Scientific Report on the meeting Groups, Graphs and Random Walks Application Reference: 5296

1 Summary

This workshop was organized by Tullio Ceccherini-Silberstein (Università del Sannio di Benevento), Maura Salvatori (Università di Milano Bicocca) and Ecaterina Sava-Huss (Techinsche Universität Graz) and took place in the Palazzone of the Scuola Normale Superiore di Pisa located in Cortona, Italy, from June 2nd to June 6th 2014. In this occasion, we celebrated Wolfgang Woess' 60th anniversary. Wolfgang Woess has been a great teacher, collaborator and colleague for almost all participants. The theme of the talks were, therefore, extremely close to Woess' area of research which is quite wide and interdisciplinary, ranging from the Geometric and Combinatorial Theory of Infinite Groups and of Infinite Graphs to Potential Analysis, from Probability (Markov Chains and Stochastic Processes (in particular Random Walks) and Percolation) to Theoretical Computer Science. The sponsors where the Italian *Istituto Nazionale di Alta Matematica "Francesco Severi"* (INdAM) and RGLIS.

2 Description of the scientific content and discussions

The workshop brought together scholars and researchers from all over the world (USA, Canada, Japan, Astralia, Iran and EUROPE: Austria, France, Switzerland, Denmark, Germany, Greece, U.K., Poland, Hungary, Netherlands, and Italy) working in the fields of Harmonic and Functional Analysis, Probability, Geometric and Combinatorial Group Theory (with emphasis in Asymptotic Methods), Graph Theory, Percolation, and Theoretical Computer Science. Half of the partcipants were senior researchers, mostly colleagues and collaborators of Wolfgang Woess (to whom the workshop was dedicated, honoring is furthcoming 60th birthday) and the other half consisted of young fellows, mostly current PhD students, Postdoctoral fellows and younger former students of Woess, working at the TU Graz under the supervision of Wolfgang Woess. The workshop was very successful since it brought a deep connection between the more mature researchers and the younger ones with a continuous interplay between the respective researches.

There where three special invited lectures: by Massimo Picardello (Università di Roma "Tor Vergata"), by Laurent Saloff-Coste (Cornell University) and by Vadim Kaimanovich (Univeristy of Ottawa). The one delivered by Massimo Picardello, served as an opening to the meeting and focused on the research of Wolfgang Woess describing his very first contacts with the Italian school of Harmonic Analysis, led by Alessandro Figà Talamanca (Università di Roma "La Sapienza"), and his first collaboration with Picardello and Maurizio Soardi (Università di Milano) which eventally led to his permanent position as a full professor of Probability in the Università di Milano (In 1999, Woess moved to TU Graz, where he presently is the Chair of the Institut für Mathematische Strukturtheorie (Math C)).

The remaining 29 talks are as follows (including the abstracts):

(1) Laurent Bartholdi: *Growth and Poisson boundary*

The Poisson boundary of a random walk (given by a one-step measure) describes the tail events of the walk. If the random walk takes place on a group G, then the Poisson boundary is very much connected to classical invariants of G: if the boundary is trivial, then the group generated by the measure's support is amenable. If the boundary is non-trivial for a finitely supported measure, then G has exponential growth. The connection between growth and Poisson boundary, however, is still mysterious. Kaimanovich and Vershik asked in 1983 whether the converse holds;

namely, whether there exist groups of exponential growth such that all measure with finite support have trivial boundary. In joint work with Anna Erschler, I will construct such an example. It is based on "permutational wreath products". I will outline a few other consequences of the construction to geometric group theory.

(2) Alexander Bendikov: *On the spectrum of randomly perturbated hierarchical Laplacian*

Let (X, d) be a locally compact separable ultrametric space. Given a measure m on X and a function $C(B)$ defined on the set of all non-singleton balls B of X we consider the hierarchical Laplacian $L = L_C$. The operator L is closely related to the concept of the hierarchical lattice of F.J. Dyson, *Existence of a phase-transition in a one-dimensional Ising ferromagnet*, Comm. Math. Phys. **12** (1969). L acts in $L^2(X, m)$, is essentially self-adjoint and has a purely point spectrum. Choosing a sequence $\{\varepsilon(B)\}$ of i.i.d. we define the perturbated function $C(B, \omega)$ and the perturbated hierarchical Laplacian $L^\omega = L_{C(\omega)}$. We study the normalized arithmetic means $\overline{\lambda}(\omega)$ of the L^ω -eigenvalues. Under some mild assumptions $\overline{\lambda}(\omega)$ converges to $N(0, 1)$ in law. We also give examples where the normal convergence fails. We prove existence of the integrated density of states. We introduce the empirical process $N(\omega, I)$ of the L^ω-eigenvalues and, assuming that the density of states is continuous, we prove that the finite dimensional distributions of N converge to the Poisson process. As an example we consider random perturbations of the Vladimirov operator acting in $L^2(X, m)$, where $X = \mathbb{Q}_p$ is the ring of p-adic numbers and m is the Haar measure. This is joint work with Stanislav A. Molchanov (NCU at Charlotte).

(3) Daniela Bertacchi: *Branching random walks on the percolation cluster of $\mathbb{Z}^d$*

In this talk we prove that under the assumption of quasi-transitivity, if a branching random walk on $\mathbb{Z}^d$ survives locally (at arbitrarily large times there are individuals alive at the origin), then so does the same process when restricted to the infinite percolation cluster $\mathcal{C}_\infty$ of a supercritical Bernoulli percolation. When no more than k individuals per site are allowed, we obtain the k-type contact process, which can be derived from the branching random walk by killing all particles that are born at a site where already k individuals are present. We prove that local survival of the branching random walk on $\mathbb{Z}^d$ also implies that for k sufficiently large the associated k-type contact process survives on $\mathcal{C}_\infty$. This implies that the strong critical parameters of the branching random walk on $\mathbb{Z}^d$ and on $\mathcal{C}_\infty$ coincide and that their common value is the limit of the sequence of strong critical parameters of the associated k-type contact processes. These results are extended to a family of restrained branching random walks, that is branching random walks where the success of the reproduction trials decreases with the size of the population in the target site.

(4) Sara Brofferio: *On unbounded invariant measures of stochastic dynamical systems*

We consider stochastic dynamical systems $X_n = \Psi_n(X_{n-1})$, where Ψ_n are i.i.d. random continuous transformations of $\mathbf{R}$. We assume that $\Psi_n(x)$ behave asymptotically like $A_n x$, for some random positive number A_n. The main example is the stochastic affine recursion $X_n = A_n X_{n-1} + B_n$. Our aim is to describe invariant Radon measures of the process $\{X_n\}$ in the critical case, when $\mathbf{E} \log A = 0$. Under optimal assumptions, we prove that those measures behave at infinity like dx/x. In the proof we strongly use some properties of random walks on the affine group. The talk will be based on a joint paper with Dariusz Buraczewski.

(5) Elisabetta Candellero: *Ends of critical branching random walks*

In this talk we will talk about "ends" of graphs and "trace" of a branching random walk (BRW). We will discuss examples showing that, on some one-ended Cayley

graphs, the trace of a transient (symmetric) BRW has infinitely many ends almost surely. (Joint work with Matthew Roberts, University of Bath).

(6) Johannes Cuno: *Random walks on Baumslag-Solitar groups*
(joint work with Ecaterina Sava-Huss) Baumslag-Solitar groups are given by $\mathrm{BS}(m,n) \cong \langle\, a, b : ab^m = b^n a \,\rangle$. First, we give a short introduction to these groups and illustrate some of their basic properties. We are particularly interested in $\mathrm{BS}(2,3)$, which is non-amenable. Let us consider a random walk on this group and, for simplicity, let us assume that our random walk is driven by a measure μ whose support consists of the standard generators a and b and their inverses. Our goal is to describe the Poisson boundary associated to the random walk geometrically. Depending on whether $\mu(a)$ and $\mu(a^{-1})$ agree or not, this goal is reached in two different ways.

(7) Daniele D'Angeli: *Topological and isometric properties of the zig-zag product of graphs*
The Zig-zag product of graphs was introduced by Reingold, Vadhan & Wigderson (2002) in order to find a pure combinatorial construction of expander graphs. In this talk I present some results obtained in collaboration with A. Donno and E. Sava-Huss about some topological properties (connectedness, number of ends) and the isomorphism problem for these graphs. I also discuss an interesting application to Schreier graphs of self-similar groups.

(8) Behrang Forghani: *Asymptotic Entropies of Transformed Random Walks*
Given a random walk on a countable group, any Markov stopping time gives rise to a new random walk on the same group. We will show that the asymptotic entropy (rate of escape) of such transformations are equal to the asymptotic entropy (rate of escape) of the original random walk times the expectation of the stopping time. The proof is based on the fact that the Poisson boundaries of these random walks are the same.

(9) Agelos Georgakopoulos: *Group Walk Random Graphs*
Recent work on Dirichlet harmonic functions on a graph lead to introducing the "effective conductance measure", which lives on the square of the Poisson boundary and can be thought of as a continuous analogue of effective conductance. Its construction motivates introducing a rich family of finite random "geometric" graphs. I will give a gentle introduction to this topic. This is partly joint work with V. Kaimanovich.

(10) Wilfried Huss: *Recurrence and transience of rotor-router walks*
(joint work with Ecaterina Sava-Huss) Rotor-router walks are deterministic analogues of random walks on graphs, and theyh have received considerable interest in recent years. In this talk we introduce a notion of recurrence and transience of rotor-router walks. After an overview of some past results, we present a theorem regarding the recurrence of rotor-router walks with random initial rotor configuration on directed covers of finite graphs.

(11) Wilfried Imrich: *Infinite Graphs with Finite 2-Distinguishing Cost*
(joint work with Debra Boutin) This note was written after a discussion with Wolfgang Woess about the size of the point stabilizers in connected, locally finite, infinite graphs earlier this year. It continues work of Boutin on the cost of 2-distinguishing finite graphs. A graph G is said to be 2-distinguishable if there is a labeling of the vertices with two labels such that only the trivial automorphism preserves the labels. Call the minimum size of a label class in such a labeling of G the cost of 2-distinguishing G. It is shown that the connected, locally finite, infinite graphs with finite 2-distinguishing cost are exactly those with countable automorphism group. Furthermore, in such graphs the cost is less than three times the size of a smallest basis. For graphs of linear growth even sharper bounds are possible.

(10) Vadim Kaimanovich: *Invariance, reversibility and the modular cocycle*
Although the now famous formula for the modular function of the group of automorphisms of a transitive graph can be traced back to the works of Schlichting and Trofimov, it is in the 1990 paper by Soardi and Woess that this formula was first explicitly used in the context of stochastic analysis on graphs. In this talk we will explain how this formula can be used in order to define a natural "modular cocycle" on the space of rooted infinite graphs, and show that it is this cocycle that plays the crucial role in the interrelations between the notions of invariance, unimodularity and reversibility for random graphs.
(12) Anders Karlsson: *Spectral invariants of graphs and the Riemann zeta function*
Carleman and other people considered zeta functions formed out of Laplace eigenvalues for domains or compact Riemannian manifolds. In this talk I will discuss spectral zeta functions of graphs and their asymptotics for certain sequences of finite graphs. The Riemann zeta function appears in error terms and a graph zeta conjecture will be seen equivalent to the Riemann hypothesis.
(13) Motoko Kotani: *Discrete Geometric Analysis applied to structural understanding of Materials*
I will present activities at AIMR Tohoku University, where mathematics and materials science collaborate to understand mechanism of materials properties and their local/global structure. Mathematics provides views in terms of discrete geometric analysis, topology, and dynamical systems. There are several emerging results in the direction.
(14) Steve Lalley: *Stochastic models of spatial epidemics and hybrid percolation*
(15) Francois Ledrappier: *Local Limit Theorem in negative curvature*
Following the schemes and the ideas of Gouzel's and Lalley's proof of the LLT for Random Walks, we prove the Local Limit Theorem for the heat kernel on the universal cover of a compact negatively curved manifold. This is joint work with Seonhee Lim.
(16) Florian Lehner: *Breaking graph symmetries by random colourings*
A colouring of a graph G is called distinguishing if it is not preserved by any non trivial automorphism of G. Equivalently, a colouring is distinguishing if its stabiliser in the automorphism group is trivial, i.e., it only consists of the identity. Tucker conjectured that, if each automorphism of a locally finite graph moves innitely many vertices, then there is a distinguishing 2-colouring of G. We investigate random 2-colourings of locally finite graphs and their stabilisers.
(17) Franz Lehner: *Graph products and combinatorics of generating functions*
The Green's functions of various products of graphs can be computed by manipulations of generating functions. In joint work with O. Arizmendi, T. Hasebe and C. Vargas we exhibit combinatorial interpretations and connections behind these manipulations.
(18) Daniel Lenz: *Schreier graphs and quasicrystals*
We study the spectral theory of Lapacians associated to the Schreier graphs arising from the action of Grigorchuks group on the boundary of the tree. These graphs can be encoded by a substitution subshift over a finite alphabet. This makes it possible to apply methods developed in the study of quasicrystals. We obtain Cantor spectrum of Lebesgue measure zero as well as absence of point spectrum. (Joint work with R. Grigorchuk and T. Nagnibeda)
(19) Sebastian Müller: *Random walks - from periodic trees to hyperbolic groups*
We comment on how some results of Tatiana Smirnova-Nagnibeda and Wolfgang Woess for random walks on periodic trees (or trees with finitely many cone types) can be carried over to random walks on Gromov hyperbolic groups.

(20) Tatiana Nagnibeda: *Lamplighters, spider-web and de Brujin graphs*
We'll discuss structural and spectral properties of three well-known families of graphs arising independently in group theory, in statistical physics and in genetics. It turns out that all three families are closely related. The talk is based on a joint work with R. Grigorchuk and P.-H. Leemann.

(21) James Parkinson: *Random walks on buildings*
This talk gives an overview of results concerning random walks on buildings and the associated Lie groups and Kac-Moody groups. Our emphasis is on the affine and hyperbolic cases. We discuss the similarities and differences in these cases, and outline the main techniques used to analyse random walks for each case. This touches on collaborations between the speaker and L. Gilch, S. Müller, B. Schapira, and W. Woess.

(22) Marc Peigné: *Margulis function and rigidity for non uniform lattices in negative curvature*
(joint work with F. Dal'bo, J.C. Picaud and A. Sambusetti) In 1969, G. Margulis proved that the volume of the ball of center x and radius R on the universal covering $\tilde{X}$ of a negatively curved and compact manifold $X = \tilde{X}/\Gamma$ is equivalent to $m(x)e^{hR}$ as $R \to +\infty$, where h is the volume entropy of $\tilde{X}$ and $m(x)$ is the so-called *Margulis function* at point x. We relax here the hypothesis on Γ, just assuming that the volume of $X = \tilde{X}/\Gamma$ is finite, and we prove that Margulis result remains valid when the curvature is 1/4-pinched. The first main ingredient is a important work by Th. Roblin about the asymptotic behavior of the orbital function of Γ ; the second one is a rigidity theorem "à la Besson-Courtois-Gallot" whose main consequence is that, in the case of 1/4-pinched curvature, the Poincaré exponent of Γ is strictly greater than the one of its parabolic subgroups.

(23) Laurent Saloff-Coste: *Random walks driven by measures supported on powers of generators*
(Joint work with Tianyi Zheng) Given a group G, a generating tuple $(s_1, .., s_k)$, and for each index i, a measure m_i on the integers, consider the walk which moves as follows. Pick i in $1, ..., k$ uniformly at random. Pick an integer n according to the measure m_i and move from the current position x to $y = xs_i^n$. In other words, this random walk takes random steps along the (finite or) countable one parameter groups associated with the generators $s_1, ..., s_k$. For a marked finitely generated group, this describes a natural collection of "spread-out" random walks allowing for long jumps. I will describe detailled results in the context of nilpotent groups.

(24) Norbert Seifter: *Big subsets with small boundaries in a graph with a vertex-transitive group of automorphisms*
(joint work with Vladimir Trofimov) The theory of ends of groups G and graphs Γ with vertex-transitive groups of automorphisms can be regarded as a theory of the Boolean algebras of subsets of G or the vertex set of Γ with finite boundaries, considered modulo finite subsets. We develop a more general approach towards "ends", replacing infinite subsets with finite boundaries by certain "big" subsets with "small" boundaries.

(25) Toshikazu Sunada: *Random walks, Diophantine problems, and rational points in complex quadrics*
A certain Diophantine problem and 2D crystallography are linked through the notion of standard realizations which was introduced originally in the study of random walks. In the discussion, a complex projective quadric is associated with a finite graph. "Rational points" on this quadric turns out to be related to standard realizations of 2D crystal structures.

(26) András Telcs: *The first step of the Brownian Sheep*
In our talk we discuss isoperimetric inequalities for weighted graphs. In particular

we present characterization of extremal sets for the capacity and expected lifetime of the random walks.

(27) Christoph Temmel: *Clique decompositions of infinite chordal graphs*
(joint with Florian Lehner) We investigate clique trees of infinite, locally finite chordal graphs. Our key tool is a bijection between the set of clique trees and the product of local families of finite trees. This enables us to enumerate all clique trees of a chordal graph. It also induces a local projection onto clique trees of finite chordal graphs, allowing us to lift various classic properties of clique trees of finite graphs to infinite clique trees. The resulting decompositions are of interest in statistical mechanics and for automorphism-invariant colorings of quasi-transitive infinite chordal graphs.

(28) Carsten Thomassen: *Orientations and decompositions of graphs*
Latin squares, Steiner triple systems and block designs are structures that can be expressed as graph decompositions. A result of Dehn on rigidity of convex polyhedra motivated an early result on claw decompositions of graphs. In this lecture we focus on the interplay between graph decomposition and graph flow, for example Tutte's flow conjectures. Special emphasis will be on the recent solution of the so-called weak 3-flow conjecture formulated by Jaeger in 1988.

(29) Fabio Zucca: *Fixed points of generating functions and strong local survival of Branching Random Walks*
Survival probabilities of branching random walks can be seen as fixed points of a (possibly infinite-dimensional) generating function. In the irreducible, finite case there are at most two fixed points, but this is not true if the state space is infinite, even if the process is irreducible. This leads to many interesting relations between global survival and local survival probabilities. In particular, for a generic continuous-time branching random walk, the so-called "strong local survival" is not a monotone property with respect to the reproduction rates. This is a joint work with D.Bertacchi.

3 Assessment of the results and impact of the event on the future directions of the field

The workshop brought together mature and young researchers from all over the world and in particular from EUROPE (Austria, France, Switzerland, Denmark, Germany, Greece, U.K., Poland, Hungary, Netherlands, and Italy). Although it was originally intended to celebrate Wolfgang Woess' 60th anniversary, the final task, fully accomplished, was to provide a surce of education and encouragement for young researchers at the earliest stage of their carreer. The lectures given by leading experts where coupled with shorter talks by younger researchers. All these witnessed the richness and the mathematical beauty of the different fields involved in this workshop: Harmonic and Functional Analysis, Probability, Geometric and Combinatorial Group Theory (with emphasis in Asymptotic Methods), Graph Theory, Percolation, and Theoretical Computer Science. As an additional confirmation of the success of the meeting, I can announce that a selection of contributions will be collected as conference proceeding most likely in the London Mathematical Society Lecture Note Series, published by the Cambridge University Press.

4A Programme of the meeting

Monday, June 2, 2014
8:30 - 10:00 Registration
10:00 -10:30 Massimo Picardello (Università Roma Tor Vergata)
10:30 - 11:15 Vadim Kaimanovich (Ottawa)
11:30 - 12:00 Sebastian Müller (Marseille)
12:00 - 12:45 James Parkinson (Sydney)
13:00 - 15:00 Lunch
15:00 - 15:30 Florian Lehner (Graz)
15:30 - 16:15 Carsten Thomassen (Copenhagen)
16:20- 17:00 Coffee Break
17:00 - 17:30 Wilfried Imrich (Leoben)
17:30 - 18:00 Christoph Temmel (Amsterdam)

Tuesday, June 3, 2014
9:00 - 9:45 Laurent Saloff-Coste (Cornell)
10:00 - 10:30 Wilfried Huss (Graz)
10:30 -11:00 Coffee Break 11:00 - 11:45 Toshikazu Sunada (Tokyo)
12:00 - 12:45 Motoko Kotani (Tohoku)
13:00 - 15:00 Lunch 15:00 - 15:30 Fabio Zucca (Politecnico Milano)
15:30 - 16:15 Steve Lalley (Chicago)
16:20- 17:00 Coffee Break 17:00 - 17:30 Elisabetta Candellero (Warwick)
17:30 - 18:00 Daniela Bertacchi (Milano Bicocca)

Wednesday, June 4, 2014
9:00 - 9:45 Tatiana Nagnibeda (Geneva)
10:00 - 10:30 Daniele D'Angeli (Graz)
10:30 -11:00 Coffee Break
11:00 - 11:45 Daniel Lenz (Jena)
12:00 - 12:45 Anders Karlsson (Geneva)
13:00 - 15:00 Lunch

Thursday, June 5, 2014
9:00 - 9:45 Agelos Georgakopoulos (Warwick)
10:00 - 10:30 Andras Telcs (Budapest)
10:30 -11:00 Coffee Break
11:00 - 11:45 Franz Lehner (Graz)
12:00 - 12:45 Sara Brofferio (Paris)
13:00 - 15:00 Lunch
15:00 - 15:30 Behrang Forghani (Ottawa)
15:30 - 16:15 Laurent Bartholdi (Gttingen)
16:20- 17:00 Coffee Break
17:00 - 17:30 Johannes Cuno (Graz)
19:30 - 00:00 Conference Dinner

Friday, June 6, 2014
9:00 - 9:45 Marc Peigné (Tours)
10:00 - 10:30 Norbert Seifter (Leoben)
10:30 -11:00 Coffee Break
11:00 - 11:45 Francois Ledrappier (Paris)
12:00 - 12:45 Alexander Bendikov (Wroclaw).

1. 4B Full list of speakers (*) and participants

(1*) Laurent Bartholdi (Georg-August-Universität zu Göttingen)
(2*) Alexander Bendikov (University of Wroclaw)
(3*) Daniela Bertacchi (Milan)
(4) Tetiana Boiko (TU Graz)
(5*) Sara Brofferio (Université Paris Sud)
(6*) Elisabetta Candellero (University of Warwick)
(7) Tullio Ceccherini-Silberstein (Sannio, Italy)
(8) Jan Czajkowski (TU Graz)
(9) Wojciech Cygan (Wroclaw University, Poland)
(10*) Johannes Cuno (TU Graz)
(11*) Daniele D'Angeli (TU Graz)
(12) Wiktor Ejsmont (TU Graz)
(13) Alessandro Figà-Talamanca (Sapienza Università di Roma)
(14*) Behrang Forghani (Ottawa)
(15*) Agelos Georgakopoulos (University of Warwick)
(16) Lorenz Gilch (TU Graz)
(17*) Wilfried Huss (TU Graz)
(18*) Wilfried Imrich (Uni Leoben, Austria)
(19*) Vadim Kaimanovich (Ottawa)
(20*) Anders Karlsson (Université de Genève)
(21) Judith Kloas (TU Graz)
(22*) Motoko Kotani (Tohoku University, Sendai)
(23) Gabriella Kuhn (Milano)
(24*) Steve Lalley (University of Chicago)
(25*) Francois Ledrappier (University of Notre Dame)
(26*) Florian Lehner (TU Graz)
(27*) Franz Lehner (TU Graz, Austria)
(28*) Daniel Lenz (Friedrich-Schiller Universität Jena, Germany)
(29) Leonede de Michele (Milano)
(30*) Sebastian Müller (Université d'Aix-Marseille I)
(31*) James Parkinson (University of Sydney)
(32*) Marc Peigné (Universit François Rabelais, Tours, France)
(33*) Massimo Picardello (Rome)
(34) Maura Salvatori (Milan)
(35*) Laurent Saloff-Coste (Cornell)
(36) Ecaterina Sava-Huss (TU Graz)
(37) Konrad Schrempf (TU Graz)
(38*) Norbert Seifter (Montanuniversität Leoben)
(39*) Tatiana Smirnova-Nagnibeda (Université de Genève)
(40*) Toshikazu Sunada (Meiji University, Tokyo)
(41*) Andras Telcs (Budapest)
(42*) Christoph Temmel (VU Amsterdam)
(43*) Carsten Thomassen (Technical University of Denmark, Copenhagen)
(44) Elmar Teufl (Eberhard-Karls-Universität Tübingen)
(45) Filippo Tolli (Università Roma Tre)
(46) Aljosa Volcic (Calabria)
(47) Wolfgang Woess (TU Graz)
(48*) Fabio Zucca (Politecnico di Milano).