FINAL REPORT FOR THE ESF AWARD RGLIS - SCIENCE MEETING 4703

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1. Summary

The meeting **Random Combinatorial Structures and Statistical Mechanics** was held in Venice from 6 to 10 May 2013. The venue of the meeting was the Palazzo Pesaro-Papafava, which is managed by the University of Warwick, and is ideally suited for small conferences.

The audience listened to impressive talks delivered by distinguished academics on various topics, including disordered systems, interacting stochastic particle systems, coagulation and fragmentation models, and random matrix theory. Speakers came from many different locations, including the Weizmann Institute, the Weierstrass Institute, the Universities of L'Aquila, Arizona, Bath, Bielefeld, Bonn, Bristol, Budapest, Cambridge, Darmstadt, Genève, Leiden, Montréal, Oxford, Padova, La Sapienza, Versailles, York, and Zürich. In addition, there were several speakers and participants from the Universities of Warwick and Queen Mary.

The talks and the discussions offered a unique perspective on the links between statistical mechanics, probability theory, and combinatorics. They provided numerous concrete examples where these links exist. The audience is now uniquely positioned to systematise and exploit them further.

2. Description of the scientific content of and discussion at the event

Going back to ideas of Boltzmann and Gibbs, statistical mechanics is the domain of physics that explains macroscopic behaviour starting with microscopic laws. Over the years, physicists have studied an amazing range of models with very imaginative mathematical methods. Connections with mathematics have recently grown deeper. This is especially true with probability theory and the theory of PDEs. And while some forerunners have pointed out interesting links with combinatorics for some time, the use of powerful combinatorial tools in statistical mechanics is just starting.

This meeting offered the possibility to discuss many recent advances. Interested readers are referred to the abstracts of the talks, that can be found in the programme below. The main topics dealt with random objects having a spatial nature. Their study require advanced techniques in the theory of probability. Many insights from statistical mechanics prove essential. The generating functions of certain combinatorial objects, and several counting properties, play also an important role. The novel element of the field is the use of combinatorial structures and species, and its further developments in order to fit this tool to the problems at hand.

3. Assessment of the results and impact of the event on the future direction of the field

We start with the scientific impact and conclude with the broader impact about the engagement between the universities of Warwick and Queen Mary.

Regarding the developments of mathematics, many bridges were created between the fields of probability theory, statistical mechanics, and combinatorics. Numerous discussions took place where experts of one field explained in details some concepts that were novel in another field. Conversely, people could identify and describe open problems in a field that can be studied and illuminated with techniques from other fields. The ever closer interactions between probability theory and combinatorics should be emphasised.

The meeting had a strong component of mathematicians from Warwick and Queen Mary. The impetus for the institutional collaboration comes from the Partnership between these two universities, that was established last year. The meeting was a great occasion for people to meet, to know each other, and to discuss, away from the usual burdens of the office. Many ideas of future collaboration were floated and several suggestions will be made soon.

4. Final programme of the meeting

Monday 6 May

09:30-09:50 Welcome

09:50-10:10 **Roman Kotecký** (University of Warwick)
Long range order for planar Potts antiferromagnets
We prove entropic long range order for the $q = 3$ Potts antiferromagnet on a class planar lattices at low temperatures. The proof is based on an enhanced Peierls argument (which is of independent interest even for the Ising model for which it extends the range of temperatures with proven long range order) combined with an additional percolation argument. The main ideas will be explained in the simplest zero temperature case. Based on a joint work with Alan Sokal and Jan Swart.

10:15-10:55 **Anton Bovier** (University of Bonn)
Branching Brownian motion: extremal process and ergodic theorems
Branching Brownian motion is a classical process in probability, describing a population of particles performing independent Brownian motion and branching according to a Galton Watson process. The behaviour of the maximal spread of this population has been in the focus of interest since the 1970'ies, culminating in the celebrated work of Bramson who gave a precise asymptotics of the maximal spread using a deep connection to a non-linear pde, the F-KPP equation. I will review some recent progress on the analysis of the fine structure of the population near the maximum. In particular, I will present a recent result that shows the convergence of the extremal process of BBM to a random cluster process under the empirical distribution of the process. This proves and extends a conjecture by Lalley and Sellke.
All results are joint work with L.-P. Arguin (Montréal) and N. Kistler (Marseille).

11:45-12:25 **Frank den Hollander** (University of Leiden)
Extremal geometry of a Brownian porous medium

The path of a Brownian motion on a d-dimensional torus run for time t is a random compact subset of the torus. We study the geometric properties of its complement for $d \geq 3$ in the limit as $t \to \infty$. In particular, we identify the space scale that determines its global properties, and derive a large deviation principle for the shape of its component with largest capacity. This in turn leads to large deviation principles for the largest inradius, the principal Dirichlet eigenvalue, and various related quantities. A hint is given of what is expected for $d = 2$.

12:30-12:55 **Sasha Gnedin** (Queen Mary University)

Functionals of random partitions and the generalised Erdős-Turan laws for permutations

The Erdős-Turán law for the uniform random permutation and its generalisation for Ewens' distribution is a central limit theorem for the logarithm of the order of permutation. This classical result is closely related to a CLT for the sum of logarithms of the block sizes for the partition generated by sampling from the PD/GEM random measure. We extend the result to more general permutations and partitions associated with the stick-breaking division of the unit interval. Depending on properties of the stick-breaking factor we show that the normal and some other stable distributions may occur in the limit. Joint work with Alexander Iksanov and Alexander Marynych.

15:00-15:40 **Davide Gabrielli** (Universita l'Aquila)

Large deviations for the empirical flow of continuous time Markov chains

We consider a continuous time Markov chain on a countable state space and prove a joint large deviation principle for the empirical measure and the empirical flow, which accounts for the total number of jumps between pairs of states. By projection, we recover the Donsker–Varadhan large deviation principle for the empirical measure.We discuss some applications. Joint work with L. Bertini, A. Faggionato.

16:15-16:35 **Roger Tribe** (University of Warwick)

Annihilating Brownian motions on R is an extended Pfaffian point process.

I will sketch the derivation of the stated result and discuss the relation of the Pfaffian point process and the real Ginibre random matrix ensemble.

16:35-16:55 **Oleg Zaboronski** (University of Warwick)

Ginibre process

We study the statistics of real eigenvalues for matrix-valued Brownian motions (Ginibre process). We discuss a connection between Ginibre process and annihilating Brownian motions in one dimension.

Tuesday 7 May

09:30-10:10 **Yvan Velenik** (University of Geneva)

On the Gibbs states of the 2d Potts model

Determining the set of all the (infinite-volume) Gibbs measures associated to a statistical mechanical system is a very difficult task, that has been accomplished only in very few cases, the most famous being the description, by Aizenman and Higuchi in the late 1970s, of the set of all Gibbs states associated to the 2-dimensional Ising model. I'll present a recent extension of this result to the 2-dimensional Potts model. This extension relies on a new approach to this type of problems, which, in addition to being more robust, provides a much better picture of the underlying phenomenon. An interesting feature of this approach is that it also provides quantitative results for large finite systems.

The talk will be based on a joint work with L. Coquille, H. Duminil-Copin and D. Ioffe.

10:15-10:55 **Dirk Zeindler** (Universitaet Bielefeld)

Asymptotic statistics of cycles in surrogate-spatial random permutations

We propose an extension of the Ewens measure on permutations by choosing the cycle weights to be asymptotically proportional to the degree of the symmetric group. This model is primarily motivated by a natural approximation to the so-called spatial random permutations recently studied by V. Betz and D. Ueltschi (hence the name "surrogate-spatial"), but it is of substantial interest in its own right. We show that under the suitable (thermodynamic) limit both measures have the similar critical behaviour of the cycle statistics characterized by the emergence of infinitely long cycles. Moreover, using a greater analytic tractability of the spatial-surrogate model, we obtain a number of new results about the asymptotic distribution of the cycle lengths (both small and large) in the full range of subcritical, critical and supercritical domains. In particular, in the supercritical regime there is a parametric "phase transition" from the Poisson–Dirichlet limiting distribution of ordered cycles to the occurrence of a single giant cycle. Our techniques are based on the asymptotic analysis of the corresponding generating functions using Pólya's Enumeration Theorem and complex variable methods.

11:45-12:25 **Alan Hammond** (University of Oxford)

The phase transition to infinite cycles in the random stirring model on trees

We establish that the phase transition for infinite cycles in the random stirring model on an infinite regular tree of high degree is sharp. That is, we prove that there exists d_0 such that, for any d at least d_0, the set of parameter values at which the random stirring model on the rooted regular tree with offspring degree d almost surely contains an infinite cycle consists of a semi-infinite interval.

12:30-12:55 **Boris Khoruzhenko** (Queen Mary University)

How many eigenvalues of a truncated orthogonal matrix are real?

This talk is about eigenvalue distributions of truncated random orthogonal matrices. Consider a matrix chosen at random from the orthogonal group $O(N)$. Truncate it by removing L columns and L rows. The resulting matrix is a random contraction with eigenvalues inside the unit disk in the complex plane. It turns out that it is possible to obtain the joint probability distribution of eigenvalues of truncated Haar orthogonals in closed form and also all marginal distributions. There are (at least) two interesting regimes in this limit: (i) weak non-orthogonality when $L = O(1)$, e.g. $L = 1$, and (ii) strong non-orthogonality when L is proportional to N, e.g. $L = N/2$. One of the striking features of eigenvalue distribution for large real random matrices is the accumulation of eigenvalues on the real line. For truncated orthogonals in the regime of strong non-orthogonality, the (expected) number of real eigenvalues grows logarithmically with N and their densi ty is proportional to $1/(1-x^2)$ away from the two accumulation points at $x = \pm 1$. This behaviour is similar to that of real roots of random Kac polynomials. In the regime of strong non-orthogonality one recovers the characteristic features of the real Gaussian matrices (Ginibre ensemble), in particular the number of real eigenvalues scales with the square root of matrix dimension. This talk is based on a joint work with Hans-Juergen Sommers and Karol Zyczkowski.

15:00-15:40 **Sabine Jansen** (Universiteit Leiden)

Many-species Tonks gas

We consider a mixture of non-overlapping rods of different lengths ℓ_k moving on the continuous line. We compute the expansion of the infinite volume pressure in powers of the activities z_k and the densities ρ_k that generalize well-known expressions for the single-species Tonks gas, and give necessary and sufficient conditions for the convergence of the expansions. This provides an explicit example against which to test known cluster expansion criteria, and illustrates that for non-negative interactions, the virial expansion can converge in a domain much larger than the activity expansion. The material complements recent work with S. Tate, D. Tsagkarogiannis and D. Ueltschi.

16:15-16:55 **Cécile Mailler** (University of Versailles)

Smoothing equations for large two-colour Pólya urns

This talk will focus on large two-colour Pólya urns. From the study of the asymptotic behaviour of such an urn arises a random variable denoted by W. The underlying tree structure of the urn permits to see W as the solution in law of a fixed point equation, from which we can deduce information about its moments, or about the existence of a density. This work can be done on the discrete urn itself, or on its continuous time embedding. Though the two variables W (arisen from discrete or continuous time) are different, they are related by connexions, which often permit to translate results from one W to the other. This work is a collaboration with Brigitte Chauvin and Nicolas Pouyanne.

17:00-17:20 **Volker Betz** (Universities of Darmstadt and Warwick)

Planar spatial random permutations

Spatial random permutations in two dimensions appear to be a rich an intriguing probabilistic model, with connections to mean curvature motion, the Bose gas, random fractal and possibly SLE. I will give an introduction into the model and explain some special cases where the connections above are evident. Furthermore, I will show numerical evidence for the validity of these connections also in more general situations.

Wednesday 8 May

09:30-10:10 **Ofer Zeitouni** (Weizmann Institute)

Slowdown for branching Brownian motions

For fixed large T consider a branching Brownian motion whose diffusivity (or branching rate) depend on time ina macrsocopic way, ie as a function of t/T for time $0 < t < T$. We will describe the slowdown effect on the speed of front propagation and show that it is roughly $cT^{1/3}$ for the case of strictly decreasing variance profile, with an explicit constant c related to the Airy equation.

Joint work with Ming Fang and with Pascal Maillard.

10:15-10:55 **Balint Tóth** (Universities of Bristol and Budapest)

Two routes to superdiffusivity

I will survey recent results on superdiffusive scaling limits (with logarithmic corrections) for 2d random motions with long memory. In self-repelling random motions (also called Brownian polymer model) and in some random walks in random environments the superdiffusivity is built up by picking up velocity autocorrelation from the past trajectory. In the Boltzmann-Grad limit of the periodic Lorentz gas the superdiffusive behaviour is due to the fat tail of the free flight distribution. (Based on joint work with Benedek Valko, respectively, Jens Marklof)

11:45-12:25 **Nick Ercolani** (University of Arizona)

Nonlinear evolution equations in the combinatorics of random maps

In this talk we will discuss some developments that bring to bear methods for studying systems of conservation laws on the asymptotic analysis of generating functions for "maps". A "random map" is a random topological tessellation of a Riemann surface. These are combinatorial objects that first arose in attempts to solve the four color problem but soon thereafter took on a life of their own. Subsequently, physicists working on the unification of the "strong" and "weak" forces discovered that generating functions for maps emerge naturally in the enumeration of Feynman diagrams for unitary gauge theories.

Our recent work provides the means for the explicit evaluation of map generating functions in terms of closed form solutions of the aforementioned conservation laws. These conservation laws are certain continuum limits of the Toda lattice differential equations in which the time variables are coupling coefficients for unitary random matrix ensembles. These generating functions have potential relevance for developing statistical mechanics on random lattices.

12:30-12:55 **Neil O'Connell** (University of Warwick)

Geometric RSK correspondence, Whittaker functions and random polymers

In recent work [O'C 2009, Corwin-O'C-Seppalainen-Zygouras 2011] we have found some surprising connections between geometric variants of the RSK correspondence and GL(n,R) Whittaker functions, thus establishing an underlying algebraic / integrable structure for the study of discrete (or semi-discrete) random polymer models. In this talk, I will describe some more recent work with Timo Seppalainen and Nikos Zygouras which provides an elementary explanation for this connection. The main result uses a new description of A.N. Kirillov's (2000) geometric lifting of the RSK map as a composition of 'local moves', and provides an 'algebraic-combinatorial' framework for the study of random polymers on a lattice with log-gamma weights, including models with symmetry constraints. The local move description is based on Noumi-Yamada's (2004) geometric row insertion algorithm. We also obtain new 'combinatorial' proofs (and generalisations) of three Whittaker integral identities due to Stade (2001, 2002).

Thursday 9 May

09:30-10:10 **Chris Hughes** (University of York)

Extreme behaviour

There has much interest in finding the true rate of growth of the Riemann zeta function, and this has led to studies on the extreme values of characteristic polynomials. This talk will survey some recent, and not so recent, results in this area.

10:15-10:55 **Jean Bertoin** (University of Zrich)

Almost giant clusters for percolation on large trees with logarithmic heights

We consider Bernoulli bond percolation on a tree with size n?1, with a parameter p(n) that depends on the size of that tree. Our purpose is to investigate the asymptotic behavior of the sizes of the largest clusters for appropriate regimes. We shall first provide a simple characterization of tree families and percolation regimes which yield giant clusters, answering a question raised by David Croydon. In the second part, we will review briefly recent results concerning two natural families of random trees with logarithmic heights, namely recursive trees and scale-free trees. We shall see that the next largest clusters are almost giant, in the sense that their sizes are of order n/lnn, and obtain precise limit theorems in terms of certain Poisson random measures. A common feature in the analysis of percolation for these models is that, even though one addresses a static problem, it is useful to consider dynamical versions in which edges are removed, respectively vertices are inserted, one after the other in certain order as time passes.

11:30-12:10 **Louis-Pierre Arguin** (University of Montreal)

Poisson-Dirichlet statistics for the extremes of log-correlated Gaussian fields

Gaussian fields with logarithmically decaying correlations, such as branching Brownian motion and the 2D Gaussian free field, are conjectured to form a new universality class of extreme value statistics (notably in the work of Carpentier & Ledoussal and Fyodorov & Bouchaud). This class is the borderline case between the class of IID random variables, and models where correlations start to affect the statistics. In this talk, I will describe a general approach based on rigorous works in spin glass theory to describe universal features of the Gibbs measure of these Gaussian fields. This is joint work with Olivier Zindy.

12:15-12:55 **Alessandra Faggionato** (Universita La Sapienza)

Nonequilibrium dynamics in the East model

The East model is a one dimensional interacting particle system with kinetically constrained jumps: the dynamics is of Glauber type and a particle can flip only if there is a vacancy on its left. Although the equilibrium state is trivial, the dynamics present several interesting features, making the East model particularly suited as simplified model of glasses. In this talk we recall some recent rigorous results obtained for the East model (in collaboration with P. Chleboun, F. Martinelli, C. Roberto and C. Toninelli), describe some open conjectures concerning the dynamics near to the relaxation time (Aldous-Diaconis conjecture and the superdomain dynamics of Evans-Sollich) and mention some higher dimensional extensions.

15:00-15:40 **Peter Mörters** (University of Bath)

Condensation in models of selection and mutation

We describe the onset of condensation in the simple model for the balance between selection and mutation introduced by Kingman in terms of a scaling limit theorem. Loosely speaking, we find that the wave moving towards genes of maximal fitness has the shape of a gamma distribution. We conjecture that this wave shape is a universal phenomenon that can also be found in a variety of more complex models. The talk is based on joint work with Steffen Dereich (Münster).

16:15-16:55 **Geoffrey Grimmett** (University of Cambridge)

Percolation of finite clusters

The infinite cluster of bond percolation is presumed to have very low density just above the critical point. How extensive is its complement, does it contain an infinite connected component? An answer will be presented, and its relationship to the lace expansion will be discussed. (joint work with Ander Holroyd and Gady Kozma)

Friday 10 May

09:30-10:10 **Wolfgang König** (WIAS / TU Berlin)

A Variational Formula for the Free Energy of a Many-Boson System

We consider N bosons in a box in $\mathbb{R}^d$ with volume N/ρ under the influence of a mutually repellent pair potential. The particle density $\rho \in (0, \infty)$ is kept fixed. Denote by $\mathcal{H}_N$ the corresponding Hamilton operator. The symmetrised trace of $\mathrm{e}^{-\beta\mathcal{H}_N}$ describes the bosons at positive temperature $1/\beta$. The existence of the limiting free energy, $f(\beta, \rho)$, is well known, as well as the fact that its value does not depend on the boundary condition.

Our main result is the identification of $f(\beta, \rho)$ in terms of an explicit variational formula, for all sufficiently small β. The main tools are a description in terms of a marked Poisson point process and a large-deviation analysis of the stationary empirical field. The formula in particular describes the asymptotic cycle structure that is induced by the symmetrisation in the Feynman-Kac formula. Our identification concerns the non-condensation phase, where 'infinitely long' cycles do not contribute.

We also derive a related variational formula as a general upper bound for $f(\beta, \rho)$ for any β, which we hope to use in future work to distinguish between the non-condensation phase and the condensation phase.

(joint work with Stefan Adams and Andrea Collevecchio)

10:15-10:55 **James Martin** (University of Oxford)

Mean-field forest fire models

Consider the following extension of the Erdős-Rényi random graph process; in a graph on n vertices, each edge arrives at rate 1, but also each vertex is struck by lightning at rate λ, in which case all the edges in its connected component are removed. Such a "mean-field forest fire" model was introduced by Ráth and Tóth. For λ scaling with n in an appropriate way, the model exhibits self-organised criticality. We describe a scaling limit in an appropriate regime, in terms of a multiplicative coalescent with an added "deletion" mechanism, and we investigate the limit of the process as seen from a typical vertex.

11:45-12:25 **Gady Kozma** (Weizmann Institute)

Random points in the metric polytope

We investigate a random metric space on n points constrained to have all distances smaller than 2, or in other words, we take a random point from the Lebesgue measure on the intersection of the so-called metric polytope with the cube $[0, 2]^{n(n-1)/2}$. We find that, to a good precision, the distances behave simply like i.i.d. numbers between 1 and 2. The proof uses an interesting mix of entropy methods and the Szemeredi regularity lemma (all terms will be explained in the talk). Joint work with Tom Meyerovitch, Ron Peled and Wojciech Samtoij.

12:30-12:55 **Stephen Tate** (University of Warwick)

Multispecies virial expansion

The notion of a cluster and virial expansion for systems where there is only one type of particle is reasonably well understood from the work of Mayer and his collaborators in the 1930s. However, what is less understood is the impact of having such expansions but with more activity and density parameters for different constituent particles. This question was answered somewhat by Fuchs (1942), who covers the case of finitely many species. He introduces the generalised radii of convergence of the virial expansion and uses techniques which foreshadow the Lagrange-Good inversion (1965). The Lagrange Good inversion is used to invert the cluster expansion relationships for pressure and density, to achieve the virial expansion of pressure in terms of many density parameters. The technique is general and algebraic. We can treat all series as formal and get a relationship on the coefficients. We investigate appropriate convergence criteria for the virial expansion we achieve from this, which I present and then explain where it comes from with regards to the multivariable Lagrange-Good inversion formula. For regular virial expansions there is the interpretation of the coefficients being irreducible integrals or the integrals corresponding to 2-connected graphs. We achieve the analogue of this through the dissymmetry theorem. I will briefly explain the main purpose behind the dissymmetry theorem and how it connects with the Lagrange Inversion. I will end with some ideas on what still needs to be achieved after this work and what questions have been left over. This is joint work with Sabine Jansen, Dmitrios Tsagkarogiannis, and Daniel Ueltschi.

15:00-15:40 **Alessandra Bianchi** (University of Padova)

Random walk in a one-dimensional Levy random environment

In this talk we introduce a model for a one-dimensional Levy-Lorentz gas corresponding to a random walk in random environment on the line. The environment is provided by a renewal point process with inter-distances having a Levy-type distribution, that can be seen as a set of randomly arranged static scatterers. We investigate the quenched behavior of the walk and provide asymptotic results about its distribution and moments. In particular we show that, contrary to the annealed case, the quenched behavior of the motion is diffusive.

16:15-16:35 **Dudley Stark** (Queen Mary University)

Developments in the Khintchine-Meinardus probabilistic method for asymptotic enumeration

A theorem of Meinardus provides asymptotics of the number of weighted partitions under certain assumptions on associated ordinary and Dirichlet generating functions. The ordinary generating functions are closely related to Euler's generating function $\prod_{k=1}^{\infty} S(z^k)$ for partitions, where $S(z) = (1-z)^{-1}$. By applying a method due to Khintchine, we extend Meinardus' theorem to find the asymptotics of the coefficients of generating functions of the form $\prod_{k=1}^{\infty} S(a_k z^k)^{b_k}$ for sequences a_k, b_k and general $S(z)$. We also reformulate the hypotheses of the theorem in terms of generating functions. This allows us to prove rigorously the asymptotics of Gentile statistics and to study the asymptotics of combinatorial objects with distinct components. This is joint work with Boris Granovsky.

16:35-16:55 **Nikos Zygouras** (University of Warwick)

Continuum limits of pinning and copolymer models

The Disordered Pinning model is concerned with the interaction of a polymer path (in its simplest form a one dimensional simple random walk) with a defect line, where disorder is present. The Copolymer model is concerned with the behavior of a polymer path, placed in the interface of two solutions. In the Copolymer case, disorder is placed along the bonds of the polymer. Scaling appropriately the variance and the mean of the disorder we obtain con- tinuum limits of the partition functions of the two models in the form of a Wiener chaos expansion. The existence of such nontrivial scaling limit provides a new inter- pretation of the so called disorder relevant and irrelevant regime and helps to shed some light on the phase diagrams of the models at the "weak coupling regime". Based on work (in progress) with F. Caravenna and R.F. Sun.