

FINAL REPORT ON THE WINTER SCHOOL “STOCHASTIC ANALYSIS OF SPATIALLY EXTENDED MODELS”

APPLICATION REFERENCE NO: 5715

1: SUMMARY

The aim of the winter school 2015 “Stochastic Analysis of Spatially Extended Models” held at Technische Universität Darmstadt from March 23, 2015 to March 27, 2015 was to give young researchers in Germany and Europe the opportunity to catch up with recent progress in two very important and active branches of modern probability theory, namely, branching Brownian motion and the theory of regularity structures.

The core of the school consisted of two mini courses each comprising five lectures of 90 minutes. The topics of the first course were branching Brownian motion and branching random walks. It was delivered by Zhan Shi (Université Paris VI). The second one dealt with regularity structures and stochastic partial differential equations and was given by Hendrik Weber (University of Warwick).

In addition to the mini courses, there were four accompanying lectures held by invited speakers. It was intended that some of the lectures go deeper into the topics of the mini courses while others broaden the picture by focussing on related models. The invited speakers were Massimiliano Gubinelli (Université Paris Dauphine), Simon C. Harris (University of Bath), Nicola Kistler (Goethe-Universität Frankfurt), and Nicolas Perkowski (HU Berlin) Their lectures covered singular stochastic partial differential equations and paracontrolled distributions, branching Brownian motion with absorbing barriers and the F-KPP equation, a multiscale refinement of the second moment method and its application to branching Brownian motion, the two-dimensional Gaussian free field, and the cover time of a planar Brownian motion on a torus, and the two-dimensional parabolic Anderson model.

Moreover, participants could deliver talks of a duration of 25 minutes to present their own research. In total, there were three afternoon sessions each comprising two such talks. These talks were given by PhD students, young postdocs and a guest professor. Two out of the three sessions were very close to the main topics of the winter school focussing on different aspects of stochastic partial differential equations.

Finally, each participant who did not give a regular talk of 25 minutes was asked to give a short presentation of 10 minutes to introduce the other participants to her or his field of research. Five participants took this opportunity and there were was one short talk session with comprising their five short presentations.

2: DESCRIPTION OF THE SCIENTIFIC CONTENT

The main contents of the winter school “Stochastic Analysis of Spatially Extended Models” were branching Brownian motion and the branching random walk, and regularity structures and stochastic partial differential equations.

Zhan Shi gave an elementary introduction to branching Brownian motion and branching random walks. The simple but useful tool of the spinal decomposition was explained in detail. He further presented an overview of classical as well as the most recent results on the asymptotics of extreme values of these processes. Connections to and recent developments in a few related models, such as branching processes in the presence of a selection criterion, or biased random walks on trees have been presented.

In his mini course, Hendrik Weber reported on recent progress in the theory of stochastic partial differential equations. In particular, he explained some ideas behind the theory of “Regularity structure” developed recently by M. Hairer. This theory gives a way to study well-posedness for a class of stochastic partial differential equations that could not be treated previously. Prominent examples include the KPZ equation as well as the dynamic Φ_3^4 model. Such equations were treated previously as perturbative expansions. Roughly speaking the theory of regularity structures provides a way to truncate this expansion after finitely many terms and to solve a fixed-point problem for the “remainder”. The key ingredient, a new notion of “regularity” which is based on the terms of this expansion, was presented. Hendrik Weber also discussed how these ideas can be used to study scaling limits of interacting particle systems.

Massimiliano Gubinelli’s lecture entitled “Singular Stochastic PDEs and paracontrolled distributions” dealt with non-linear evolution problems perturbed by singular noise sources which arise naturally as scaling limits of certain microscopic evolutions or homogenisation problems. The parabolic Anderson model, the Kardar-Parisi-Zhang equation and the stochastic quantisation equation are examples of such systems. Solving (or even giving a meaning to) these equations require a detailed understanding of the propagation of the stochastic perturbations via the non-linear evolution. Gubinelli explained how ideas and tools from harmonic analysis can be useful in this analysis and in the related problem of studying the convergence of small scale models to their scaling limits.

Simon C. Harris presented recent results on the survival near criticality in a branching Brownian motion model with killing. In particular, he discussed the asymptotic survival probability of BBM with killing at the boundaries of a strip as the strip’s width decreases down to criticality, making use of a probabilistic decomposition of the branching process into a ‘blue tree’ of immortal particles which is dressed with red trees that each eventually become extinct.

Nicola Kistler’s lecture discussed the second moment method in the context of extremes of large combinatorial structures. He presented a refinement of the method which is particularly efficient in order to tackle the extremes of correlated random fields where multiple scales can be identified. The refinement relies on elementary steps only, and applies equally well to a number of models, such as branching Brownian motion, the 2-dim Gaussian free field, the cover time by planar Brownian motion, etc.

Nicolas Perkowski presented recent results in his talk showing that the two-dimensional lattice parabolic Anderson model (PAM) with small potential converges under parabolic rescaling weakly to the continuum PAM, universally for all centered i.i.d. potentials with sufficiently many

moments. He also showed milestones of the proof based on paracontrolled distributions, which also extends to certain nonlinear generalizations of the PAM.

Finally, in the talks given by participants, the topics were mainly stochastic partial differential equations.

3: ASSESSMENT OF THE RESULTS AND IMPACT

Besides the two main lecturers and the four invited speakers, the winter school “Stochastic Analysis of Spatially Extended Models” was attended by 30 participants, mostly PhD students and young postdocs from Europe. They were given extensive insights into two modern fields of probability by two experts, Zhan Shi and Hendrik Weber. On the one hand, this has extended the professional knowledge and technical toolkit of the participants. On the other hand, the young researchers got a better overview over open problems and possible further directions of research. From the additional lectures, they got an impression of active fields in probability theory. Moreover, the workshop has brought many scientists together, especially young scientists with some leading experts, and has provided a forum for the exchange of ideas, initiation and conduct of joint research projects.

ANNEX 4A: MEETING PROGRAMME

MONDAY, 23. MARCH 2015

| Time | Speaker | Title of Talk |
|-------------|-------------------------|--|
| 09:00-10:30 | Zhan Shi | <i>Mini course: Branching Brownian motion</i> |
| 10:30-11:00 | | –Coffee Break– |
| 11:00-12:30 | Hendrik Weber | <i>Mini course: Regularity structures and SPDEs</i> |
| 12:30-14:00 | | –Lunch Break– |
| 14:00-15:00 | Massimiliano Gubinelli | <i>Singular Stochastic PDEs and paracontrolled distributions</i> |
| 15:00-15:25 | André de Oliveira Gomes | <i>First exit times for jump diffusions</i> |
| 15:30-15:55 | Stephen J. Tate | <i>Virial expansion bounds from tree partition schemes with nonlocal drift</i> |
| 16:00-16:30 | | –Coffee Break– |
| 16:30-17:30 | Short talks | Cannizaro, Hammami, Sivak, von der Lüche, Wresch |
| 17:30-19:00 | Cheese & Wine | |

TUESDAY, 24. MARCH 2015

| Time | Speaker | Title of Talk |
|-------------|-----------------|--|
| 09:00-10:30 | Zhan Shi | <i>Mini course: Branching Brownian motion</i> |
| 10:30-11:00 | | –Coffee Break– |
| 11:00-12:30 | Hendrik Weber | <i>Mini course: Regularity structures and SPDEs</i> |
| 12:30-14:00 | | –Lunch Break– |
| 14:00-15:00 | Simon C. Harris | <i>Branching Brownian motion with killing</i> |
| 15:00-15:25 | Jonas M. Tölle | <i>Stability and rescaling of singular nonlinear SPDEs</i> |
| 15:30-15:55 | Martin Redmann | <i>SPA applied to SPDEs</i> |
| 16:00-16:30 | | –Coffee Break– |

WEDNESDAY, 25. MARCH 2015

| Time | Speaker | Title of Talk |
|-------------|-------------------|---|
| 09:00-10:30 | Zhan Shi | <i>Mini course: Branching Brownian motion</i> |
| 10:30-10:45 | | –Coffee Break– |
| 10:45-12:15 | Hendrik Weber | <i>Mini course: Regularity structures and SPDEs</i> |
| 12:15-12:30 | | –Coffee Break– |
| 12:30-13:30 | Nicolas Perkowski | <i>An invariance principle for the 2d parabolic Anderson model with small potential</i> |
| 13:30-15:00 | | –Lunch Break– |
| 15:00-18:00 | | –Free Afternoon– |

THURSDAY, 26. MARCH 2015

| Time | Speaker | <i>Title of Talk</i> |
|--------------------|-------------------|---|
| 09:00-10:30 | Zhan Shi | <i>Mini course: Branching Brownian motion</i> |
| 10:30-11:00 | | –Coffee Break– |
| 11:00-12:30 | Hendrik Weber | <i>Mini course: Regularity structures and SPDEs</i> |
| 12:30-14:00 | | –Lunch Break– |
| 14:00-15:00 | Nicola Kistler | <i>A multiscale refinement of the 2nd moment method</i> |
| 15:00-15:25 | Masato Hoshino | <i>KPZ equation with fractional derivatives of white noise</i> |
| 15:30-15:55 | Dai Noboriguchi | <i>A kinetic formulation for stochastic scalar conservation laws with boundary conditions</i> |
| 16:00-16:30 | | –Coffee Break– |
| 19:00-22:00 | Conference dinner | |

FRIDAY, 27. MARCH 2015

| Time | Speaker | <i>Title of Talk</i> |
|--------------------|----------------|---|
| 09:00-10:30 | Zhan Shi | <i>Mini course: Branching Brownian motion</i> |
| 10:30-11:00 | | –Coffee Break– |
| 11:00-12:30 | Hendrik Weber | <i>Mini course: Regularity structures and SPDEs</i> |
| 12:30-14:00 | | –Lunch Break, End of Workshop– |

ANNEX 4B: SPEAKERS AND PARTICIPANTS¹

Frank Aurzada TU Darmstadt
Volker Betz TU Darmstadt
Ann-Kathrin Bott TU Darmstadt
Matthias Birkner Universität Mainz
Giuseppe Cannizzaro TU Berlin
Alexander Dalinger TU Darmstadt
Andrej Depperschmidt Universität Freiburg
Massimiliano Gubinelli Paris Dauphine
Mohamed Ali Hammami University of Sfax
Simon C. Harris University of Bath
Masato Hoshino Tokyo University
Aref Jeribi University of Sfax
Nicola Kistler Goethe-Universität Frankfurt
Alexander Kreiß TU Darmstadt
Lisa Kristl TU Darmstadt
Jan-Erik Lübbers TU Darmstadt
Katharina von der Lühe Universität Bielefeld
Martin Maiwald Universität Münster

Matthias Meiners TU Darmstadt
Christian Mönch TU Darmstadt
Florian Müller TU Darmstadt
Dai Noboriguchi Waseda University
André de Oliveira Gomes HU Berlin
Nicolas Perkowski HU Berlin
Martin Redmann MPI Magdeburg
Martin Saal TU Darmstadt
Helge Schäfer TU Darmstadt
Zhan Shi Université Paris VI
Iryna Sivak École polytechnique
Stephen James Tate Imperial College London
Reinhard Tent TU Darmstadt
Jonas Tölle Universität Bielefeld
Pavlos Tsatsoulis University of Warwick
Stefan Walter TU Darmstadt
Hendrik Weber University of Warwick
Lukas Wresch Universität Bielefeld

¹Names of speakers are underlined.