 graduate students. The appendix provides the programme of the meeting and a complete list of talks, speakers and participants.

2 Description of scientific content

We highlight some of the scientific content of the meeting. Some of the scientific background was outlined in the earlier application for funding.

2.1 Masterclass (Aug 12-16)

The five days consisted of 15 hours of talks forming complete exposition of the recent work of Bridgeman, Canary, Labourie, and Sambarino ([1]).

Moduli spaces can be thought of as parameter spaces of representations, with the classical example being the Teichmüller space of a surface S of hyperbolic type, which is the space of representations of its fundamental group $\pi_1(S)$ into $PSL(2,\mathbb{R})$. More generally, one can consider the space of surface-group representations into $SL(n,\mathbb{R})$ or higher rank Lie groups. A fundamental result of Hitchin ([2]) asserts that this has a connected, contractible component that is the analogue of Teichmüller space. In this context the new work defines a Riemannian metric on the Hitchin component that is invariant under the action of the mapping class group of S, and restricts to the usual Weil-Petersson metric on the Fuchsian locus.

This work hinges on a certain dynamical characterization of representations in the Hitchin component, introduced by Labourie and developed by Guichard, Wienhard and others. In brief, for ρ a Hitchin representation, one considers the flag space of the eigenvalue spectrum of the image and obtains a ρ-equivariant "limit map" from the Gromov boundary $\partial_\infty \pi_1(S)$ to $\mathbb{RP}^n$, which can be considered an analogue of the "quasi-circle" for a quasifuchsian surface-group representation in $PSL_2(\mathbb{C})$. Moreover, the geodesic flow of the word-hyperbolic group $\pi_1(S)$ induces an Anosov flow on the flat $\mathbb{R}^n$-bundle arising from ρ, which turns out to be conjugate to a Hölder reparametrization of the geodesic flow. Powerful mathematical tools from dynamical systems theory, inspired by ideas from statistical physics (*e.g.* [3]) can then be brought in, to show there is then a "pressure" function on the Hitchin component with a symmetric non-negative-definite Hessian. A further argument involving trace identities shows that this Hessian is in fact positive definite, and hence can be considered to be a Riemannian metric tensor.

The technical obstacles and details were considerable, in particular establishing analyticity of the various dynamical quantities involved. The Masterclass provided ample opportunity for a thorough discussion.

It should be noted that the usual Weil-Petersson metric on Teichmüller space also has an analogous construction due to Wolpert ([4]). This WP metric is well-known to be compatible with the symplectic and Kähler structure on moduli space, which has proved crucial in relation to various applications in string theory and related mathematics. The question of an analogous compatibility of the new pressure metric with the symplectic structure for moduli spaces for SL_n for $n > 2$ naturally arises.

2.2 Higgs Bundles Day (Aug 17)

The result of Hitchin mentioned in the previous section originally arose from the theory of Higgs bundles which developed in the 1980s as a non-abelian Hodge theory, and has since proved to be of spectacular importance.

The 3-hour crash course by Oscar Garcia-Prada (Madrid) was an incisive introduction to the subject by one of the worlds leading experts in the field. He discussed Higgs bundles from a slightly more general point of view, and in remarkable way led up to forefront of research, including some of his own, involving such bundles for groups of Hermitian type.

This was a valuable inclusion in the program, and was very helpful to the participants specializing in other aspects of moduli space theory.

2.3 Conference (Aug 19-22)

Designed to complement the techniques learned in the Masterclass, and announce new applications and developments, the conference featuring 16 talks (listed in the Appendix) was a remarkable success.

The talks, by many distinguished speakers, covered a broad range of topics, with a focus on the interplay between the dynamical, geometric and algebraic themes central to the study of moduli spaces. The content of the talks related to :

- Thermodynamic formalism and dynamics in moduli space (Canary, Ledrappier, Pollicott, Sharp).

- Geometry of Higgs bundles and the Hitchin-Kobayashi correspondence (Garcia-Prada, Guichard, Oliveira, Biswas, Riera).

- Cluster algebras, TQFTs and representation varieties (Shapiro,Kashaev, Guichard, Dimofte).

- Convex projective structures and the SL_3 character varieties (Wolpert, Wolf).

- Hyperbolic geometry and Teichmüller theory (McShane, Dimofte, Masur).

References

[1] Bridgeman, Canary, Labourie and Sambarino, *The pressure metric for convex representations.* Arxiv preprint: `http://arxiv.org/abs/1301.74591978`

[2] Nigel Hitchin, *Lie groups and Teichmüller space.* Topology, Vol. 31 No. 3, 1992.

[3] David Ruelle, *Thermodynamic formalism.* Addison-Wesley, 1978.

[4] Scott Wolpert, *Thurston's Riemannian metric for Teichmüller space.* Journal of Diff. Geom. 23 No. 2, 1986.

3 Assessment of impact

As briefly outlined in §2, the meeting was an intensive yet broad discussion of several new aspects of the study of moduli spaces. The structure of the program (see the Appendix) offered ample time for discussion, and provided a enormous learning experience for the participants, which included graduate students and eminent researchers from various fields (also see the Appendix).

The existence of the new "pressure metric" that was the subject of the Masterclass (see §2.1) leads to many questions:

Question 1. *Are the geodesics in the pressure metric of infinite length in certain directions?*

Question 2. *Is the group of isometries precisely the mapping class group?*

Question 3. *Does the metric have negative curvature?*

The event was a tremendous opportunity for young researchers to get acquainted with the above questions, that are likely to fuel much research in the coming years.

One would like to particularly highlight here that discussions during the conference, that included an open-ended question-and-answer session, led to some conjectures regarding the following question:

Question 4. *Is the pressure metric a Kähler metric?*

The symplectic structure on moduli spaces has been an essential tool in the geometry of Higgs bundles, and this conference provided a unique opportunity for specialists in that field to interact with those familiar with dynamical aspects.

It is only inevitable that this interaction would lead to further productive collaboration and important advances in the future.

A Appendix: Organizational details

A.1 Programme of the meeting

Daily schedule for the Masterclass:

5 days, 4 lectures each day with timings 10:00-10:45, 11:15-12:00, 14:00-14:45, 15:00-16:00, and Lunch from 12:00 -14:00.

Social programme:

Monday, August,12 at 18:00. Social networking dinner,
Thursday, August, 15 at 18:00. Special dinner.

International conference:

5 days, 4 one-hour talks each day with timings 9:30-10:30, 11:00-12:00, 14:00-15:00, 15:30-16:30. The social programme also consisted of a Social networking dinner on Monday, August 19 and a Special dinner on Wednesday, August 21.

A.2 List of speakers and talks

Masterclass:

- Martin Bridgeman (Boston College),

- Dick Canary (Univ. of Michigan)

- Andres Sambarino (Paris-Sud 11)

Higgs Bundles Day:

- Oscar Garcia-Prada (Madrid)

International Conference:

- Mark Pollicott (Univ. of Warwick)

- Richard Sharp (Univ. of Warwick)

- Andre Gama Oliveira (UTAD)

- Scott A. Wolpert (Univ. of Maryland)

- Ignasi M. Riera (Univ. of Barcelona)

- Greg McShane (Institut Fourier)

- Rinat Kashaev (Univ. of Geneva)

- Michael Wolf (Rice Univ.)

- Olivier Guichard (Paris-Sud)

- Howard Masur (Univ. of Chicago)

- Indranil Biswas (TIFR)

- Tudor Dimofte (IAS Princeton)

- Francois Ledrappier (Univ. of Notre Dame)

- Richard Canary (Univ. of Michigan)

- Michael Shapiro (Michigan State Univ.)

- Ignasi Mundet Riera (Univ. de Barcelona)

List of talks (day by day):

- *The pressure metric on the Hitchin component* by Dick Canary

- *Involutions of the moduli space of Higgs bundles and branes* by Oscar Garcia-Prada

- *Regularity of the entropy of certain random walks* by Francois Ledrappier

- *Cluster algebra compatible with Cremmer-Gervais Poisson-Lie bracket on SL(n)* by Michael Shapiro

- *Products of twists, geodesic-lengths, shears and projective twist-bulge deformations* by Scott A. Wolpert

- *Weil Petersson metrics and Thermodynamic Formalism* -Mark Pollicott

- *A new formulation of the Teichmüller TQFT* by Rinat Kashaev

- *(q-)Teichmüller Theory from 3d* by Tudor Dimofte

- *Geometric rank of Teichmller space* by Howard Masur

- *The Zariski closure of Hitchin representations* by Olivier Guichard

- *Polynomial Pick forms for affine spheres and real projective polygons* by Mike Wolf

- *A Weil-Petersson metric for graphs* by Richard Sharp

- *Small dilatation pseudo Anosovs* by Greg Mcshane

- *Components of moduli spaces of Higgs bundles* by Andr Gama Oliveira

- *Geometry of the Quot scheme* by Indranil Biswas

- *On the Cayley and Hitchin-Kobayashi correspondences* by Ignasi Mundet Riera

A.3 List of participants for the QGM masterclass & conference 12-22 Aug 2013

List of Participants	Institution/Place	Arrival/Departure
Subhojoy Gupta	AU/Caltech	11.08-22.08 2013
Tobias Kildetoft	AU	11.08-22.08 2013
Henning Haahr Andersen	AU	11.08-22.08 2013
Gregor Masbaum	AU (Visiting professor)	11.08-22.08 2013
Peter Zograf	AU	11.08-22.08 2013
Alexei Venkov	AU	11.08-22.08 2013
Jørgen Ellegaard A.	AU	11.08-22.08 2013
Søren Fuglede Sørensen	AU	11.08-22.08 2013
Simone Marzioni	AU	11.08-22.08 2013
Jens Kristian Egsgaard	AU	11.08-22.08 2013
Jens-jakob Kratmann Nissen	AU	11.08-22.08 2013
Niels Leth Gammelgaard	AU	11.08-22.08 2013
Andrew Swann	AU	11.08-22.08 2013
Paolo Masulli	AU	11.08-22.08 2013
Troels Bak Andersen	AU	11.08-22.08 2013
Alan Lai	Caltech	11.08-22.08 2013
Lien-Yung Kao	Indiana	10.08-24.08 2013
Qiongling Li	Rice	11.08-24.08 2013
Yasushi Kasahara	Japan	11.08-23.08 2013
Vincent Alberge	Strasbourg	11.08-23.08 2013
Brian Collier	UIUC	11.08-23.08 2013
Tengren Zhang	Michigan	11.08-23.08 2013
Nariya Kawazumi	Tokyo	11.08-22.08 2013
Sanjay Singh	Mumbai	12.08-23.08 2013
Binbin Xu	Grenoble	11.08-23.08 2013
Zhe Sun	Paris	11.08-23.08 2013
Sourav GHOSH	Paris	11.08-23.08 2013
Polina Vytnova	Warwick	11.08-23.08 2013
Yusuke Kuno	Japan	11.08-23.08 2013
François Ledrappier	Indiana	11.08-22.08 2013
Daniele Alessandrini	Heidelberg	11.08-23.08 2013
Gye-Seon Lee	Heidelberg	11.08-23.08 2013
Jakob blaavand	Oxford	12.08-23.08 2013
Steven Bradlow	UIUC	17.08-21.08 2013
Nuno Romao	Max Planck	17.08-23-08 2013
Joonhyung Kim	Heidelberg	17.08-22.08 2013

Junyeol Kim		18.08-22.08 2013
Roland Van der Veer		18.08-22.08 2013
Xavier Morvan		18.08-23.08 2013
Dick Canary	Michigan	11.08-22.08 2013
Andrés Sambarino	Paris	11.08-22.08 2013
Martin Bridgeman	Boston	11.08-17.08 2013
Mark Pollicott	Warwick	11.08-23.08 2013
Richard Sharp	Warwick	18.08-23.08 2013
Scott A. Wolpert	Maryland	13.08-21.08 2013
Andre Gama Oliveira	Barcelona	18.08-23.08 2013
Ignasi Mundet i Riera	Barcelona	21.08-23.08 2013
Greg McShane	Grenoble	11.08-23.08 2013
Rinat Kashaev	Paris	18.08-24.08 2013
Mike Wolf	Houston	18.08-23.08 2013
Olivier Guichard	Paris	19.08-22.08 2013
Howard Masur	Chicago	18.08-22.08 2013
Oscar Garcia-Prada	Madrid	16.08-23.08 2013
Michael Shapiro	Michigan	11.08-23.08 2013
Indranil Biswas	Mumbai	18.08-23.08 2013
Tudor Dimofte	Princeton	18.08-22.08 2013