FINAL REPORT ON THE CONFERENCE "BRANCHING DIFFUSIONS AND GAUSSIAN FREE FIELDS, IN PHYSICS, PROBABILITY, AND NUMBER THEORY" JUNE 10TH-14TH 2013, MARSEILLE (FRANCE)

1. Summary

The conference was organized by Yan Fyodorov (Queen Mary and Westfield College, London), Véronique Gayrard (Aix-Marseille University and CNRS), and Nicola Kistler (Jean Morlet Chair 2013 / CIRM Luminy and Aix-Marseille University). It took place on the Campus Saint-Charles in Marseille and brought together about 50 participants working in a number of disparate fields. The scientific aspects of the conference, as well as the assessment, are provided below. Additional information is available on the conference website:

http://www.jean-morlet-2013.org/conference—general-informations.html

The meeting would have never taken place without the financial support of the European Science Foundation through the RGLIS-research network. We would thus like to take this opportunity to express our sincere gratitude to ESF and the steering committee of RGLIS. Many thanks also to the ESF staff for the smooth collaboration, and in particular to Mme Durant for her much appreciated flexibility when dealing with our requests.

London/Marseille, June 2013

Y. Fyodorov, V. Gayrard, and N. Kistler

2. Description of the scientific meeting

The meeting brought together scholars working on fields as disparate as physics, probability and number theory, all sharing logarithmically correlated random variables as common thread. It is in fact an exciting time for researchers working on these fields, as new connections and important breakthroughs are appearing on a "weekly basis".

In particular we had contributions on the extremes of branching Brownian motions (Arguin), 2-dim Gaussian free fields (Bramson, Ding, Louidor, Zeitouni), and more generally log-correlated random variables (Zindy). Maillard addressed in his talk the issue of the convergence of the derivative martingale in the case of branching diffusions (with or without selection), an object which plays a fundamental role in the analysis of (approximate) hierarchal models. Structures similar to those arising in the extremal process of branching Brownian motion are believed to be universal: beside the case of the 2-dim Gaussian free field, one further evidence supporting this claim comes from the topic of nested conformal loops, a subject discussed by Aïdékon; his talk suggested yet another link, namely with Schramm-Loewner Equations and conformal field theory. Another field

where similar structures play an important role is that of Mandelbrot cascades, out of which one can construct multi-fractal random measures which are supposed to play a crucial role in the theory of 2-dim quantum gravity: a number of participants (Barral, Kupiainen, Nikula, Webb) presented beautiful contributions on these subjects. The role of the 2-dim Gaussian free field in the Liouville quantum gravity has been also addressed (Duplantier), and results establishing the validity of the celebrated KPZ formula have been presented. The KPZ formula was also the subject of Vargas' talk, whereas Rhodes explained the construction of Liouville Brownian motion.

Another topic addressed during the conference was that of Dynkin isomorphisms (Eisenbaum, Rosen) and the issue of cover times of random walks on large graphs (Belius, Comets, Lee): somewhat surprisingly, these fields are intimately related to the topic of the conference through an important recent work showing deep connections with the behavior of the extremes of Gaussian free fields. One ingredient of the Dynkin isomorphisms are the local times of random walks, which are of fundamental importance in the study of self avoiding random walks: pertaining to this subject, Bolthausen presented recent results leading to an invariance principle in large enough dimensions.

A comprehensive overview of the broad field of random matrices and directed polymers was presented by Bouchaud.

Other participants showed most interesting applications of log-correlated random fields in physics and mathematical finance (Muzy). Gaussian free fields also show up in the study of the XOR-Ising model (de Tilière). Very much in the spirit of disordered systems, one may as well study the Gaussian free field pinned in a random media: this topic was the subject of a talk by Coquille.

On the physics side, Derrida explained the role of hierarchal structures, and the emergence of the Bolthausen-Sznitman coalescent in simple models of evolutions. Le Doussal presented some general methods to address the question of the behavior of the extremes of log-correlated random fields, and the role played by the so-called freezing transition, a fundamental aspect of certain models in the 1-RSB class; a similarly intriguing behavior was discussed by Rosso for the case of the Burgers equation. Ostrovsky highlighted some steps towards the derivation of the limit lognormal law. Simm presented studies on fractional Brownian motion with Hurst index $H = 0$, whereas Gruzberg explained the role of multi-fractal measures in the problem of Anderson localization.

Finally, mesmerizing links between branching diffusions/log-correlated fields and number theory were presented by Bourgade and Keating.

3. Assessment

We believe the timing of the meeting could not have been any better. In particular, the conference brought together scholars working in disparate fields which - thanks to the spectacular ongoing research activity - are getting "closer and closer": the realization of this fact was extremely appreciated by the participants, among the foremost researchers in their own fields, who emphasized how much they enjoyed to see how (and to which extent) certain mathematical structures encountered in their research show up in domains which might look at first sight quite far apart. The beautiful talks provided by the lecturers exposed the audience to a common background which is crystallizing at steady pace, and

there are good reasons to believe that the ensuing insights will lead to fruitful interactions in the years to come.

4. Final program of the meeting

4.1. **Timetable.**

	Mon	Tue	Wed	Thu	Fri
0845 - 0930	registration	breakfast	breakfast	breakfast	breakfast
0930 - 1015	Derrida	Duplantier	Lee	Keating	Maillard
1015 -1100	Bramson	Rhodes	Belius	Bourgade	Coquille
1100 - 1130	coffee break	coffee break	coffee break	coffee break	coffee break
1130 - 1215	Ding	Vargas	Rosen	Rosso	Zindy
1215 -1300	Zeitouni	Kupiainen	Eisenbaum	Simm	Aïdékon
1300 - 1430	lunch	lunch	lunch	lunch	closing
1430 - 1515	Le Doussal	Nikula	Ostrovsky	Bolthausen	
1515 - 1600	Bouchaud	Webb	Muzy	Comets	
1600 -1630	coffee break	coffee break	coffee break	coffee break	
1630 -1715	Louidor	Barral	Gruzberg		
1715 - 1800	Arguin	de Tilière			

4.2. **Talks.** The abstracts could not be included in this dociment because of the limitation on the total number of pages: they are available on the conference's website.

Élie Aïdékon, UPMC Paris: The extremal process in nested conformal loops.

Louis-Pierre Arguin, University of Montreal: Law and Ergodicity of the Extremal Process of branching Brownian motion.

Julien Barral, Universite Paris 13: Multifractal analysis of Mandelbrot measures and related questions.

David Belius, ETH Zurich: Gumbel fluctuations for the cover time of the discrete torus.

Erwin Bolthausen, University of Zurich: A new view on lace expansions and self-avoiding random walks.

Jean Philippe Bouchaud, CFM Paris: Some thoughts about random matrix theory and directed polymers.

Paul Bourgade, Harvard: Random Matrices, Strong Szegö's theorem and L functions.

Maury Bramson, University of Minnesota: Convergence in Law of the Maximum of the Two-Dimensional Gaussian Free Field, Part 1: Motivation from Branching Brownian Motion.

Francis Comets, Paris-Diderot: Large deviations for the cover time of two-dimensional torus.

Loren Coquille, University of Geneva: Discrete GFF with disordered pinning on Z^d.

Jian Ding, University of Chicago: Convergence in Law of the Maximum of the Two-Dimensional Gaussian Free Field, Part 2: Tail estimates via sparsification and MBRW.

Bernard Derrida, ENS Paris: Simple models of evolution with selection and genealogies.

Bertrand Duplantier, CEA Saclay: Schramm-Loewner Evolution and Liouville Quantum Gravity.

Nathalie Eisenbaum, Paris 6: Inequalities for permanental processes.

Ilya Gruzberg, Univesity of Chicago: Critical wave functions and their multifractal spectra.

Jon P. Keating, Bristol: Freezing and extreme values: from RMT to number theory.

Antti Kupiainen, Helsinki: Critical Mandelbrot Cascades.

Pierre Le Doussal, ENS Paris: Extrema of log-correlated fields, freezing transitions and multifractality.

James Lee, U Washington: Cover times, majorizing measures, and the Gaussian free field.

Oren Louidor, UCLA: The thinned extremal process of the 2D discrete Gaussian Free Field.

Pascal Maillard, Weizmann Institute: Branching Brownian motion martingales and FKPP travelling waves.

Jean-Francois Muzy, Universite de Corse: Non-stationary $1/f$ noise as a model for log-volatility of financial time series.

Miika Nikula, Helsinki: On geometric properties of critical lognormal multiplicative chaos.

Dmitry Ostrovsky: Towards the Limit Lognormal Law.

Rémi Rhodes, Paris Dauphine: Liouville Brownian Motion.

Jay Rosen, City University of New York: Isomorphism theorems for intersection local times of random interlacements.

Alberto Rosso, Paris Sud - Orsay: Freezing transition in decaying Burgers turbulence.

Nick Simm, Queen Mary and Westfield College: fBm with Hurst index $H = 0$ and statistics of GUE characteristic polynomials.

Beatrice de Tiliere, UPMC Paris: Loops in the XOR-Ising model.

Vincent Vargas, Paris Dauphine: Atomic Gaussian multiplicative chaos and KPZ duality.

Christian Webb, Helsinki: Some tools for proving convergence of multiplicative cascade measures.

Ofer Zeitouni, U Minnesota and Weizmann Institute: Convergence in Law of the Maximum of the Two-Dimensional Gaussian Free Field, Part 3: From tail estimates to convergence of the maximum, and characterization of the limit.

Olivier Zindy, UPMC Paris: Poisson-Dirichlet statistics for the extremes of log-correlated Gaussian fields.

5. List of Participants

1. Elie Aidekon, (M)
elie.aidekon@upmc.fr
Maître de conférences
LPMA , Université Pierre et Marie Curie (P6)
75252 PARIS Cedex 05, France

2. Louis-Pierre Arguin, (M)
arguinlp@dms.umontreal.ca
Professeur
Université de Montréal, Pavillon André-Aisenstadt
2920, Chemin de la tour, local AA-5190

Montréal (Québec) H3T 1J4, Canada

3. Julien Barral, (M)
julien.barral@math.univ-paris13.fr
Professeur
Université Paris 13, LAGA-Institut Galilée,
99 av. J.-B. Clément
93400 Villetaneuse, France

4. David Belius, (M)
david.belius@math.ethz.ch
Post doctorant
Department of Mathematics
ETH Zurich (Swiss Federal Institute of Technology)
Raemistrasse 101, 8092 Zurich
Switzerland

5. Erwin Bolthausen, (M)
eb@math.uzh.ch
Professeur
Institut fur Mathematik
Universitat Zurich
Winterthurerstrasse 190
CH-8057 Zurich
Switzerland

6. Jean Philippe Bouchaud, (M)
jean-philippe.bouchaud@cea.fr
Directeur de recherche
CFM S.A. 23/25, rue de l'Université 75007 Paris

7. Paul Bourgade, (M)
bourgade@math.harvard.edu
Professeur
Department of Mathematics Harvard University
One Oxford Street,
Cambridge, MA 02138, USA

8. Maury Bramson, (M)
bramson@math.umn.edu
Professeur
School of Mathematics,
University of Minnesota,
206 Church St. SE,

Mpls, MN 55455 USA

9. Tim van de Brug. (M)
t.vande.brug@vu.nl
Post doctorant
VU University Amsterdam
Department of Mathematics
De Boelelaan 1081a
1081 HV Amsterdam
The Netherlands

10. Laurent Chevillard, (M)
Cargé de recherche - CNRS
laurent.chevillard@ens-lyon.fr
cole Normale Supérieure de Lyon
Laboratoire de Physique, ENS Lyon

11. Francis Comets, (M)
comets@math.jussieu.fr
Professeur
Université Paris Diderot - Paris 7
F-75205 Paris Cedex 13

12. Loren Coquille, (F)
loren.coquille@unige.ch
Doctorante
Section de mathématiques
2-4 rue du Lièvre, CP 64
CH-1211 Genève 4
Switzerland

13. Jian Ding, (M)
jianding@galton.uchicago.edu
Professeur
University of Chicago Department of Statistics,
5734 S. University Avenue,
Chicago, IL 60637 USA

14. Bernard Derrida, (M)
derrida@lps.ens.fr
Professeur
Laboratoire de Physique Statistique ENS,
24 rue Lhomond 75005 Paris

15. Pierre Le Doussal, (M)
pierre.ledoussal@lpt.ens.fr
Professeur
Laboratoire de Physique Théorique,
cole Normale Supérieure,
24 rue Lhomond 75005 Paris France

16. Bertrand Duplantier, (M)
bertrand.duplantier@cea.fr
Directeur de recherches - CEA
Institut de Physique Théorique,
Orme des Merisiers batiment 774,
Point courrier 136
CEA/DSM/IPhT, CEA/Saclay
F-91191 Gif-sur-Yvette Cedex
FRANCE

17. Nathalie Eisenbaum, (F)
nathalie.eisenbaum@upmc.fr
Professeur
Laboratoire de Probabilités,
Université Paris VI - 4, Place Jussieu
75252 PARIS CEDEX 05

18. Yan Fyodorov, (M)
Y.Fyodorov@qmul.ac.uk
Professeur
Queen Mary and Westfield College
School of Mathematical Sciences,
Queen Mary University of London,
Mile End Road, London E1 4NS, UK

19. Veronique Gayrard, (F)
Directeur de recherche - CNRS
gayrard@cmi.univ-mrs.fr
CMI, Aix-Marseille Université,
39, rue F. Joliot Curie,
13453 Marseille Cedex 13
FRANCE

20. Ilya Gruzberg, (M)
gruzberg@uchicago.edu
Professeur
University of Chicago,
Department of Physics,

5720 S. Ellis Ave, Chicago,
IL 60637 USA

21. Jon P. Keating, (M)
j.p.keating@bristol.ac.uk
Professeur
School of Mathematics
University of Bristol
Bristol BS8 1TW - UK

22. Nicola Kistler, (M)
Post Doctorant, titulaire de la chaire Jean-Morlet, CIRM
CMI, Aix-Marseille Université,
39, rue F. Joliot Curie,
13453 Marseille Cedex 13
FRANCE

23. Jeffrey Kuan, (M)
jkuan@math.harvard.edu
Doctorant
Department of Mathematics
Harvard University
One Oxford Street
Cambridge MA 02138

24. Antti Kupiainen, (M)
antti.kupiainen@helsinki.fi
Professeur
P.O. Box 68 (Gustaf Hallstromin katu 2b)
FI-00014 University of Helsinki,
Finlande

25. Thi Huyen Le, (F)
huyen@cmi.univ-mrs.fr
Doctorante
CMI, Aix-Marseille Université,
39, rue F. Joliot Curie,
13453 Marseille Cedex 13
FRANCE

26. James Lee, (M)
jrl@cs.washington.edu
Professeur associé
Department of Computer Science and engineering,
Box 352350

University of Washington Seattle, WA 98195-2350 USA

27. Marcin Lis, (M)
m.lis@vu.nl
Doctorant
VU University Amsterdam
Department of Mathematics
De Boelelaan 1081a
1081 HV Amsterdam
The Netherlands

28. Oren Louidor, (M)
louidor@math.ucla.edu
Post doctorant
Mathematics Department
University of California Los Angeles
520 Portola Plaza
Math Sciences Building, Room 6148
Los Angeles, CA 90095

29. Gregory Maillard, (M)
maillard@cmi.univ-mrs.fr
Professeur
CMI, Aix-Marseille Université,
39, rue F. Joliot Curie,
13453 Marseille Cedex 13
FRANCE

30. Pascal Maillard, (M)
pascal.maillard@weizmann.ac.il
Post doctorant
Faculty of Mathematics,
Weizmann Institute of Science,
PoB 26, Rehovot 76100, Israel

31. Jean-Francois Muzy, (M)
muzy@univ-corse.fr
Professeur
Laboratoire SPE UMR 6134
CNRS-Université de Corse
Campus Grimaldi
20250 Corte France

32. Miika Nikula, (M)
Doctorant

miika.nikula@helsinki.fi
PL 68 (Gustaf Hallstromin katu 2b)
00014 Helsingin yliopisto FINLAND

33. Dmitry Ostrovsky, (M)
dm_ostrovsky@aya.yale.edu
Post doctorant
Yale University
Mathematics Dept.
PO Box 208283
New Haven, CT 06520-8283
USA

34. Etienne Pardoux, (M)
pardoux@cmi.univ-mrs.fr
Professeur
CMI, Aix-Marseille Université,
39, rue F. Joliot Curie,
13453 Marseille Cedex 13
FRANCE

35. Pierre Picco, (M)
picco@cmi.univ-mrs.fr
Directeur de recherche - CNRS
CMI, Aix-Marseille Université,
39, rue F. Joliot Curie,
13453 Marseille Cedex 13
FRANCE

36. Remi Rhodes, (M)
rhodes@ceremade.dauphine.fr
Maître de conférence
CEREMADE-Université Paris-Dauphine
Place du maréchal de Lattre de Tassigny
75775 Paris Cedex 16

37. Jay Rosen, (M)
jrosen30@optimum.net
Professeur
Department of Mathematics College of Staten Island,
CUNY, 2800 Victory Boulevard
Staten Island, NY, 10314 USA

38. Alberto Rosso, (M)
alberto.rosso@u-psud.fr

Laboratoire Physique Theorique et Modeles Statistiques
15 rue G. Clemenceau, 91405 Orsay, France

39. Jean Ruiz, (M)
ruiz@cpt.univ-mrs.fr
Directeur de recherche - CNRS
Centre de Physique Théorique
Campus de Luminy, Case 907
13288 Marseille cedex 9, France

40. Senya Shlosman, (M) CPT-CNRS Marseille
shlosman@cpt.univ-mrs.fr
Directeur de recherche - CNRS
Centre de Physique Théorique
Campus de Luminy, Case 907
13288 Marseille cedex 9, France

41. Nick Simm, (M)
n.simm@qmul.ac.uk
Doctorant
School of Mathematical Sciences,
Queen Mary University of London,
Mile End Road, London E1 4NS, UK

42. Marina Talet, (F)
talet@cmi.univ-mrs.fr
Maître de conférence
CMI, Aix-Marseille Université,
39, rue F. Joliot Curie,
13453 Marseille Cedex 13
FRANCE

43. Béatrice de Tilière (F)
Maître de conférence
beatrice.de_tiliere@upmc.fr
UPMC, LPMA
4 Place Jussieu, Tour 16-26, 2ème étage
75005 Paris

44. Vincent Vargas, (M)
vargas@ceremade.dauphine.fr
Chargé de recherche - CNRS
CEREMADE
CNRS, UMR 7534
Université Paris-Dauphine

F-75016 Paris, France

45. Thi Thuy Ha Vu, (F)
CMI, Aix-Marseille Université,
Doctorante
39, rue F. Joliot Curie,
13453 Marseille Cedex 13
FRANCE

46. Christian Webb, (M)
christian.webb@helsinki.fi
Doctorant
P.O. Box 68 (Gustaf Hllstrmin katu 2b)
FI-00014 University of Helsinki
Helsinki, Finland

47. Ofer Zeitouni, (M)
zeitouni@math.umn.edu
Professeur
Faculty of Mathematics,
Weizmann Institute of Science,
PoB 26, Rehovot 76100, Israel

48. Olivier Zindy, (M)
olivier.zindy@upmc.fr
Professeur
Laboratoire de Probabilités et Modèles Aléatoires
CNRS UMR 7599v Université Paris 6 4 place Jussieu
75252 PARIS Cedex 05 France