Final Report on the Winter School "Spatial Models in Statistical Mechanics"

Application Reference No: 5209

1: Summary

Despite the tremendous international mathematical activity in spatial stochastic models and statistical mechanics, the subject is still not widely taught in graduate level courses in Germany. The aim of the winter school 2014 "Spatial Models in Statistical Mechanics" held at Technische Universität Darmstadt from 24 February 2014 to 28 February 2014 was to improve this situation by introducing PhD students and young postdocs to two selected areas of statistical mechanics in greater depth: the Schramm-Loewner evolution, and random walks in random dynamic random environments.

The core of the school consisted of two mini courses each comprising five lectures of 90 minutes. The topics of the first course were percolation on the triangular lattice and the Schramm-Loewner evolution. It was delivered by Vincent Beffara (UMPA - ENS Lyon). The second one dealt with random walks in dynamic random environments and was given by Vladas Sidoravicius (IMPA Rio de Janeiro).

In addition to the mini courses, there were five accompanying lectures held by invited speakers. It was intended that some of the lectures go deeper into the topics of the mini courses while others broaden the picture by focussing on different models of statistical mechanics. The invited speakers were Daniel Ueltschi (University of Warwick), Markus Heydenreich (Universiteit Leiden), Alexander Fribergh (Université de Toulouse), Gábor Pete (Technical University of Budapest), and Sebastian Müller (LATP Marseille). Their lectures covered quantum spin systems, phase transition phenomena for contact processes, biased random walks on supercritical percolation clusters, near-critical and critical Bernoulli percolation on planar lattices, interacting growth processes and invariant percolation

Moreover, participants could deliver talks of a duration of 20 minutes to present their own research. In total, there were three afternoon sessions each comprising three such talks. These talks were given by PhD students and young postdocs. Two out of the three sessions were very close to the main topics of the winter school and covered percolation models and random walks in random environments. The third session had a wider focus and included topics such as recursive triangulations of the disk, the Bernoulli sieve, and forest fire models.

Finally, each participant who did not give a regular talk of 20 minutes was asked to give a short presentation of 10 minutes to introduce the other participants to her or his field of research. Almost every participant took this opportunity and there were three short talk sessions with a total of 17 presentations which altogether provided a very good overview over what models and problems young researchers in the field in Europe are currently working on.

2: DESCRIPTION OF THE SCIENTIFIC CONTENT

The main contents of the winter school "Spatial Models in Statistical Mechanics" were critical Bernoulli percolation on the triangular lattice and the Schramm-Loewner evolution, covered by the lectures of Vincent Beffara, and random walks in dynamic random environments, covered by the mini course of Vladas Sidoravicius.

Critical Bernoulli percolation on the triangular lattice is one of several critical two-dimensional random models, such as the Ising and random-cluster models, loop-erased walks, uniform spanning trees and the Gaussian free field, in which great progress has been made in recent years. All these models exhibit conformal invariance in the scaling limit, which allows to describe their interfaces as random, conformally invariant curves, namely, Schramm-Loewner evolutions (SLE) or its variants.

In the mini-course on SLE an overview of SLE was given. The proof of convergence to it for critical Bernoulli percolation on the triangular lattice was given. It was discussed what can be deduced about the discrete models from their scaling limits. One central result that was shown was that the probability that a critical percolation cluster has diameter more than R scales like $R^{-5/48}$; and that the probability for $p > p_c$ that the origin lies on an infinite cluster behaves like $(p - p_c)^{5/36}$.

Random walks are one central topic in probability theory. Classical random walk theory is very developed nowadays with complete answers to many interesting questions such as recurrence and transience, velocity, typical fluctuations, large deviations from the typical behavior, *etc*. For random walks in static random environments, some problems are completely solved while for others only partial results exist. However, for random walks in dynamic random environments, even questions that are simple for classical random walks or random walks in static random environments turn out to be very difficult and remain unsolved today.

In the mini course on random walks in dynamic random environments by Vladas Sidoravicius, some by now classical results for random walks in static random environments were reviewed. Then several models of random walks in dynamical random environments were considered and their distinguishing features were highlighted, such as adoptive vs. non-adoptive random environments. Models considered were random walks on random walks, random walks in (deterministically) growing domains in the plane, the self-avoiding walk on the plane, the rancher process, the vacuum cleaner model in d-dimensional space and in a one-dimensional tube, the cookie walk in one dimension, the Coffman-Gilbert queueing model, sandpile models, activated random walks, and diffusion limited aggregation. Known results on these processes were stated and explained and open problems were presented. Some techniques of proof were studied in greater detail such as multiscale analysis.

The lectures given by invited speakers covered different spatial models. Daniel Ueltschi in his lecture gave a review of the background for quantum spin systems and explained random loop representations that allow to express the quantum correlations in a probabilistic way. He described an extension of these representations for the ferromagnetic and antiferromagnetic Heisenberg models and described conjectures about the joint distribution of the lengths of large loops.

In Markus Heydenreich's lecture, the two-dimensional contact process and a variant that arises as a model for vegetation patterns in arid landscapes were discussed. The focus of the lecture was

on percolation under invariant measures of such processes. It was proved that the percolation transition is sharp. Implications concerning early warning signals for desertification were discussed.

Alexander Fribergh explained the proof for the sharpness of the phase transition for speed in the biased random walk on the supercritical percolation cluster on $\mathbb{Z}^d$. To be more precise, for each $d \geq 2$, and for any supercritical parameter $p > p_c$, the proof was explained of the existence of a critical strength for the bias, such that, below this value, the speed is positive, and, above the value, it is zero. The value of the critical bias was explicitly identified, and, in the sub-ballistic regime, the polynomial order of the distance moved by the particle was given. Each of these conclusions is is related to the geometry of the traps that are most effective at delaying the walk. This geometry was explained. Finally, a dynamic renormalization statement in a much stronger form than was previously known was presented.

Gábor Pete's lecture was based on Kesten's scaling relation (1987), which gives the near-critical window and the off-critical exponent in terms of critical exponents of standard Bernoulli percolation on planar lattices. He described the results of a long project with Christophe Garban and Oded Schramm in which they built the scaling limit of the near-critical percolation ensemble from the critical scaling limit.

In Sebastian Müller's talk the relation between reversible growth processes and invariant percolation was underlined. Several examples were presented where survival of interacting growth processes can be expressed in terms of invariant percolation on trees. While the case-by-case study of interacting growth processes often involves a high amount of technical effort, the approach presented in the lecture is more conceptual using "soft proofs" and might be useful in a wider set of problems.

Finally, in the talks given by participants, the topics were mainly percolation and random walks in random environments.

3: Assessment of the Results and Impact

Besides the two main lecturers and the five invited speakers, the winter school "Spatial Models in Statistical Mechanics" was attended by 39 participants, mostly PhD students and young postdocs from Europe. They were given extensive insights into two modern fields of statistical mechanics by two worldwide acknowledged experts, Vincent Beffara and Vladas Sidoravicius. On the one hand, this has extended the professional knowledge and technical toolkit of the participants. On the other hand, the young researchers got a better overview over open problems and possible further directions of research. From the additional lectures, they got an impression of active fields in statistical mechanics. Moreover, the workshop has brought many scientists together, especially young scientists with some leading experts, and has provided a forum for the exchange of ideas, initiation and conduct of joint research projects.

ANNEX 4A: MEETING PROGRAMME

MONDAY, 24. FEBRUARY 2014

Time	Speaker	*Title of Talk*
09:05-10:50	Vincent Beffara	*Mini course on Schramm-Loewner evolution*
10:50-11:15		–Coffee Break–
11:15-12:45	Vladas Sidoravicius	*Mini course on random walk in random environment*
12:45-14:00		–Lunch Break–
14:00-15:00	Daniel Ueltschi	*Random loop representations for quantum spin systems*
15:00-15:20	Simon Aumann	*Singularity of nearcritical percolation scaling limits*
15:20-15:40	Robert Jörg Fitzner	*Nearest-neighbor percolation shows mean-field behavior in* $d > 14$
15:40-16:00	Dirk Erhard	*Brownian paths homogeneously distributed in space: percolation phase transition and uniqueness of the unbounded cluster*
16:00-16:30		–Coffee Break–
16:30-17:30	Short talks	Duhart, Jahnel, Lees, Matetski, Pulvirenti, Tate
17:30-19:00	Cheese & Wine	

TUESDAY, 25. FEBRUARY 2014

Time	Speaker	*Title of Talk*
09:00-10:30	Vincent Beffara	*Mini course on Schramm-Loewner evolution*
10:30-11:00		–Coffee Break–
11:00-12:30	Vladas Sidoravicius	*Mini course on random walk in random environment*
12:30-14:00		–Lunch Break–
14:00-15:00	Markus Heydenreich	*Sharp percolation transition of contact processes*
15:00-15:20	Henning Sulzbach	*The dual tree of a recursive triangulation of the disk: convergence and fractal dimension*
15:20-15:40	Alexander Marynych	*The Bernoulli sieve: allocation scheme in a random environment*
15:40-16:00	Robert Graf	*A forest-fire model on the upper half-plane*
16:00-16:30		–Coffee Break–
16:30-17:30	Short talks	Bethuelsen, Orenshtein, Pham, Taggi, Walter
19:00-22:00	Conference dinner	

WEDNESDAY, 26. FEBRUARY 2014

Time	Speaker	*Title of Talk*
09:00-10:30	Vincent Beffara	*Mini course on Schramm-Loewner evolution*
10:30-10:45		–Coffee Break–
10:45-12:15	Vladas Sidoravicius	*Mini course on random walk in random environment*
12:15-12:30		–Coffee Break–
12:30-13:30	Alexander Fribergh	*Phase transition for the speed of the biased random walk on the supercritical percolation cluster*
13:30-15:00		–Lunch Break–
15:00-18:00		–Free Afternoon–

THURSDAY, 27. FEBRUARY 2014

Time	Speaker	*Title of Talk*
09:00-10:30	Vincent Beffara	*Mini course on Schramm-Loewner evolution*
10:30-11:00		–Coffee Break–
11:00-12:30	Vladas Sidoravicius	*Mini course on random walk in random environment*
12:30-14:00		–Lunch Break–
14:00-15:00	Gabor Pete	*Near-critical percolation versus near-critical FK-Ising*
15:00-15:20	Julien Poisat	*Quenched functional central limit theorem for random walks in random sceneries*
15:20-15:40	Aser Cortines Peixoto Neto	*Front velocity and polymers in random medium*
15:40-16:00	Elisabeth Bauernschubert	*Perturbing random walk in a random environment with cookies of maximal strength*
16:00-16:30		–Coffee Break–
16:30-17:30	Short talks	Döbler, Duraj, Höfelsauer/Weidner, Miller, Mönch

FRIDAY, 28. FEBRUARY 2014

Time	Speaker	*Title of Talk*
09:00-10:30	Vincent Beffara	*Mini course on Schramm-Loewner evolution*
10:30-11:00		–Coffee Break–
11:00-12:30	Vladas Sidoravicius	*Mini course on random walk in random environment*
12:30-14:00		–Lunch Break–
14:00-15:00	Sebastian Müller	*Interacting growth processes and invariant percolation*
15:00-15:30		–Coffee Break, End of Workshop–

ANNEX 4B: SPEAKERS AND PARTICIPANTS[1]

Aumann, Simon LMU München
Aurzada, Frank Technische Universität Darmstadt
Bauernschubert, Elisabeth Universität Tübingen
Beffara, Vincent UMPA - ENS Lyon
Bethuelsen, Stein Andreas Universiteit Leiden
Betz, Volker Technische Universität Darmstadt
Bott, Ann-Kathrin Technische Universität Darmstadt
Conijn, René Petrus VU Amsterdam
Cortines Peixoto Neto, Aser Université Paris Diderot - Paris 7
Döbler, Christian Technische Universität München
Duhart, Horacio G. University of Bath
Duraj, Jetlir LMU München
Erhard, Dirk Universiteit Leiden
Fitzner, Robert Jörg Stockholm University
Fribergh, Alexander Université de Toulouse
Furer, Dmytro Technische Universität Darmstadt
Graf, Robert LMU München
Hertel, Ida Technische Universität Darmstadt
Heydenreich, Markus Universiteit Leiden
Höfelsauer, Thomas Technische Universität München
Jahnel, Benedikt Universität Bochum
Leckey, Kevin Goethe Universität Frankfurt a. M.
Lees, Benjamin University of Warwick
Lübbers, Jan-Erik Technische Universität Darmstadt
Marynych, Alexander T. S. National University of Kiev
Matetski, Kanstantsin University of Warwick
Meiners, Matthias Technische Universität Darmstadt
Miller, Katja Technische Universität München
Mönch, Christian Technische Universität Darmstadt
Müller, Florian Technische Universität Darmstadt
Müller, Sebastian Aix-Marseille Université
Orenshtein, Tal Technische Universität München
Pete, Gábor Technical University of Budapest
Pham, Cong Zan Aix-Marseille Université
Poisat, Julien Universiteit Leiden
Pulvirenti, Elena Universiteit Leiden
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Sidoravicius, Vladas IMPA Rio de Janeiro
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Tate, Stephen James University of Warwick
Ueltschi, Daniel University of Warwick
Walter, Stefan Technische Universität Darmstadt
Weidner, Felizitas Technische Universität München

[1] Names of speakers are underlined.