« Report on the Workshop "Random Walks, random Media, Reinforcement" »

Centre Paul Langevin à Aussois, France
10th to 14th Juin 2013

1. Summary

Organizers : Nadine Guillotin-Plantard, Christophe Sabot, Jean Bérard.

The workshop took place in Paul Langevin center, CNRS, in Aussois. It brought together 42 participants mainly specialists in disordered media, interacting random walks and interacting particles systems.

It was organized around two mini-courses of Vladas Sidoravicius and Timo Seppalaïnen, 15 lectures of 35 min, 45 min or 1h, and some time left for scientific discussions.

The topics involved in the workshop have been
– Interacting particles systems (sandpile, activated random walks).
– Random polymers.
– Random walks in random environment.
– Interacting random walks (reinforced/repulsive random walks).
– And percolation, random partitions, stochastic flows.

2. Description of the scientific contents

The workshop was centered on different models of self-interacting random walks. Among these models, the focus was on particle systems (sandpile, activated random walks), random polymers, random walks in random environments, random walks on random graphs like percolation clusters or random trees, reinforced random walks, random walks in random sceneries. These models have different origins and motivations coming from physics, biology or engineering. All these models exhibit strong space/time dependence, either by definition of the model itself, or as a consequence of averaging with respect to the random environment. This feature leads to very challenging questions, far from being solved mathematically. They have been intensively studied in recent years, and form a central subject in probability. Technics involved in these topics cover a large spectrum and the two courses presented two very different approaches : the course of V. Sidoravicious explained how sophisticated multiscale analysis gives precious information on the model of activated random walk, the course of Timo Seppalainen presented how "miraculous" identities can solve difficult questions on a "solvable" model of random polymers.

2.1. Description of the course of V. Sidoravicius : "Absorbing state phase transitions : challenges for mathematicians".

In a series of 6h lectures, V. Sidoravicius focused on two chief examples of conservative, infinite-volume particle systems : the activated random walk model for reaction-diffusion and the stochastic sandpile model. He presented some novel ideas and techniques (based on multiscale analysis and sophisticated 0-1 laws) which allow to get a step forward in understanding the absorbing-state phase transition.

These models belong to a large and important class of driven-dissipative lattice systems that naturally evolve to a critical state, which is characterized by power-law distributions of the sizes of relaxation events. A paradigm example is the emergence of avalanches caused by small perturbations in sandpile models. In many mathematically interesting and physically relevant cases such systems are attracted to a stationary critical state without being specifically tuned to a critical point. In particular, it is believed that this phenomenon lies behind random fluctuations at the macroscopic scale, and creation of self-similar shapes in a variety of growth systems. Due to strong non-locality of correlations and dynamic long-range effects, classical analytic and probabilistic techniques fail in most cases of interest, making the rigorous analysis of such systems a major mathematical challenge. Among theories which attempt to explain long-ranged space-time correlations, the physical paradigm called "self-organized criticality" takes its particular place. It appears in systems whose natural dynamics drives them towards, and then maintains them at the edge of stability. However, for non-equilibrium steady states it is becoming increasingly evident that self-organized criticality is related to conventional critical behavior, namely that of an absorbing-state phase transition. This was the main topic of the course. The known examples are variations of underlying non-equilibrium systems which actually do have a parameter and exhibit critical phenomena. The phase transition in these systems arises from a conflict between a spread of activity and a tendency for this activity to die out, and the transition point separates an active and an absorbing phase in which the dynamics gets eventually extinct in any finite region.

2.2. Description of the course of Timo Seppaläinen : "Features of the log-gamma polymer model : limiting random walk in random environment, stationary polymer, fluctuation exponents".

Timo Seppaläinen gave a 6h lecture. After a brief introduction of the general d+1 dimensional lattice model of directed polymer in a random environment, T. Seppaläinen focused on properties of the explicitly solvable 1+1 dimensional log-gamma polymer model. This model corresponds to a specific choice for the law of the environment (log-gamma) ; it shows some remarkable identities and has in particular an explicit stationary version, obtained by an appropriate choice of boundary conditions, that can be used to derive various properties. T. Seppaläinen showed how ratios of point-to-point and point-to-line partition functions converge to gamma-distributed limits. One consequence of this is that the quenched polymer measure converges to a random walk in a correlated random environment. This RWRE can then be thought of as an infinitely long polymer, and it is also a positive temperature analogue of the competition interface of last-passage percolation.

2.3. Description of the 1h lectures.
The workshop was organized around the two mini-courses and 15 lectures given by permanent researchers (six 1h lectures), post-docs (three 45min lectures) and phd students (six 35min lectures).

Laurent Tournier presented a joint work with V. Sidoravicius and L. Rolla on the Greedy cleaning algorithm and gave new results on the model on $\mathbb{Z}$ and on the slab.

Bruno Schapira presented a joint work with M. Benaïm and O. Raimond on the strongly reinforced vertex reinforced random walk on the complete graph. He showed some surprising behavior of localization on sets on various sizes, using technics of stochastic algorithms.

O. Raimond considered a stochastic differential equation (SDE) on a metric directed graph, directed by a Brownian motion. This SDE is an extension of Tanaka's SDE. The solutions of this SDE are flows of kernels. He described the solutions of this SDE and explained how to construct them.

A. Fribergh presented a joint work with V. Sidoravicius and G. Ben Arous on a result of monotonicity of the speed of biased random walks on Galton-Watson trees, using a clever coupling argument.

P. Mathieu presented a joint work with J.C. Mourrat where they show aging of Glauber-type dynamics on the random energy model, in the sense that they obtain the scaling limits of the clock process and of the age process.

P. Tarrès presented new results concerning the Vertex Reinforced Jump Process (VRJP) and its relation with large deviation for Markov Jump Processes and gave a new proof of the generalized Ray-Knight theorem inspired by the VRJP.

3. Achievements

– The CNRS center in Aussois is a perfect place to stimulate discussions between participants. The scheduled was thought to let some free time at the beginning of the afternoon for scientific discussions. The workshop has been very successful in this respect.
– The course of V. Sidoravicius presented a subject which is widely open, and which leads to many challenging open questions. The course of T. Seppalaïnen presented a beautiful setting where questions which are still inaccessible for general polymers can be solved : in this sense it gives a rich environment where there is still a lot of work to do. These two courses give opportunities to participants, in particular young researchers, to find some new directions of research.
– Many young researchers (phd students or post-docs) have participated to the workshop and gave short lectures. It gave them the opportunity to learn some new topics and to talk to senior researchers and eventually get to collaborate with them.
– Multiscale analysis is a rich and sophisticated set of technics that has been proven very useful in different contexts, in particular in random walks in random environments, reinforced random walks, which are the topics of interest of most of the participants. It is in general difficult to implement, the course of V. Sidoravicius gave a clear account on the ideas leading the general strategy, and may help to find new developments.
– The short lectures gave the opportunity for this community of researchers which are working on very close topics to exchange on recent progresses.

4. PROGRAM

Lundi 10 juin

9h00 - 10h30 : T. Seppäläinen (1)

10h30 - 11h00 : Pause café

11h00 - 12h30 : V. Sidoravicius (1)

12h45 : Déjeuner

16h10 - 17h10 : P. Mathieu, Aging for the REM.

17h10 - 17h55 : C. Laurent, Ballisticity of random walk in random environment in dimension 2.

18h00 - 18h35 : S. Bethuelsen, Random Walks in Dynamic Random Environment.

18h40 - 19h15 : X. Chen, Percolation de l'accessibilité sur N-arbres.

19h30 : Diner

Mardi 11 juin

9h00 - 10h30 : T. Seppäläinen (2)

10h30 - 11h00 : Pause café

11h00 - 12h30 : V. Sidoravicius (2)

12h45 : Déjeuner

17h00 - 18h00 : P. Tarrès, TBA

18h00 - 18h45 : R. Soares dos Santos, A quenched central limit theorem for random walks in random scenery in dimension 2 .

18h50 - 19h25 : D. Kious, Stuck Walks : a conjecture of Erschler, Tóth and Werner.

19h30 : Diner

Mercredi 12 juin

8h45 - 9h45 : O. Raimond, Stochastic flows on graphs.

9h50 - 10h25 : J. Bureaux, Random vector partitions and asymptotic formulae.

10h25 - 10h55 : Pause café

10h55 - 11h40 : J. Poisat, Large deviation principles for words drawn from correlated letter sequences.

11h45 - 12h30 : L. Tournier, On the greedy cleaning problem.

12h45 : Déjeuner

Après-midi libre

19h00 : Apéritif

19h30 : Diner

<div align="center">Jeudi 13 juin</div>

9h00 - 10h30 : V. Sidoravicius (3)

10h30 - 11h00 : Pause café

11h00 - 12h30 : T. Seppäläinen (3)

12h45 : Déjeuner

16h00 - 17h00 : B. Schapira, Vertex Reinforced Random Walk on the complete graph.

17h00 - 18h00 : A. Fribergh, On the monotonicity of the speed of biaised random walk on a Galton-Watson tree without leaves.

18h05 - 18h50 : G. Faraud, Connection times in large ad-hoc mobile networks.

18h55 - 19h30 : M. Ba, Sobolev Inequality and Individual invariance Principle for diffusions in Periodic Media.

19h30 : Diner

<div align="center">Vendredi 14 juin</div>

8h45 - 10h15 : V. Sidoravicius (4)

10h15 - 10h30 : Pause café

10h30 - 12h00 : T. Seppäläinen (4)

12h00 : Déjeuner

<div align="center">5. PARTICIPANTS</div>

5.1. **Convenors.**

Jean Bérard, Villeurbanne,

Nadine Guillotin-Plantard, Villeurbanne,

Christophe Sabot, Villeurbanne,

5.2. **Speakers.**

Stein Bethuelsen, Amsterdam,

Julien Bureaux, Nanterre,

Xinxin Chen, Paris,

Renato Dos Santos, Villeurbanne,

Gabriel Faraud, Berlin,

Alexander Fribergh, Toulouse,

Daniel Kious, Toulouse,

Clément Laurent, Marseille,

Pierre Mathieu, Marseille,

Ba Moustapha, Marseille,

Julien Poisat, Leiden,

Olivier Raimond, Nanterre,

Bruno Schapira, Marseille,

Timo Seppalainen, Madison,

Vladas Sidoravicius, Rio de Janeiro,

Pierre Tarrès, Oxford,

Laurent Tournier, Villetaneuse,

5.3. **Participants.**

Omer adelman, Paris,

Anne-Laure Basdevant, Nanterre,

Daniel Boivin, Brest,

Elodie Bouchet, Villeurbanne,

Omar Boukhadra, Constantine,

Van Hao Can, Marseille,

Fabienne Castell, Marseille

Jérôme Depauw, Tours,

Jean-Marc Derrien, Brest,

Yves Derrienic, Brest,

Alexis Devulder Versailles,

Nathanaël Enriquez, Nanterre,

Yueyun Hu, Villetaneuse,

Thi Thu Hien Le, Brest,

Bastien Mallein Paris,

Françoise Pene, Brest,

Cong Zan Pham, Marseille,

Andrey Piatnitski, Narvik,

Clément Rau, Toulouse,

Arvind Singh, Orsay,

Frédérique Watbled, Vannes,

Xiaolin Zeng, Villeurbanne,