Research Networking Programmes

Science Meeting – Scientific Report

The scientific report (WORD or PDF file - maximum of seven A4 pages) should be submitted online within two months of the event. It will be published on the ESF website.

***Proposal Title**: Inhomogeneous Random Systems 2015*

Application Reference N°: 5769

1) Summary

The conference was organized by François Dunlop (LPTM, Université de Cergy-Pontoise), Thierry Gobron (LPTM, CNRS - Université de Cergy-Pontoise) and Ellen Saada (MAP5, CNRS - Université Paris Descartes)

It took place on Tuesday, January 27 and Wednesday, January 28, 2015 at the Institut Henri Poincaré, Paris, and brought together about 70 participants, mathematicians and physicists, working on disordered or random systems.
The scientific aspects of the conference are provided below. Additional information is available on the conference website:

http://irs.math.cnrs.fr

Ellen Saada, Thierry Gobron, Francois Dunlop.

2) Description of the scientific content of and discussions at the event .

The aim of this annual workshop is to bring together mathematicians and physicists working on disordered or random systems, and to discuss recent developments on themes of common interest. Each of the two days is devoted to a specific topic and organized together with a moderator, specialist of the

field. This year, the themes were “Random interface models” with Fabio Toninelli (Lyon) as moderator, and “Random tilings and surfaces” with Jérémie Bouttier (Saclay).

Random interface models:

Random surface models are ubiquitous in statistical physics, arising for instance as models for interfaces between thermodynamic phases or as height functions associated to discrete models (dimer models, etc). Among the topics that will be discussed are: equilibrium height fluctuations and stochastic interface dynamics, roughening vs. rigidity phenomena.

Random tilings and surfaces.

Counting domino or rhombus tilings of a planar domain is a classical combinatorial problem which received historical contributions from MacMahon, Kasteleyn, Temperley and Fisher, etc. It is intimately connected with statistical physics (it models the adsorption of diatomic molecules on a surface) and probability theory (how does a large uniform random tiling look like ?). Due to the constraint that tiles may not overlap, tilings are highly sensitive to boundary conditions, which are responsible for the remarkable "limit shape" phenomenon, that one can study thanks to a correspondence between tilings and discretized random surfaces. Tilings are also related to random matrices and Schur processes, owing to their "free-fermionic" nature. We will present some recent developments of the research on these models and on their natural deformations (non free-fermionic/non integrable models, aperiodic tilings...).

3) Assessment of the results and impact of the event on the future directions of the field.

This meeting brings together mathematicians and physicists, working on the same fields but with their own approach. The two themes, random interface models and random tilings and surfaces, are closely related but still very different fields and are still very active fields of research

phenomena and time delays in stochastic systems can be encountered in a variety of fields, in physical and chemical systems as well as in biology, and at various level of organization. An overview of these various aspects by specialists of the field was clearly appreciated by the participants.

4) Programme of the meeting and full list of speakers and participants

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Tuesday January 27: RANDOM INTERFACE MODELS.

Moderator: **Fabio Toninelli** (Lyon).

9.15- 9.40 Opening

9.40-10.30 **Fabio Toninelli** (Lyon): Introduction.

10.23-11.20 **Béatrice de Tilière** (Paris): Height representation of XOR-Ising loops via bipartite dimers.

11.20-11.40 Coffee Break

11.40-12.30 **Ron Peled** (Tel Aviv): Delocalization of two-dimensional random surfaces with hard-core constraints.

12.30-13.40 Lunch

13.40-14.30 **Fabio Martinelli** (Roma): Harmonic pinnacles in the discrete Gaussian model.

14.30-15.20 **Oren Louidor** (Haifa): The full extremal process of the discrete Gaussian free field in 2D.

15.20-15.40 Coffee Break

15.40-16.30 **Benoit Laslier** (Cambridge): The Glauber dynamics on lozenge tilings and other dimer models.

16.30-17.20 **Thierry Bodineau** (Palaiseau): Interface motion in disordered media.

Wednesday 28 January: RANDOM TILINGS AND SURFACES.

Moderator: Jérémie Bouttier (Saclay)

9.40-10.30 **Jérémie Bouttier** (Saclay): Introduction.

10.30-11.20 **Sylvie Corteel** (Paris): Dimers on Rail Yard Graphs.

11.20-11.40 Coffee Break

11.40-12.30 **Alessandro Giuliani** (Roma): Height fluctuations in interacting dimers.

12.30-14.00 Lunch

14.00-14.50 **Filippo Colomo** (Firenze): Arctic curves of the six-vertex model.

14.50-15.40 **Thomas Fernique** (Villetaneuse): From random to quasiperiodic tilings.

15.40-16.00 Coffee Break

16.00-16.50 **Leonid Petrov** (Charlottesville): Dynamics of random surfaces and interacting particle systems via spectral properties.

16.50-17.40 **Patrik Ferrari** (Bonn): From a 2+1 dimensional particle system to random tilings and random matrices.

Abstracts:

Fabio Toninelli (Lyon): *Introduction.*

Random surface models (discrete or continuous) are ubiquitous in statistical physics, arising for instance as models for interfaces between thermodynamic phases or as height functions associated to discrete models (dimer models, etc). In this introduction, that should serve as a motivation for the rest of the day, I will give a non-systematic overview of some open challenges in this field, both about equilibrium fluctuations and about dynamical evolution of random interfaces. In some of these directions there were important advances and exciting activity recently, as witnessed by the following talks

Thierry Bodineau (Palaiseau): *Interface motion in disordered media.*

We will first review the return to equilibrium of the Ising model when a small external field is applied. The relaxation time is extremely long and can be estimated as the time needed to create critical droplets of the stable phase which will invade the whole system. We will then discuss the impact of disorder on this metastable behavior and show that for Ising model with random interactions (dilution of the couplings) the relaxation time is much faster as the disorder acts as a catalyst. In the last part of the talk, we will focus on the droplet growth and study a toy model describing interface motion in disordered media.

Béatrice de Tilière (Paris): *Height representation of XOR-Ising loops via bipartite dimers.*

The XOR-Ising model is constructed from two independent Ising models. We show in an explicit way that loops separating clusters of spins of XOR-Ising configurations have the same law as loops in a bipartite dimer model. As a consequence, XOR-loops have the

same law as level lines of a height function which, at the critical point, converges weakly in distribution to a Gaussian free field. This is joint work with Cédric Boutillier.

Benoit Laslier (Cambridge): *The Glauber dynamics on lozenge tilings and other dimer models.*

We study the so called Glauber dynamics on dimer model where the only allowed moves are rotations of all the dimers among a single face. We will show in a relatively general setup that the time needed for the system to reach equilibrium is of order L^(2+o(1)), where L is the typical length scale of the system. The exponent 2 is optimal. More precisely, for surfaces attached to a curve drawn in some plane, we will control the mixing time for several models, including lozenge tilings and domino tilings. For surfaces attached to a general curve, we will only work with lozenges and use a weaker notion of "macroscopic" convergence.

Oren Louidor (Haifa): *The full extremal process of the discrete Gaussian free field in 2D.*

We show the existence of the limit of the full extremal process of the discrete Gaussian free field in 2D with zero boundary conditions. The limit is a clustered Poisson point process with a random intensity measure, which is conjecturally related to the critical Liouiville quantum gravity measure w.r.t. the continuous Gaussian free field. Several corollaries follow directly, e.g. a natural construction for the super-critical Gaussian multiplicative chaos and Poisson-Dirichlet statistics for the limiting Gibbs measure - both w.r.t. the CGFF. The proof is based on a novel concentric decomposition of the DGFF which effectively reduces the problem to that of finding asymptotics for the probability of a decorated non-homogenous random-walk required to stay positive. Entropic repulsion plays a key in the analysis. Joint work with M. Biskup (UCLA).

Fabio Martinelli (Roma): *Harmonic pinnacles in the discrete Gaussian model.*

The 2D Discrete Gaussian model gives each height function eta: Z^2 to Z a probability proportional to exp[-beta*H(eta)], where beta is the inverse temperature and H(eta)=sum(eta_x-eta_y)^2 sums over nearest-neighbor bonds. We consider the model at large fixed beta, where it is flat unlike its continuous analog (the Gaussian Free Field). We first establish that the maximum height in an LxL box with 0 boundary conditions concentrates on two integers M,M+1 with M ~ [(2/pi*beta)logL(loglogL)]^{1/2}. The key is a large deviation estimate for the height at the origin in Z^2, dominated by "harmonic pinnacles", integer approximations of a harmonic variational problem. Second, in this model conditioned on eta>=0 (a floor), the average height rises, and in fact the height of almost all sites concentrates on levels H,H+1 where H ~ M/sqrt(2). This in particular pins down the asymptotics, and corrects the order, in results of Bricmont, El-Mellouki and Fröhlich (1986). Finally, our methods extend to other classical surface models (e.g., restricted SOS), featuring connections to p-harmonic analysis and alternating sign matrices. Joint work with Eyal Lubetzky and Allan Sly.

Ron Peled (Tel Aviv): *Delocalization of two-dimensional random surfaces with hard-core constraints.*

We study the fluctuations of random surfaces on a two-dimensional discrete torus. The random surfaces we consider are defined via a nearest-neighbor pair potential which we require to be twice continuously differentiable on a (possibly infinite) interval and infinity outside of this interval. No convexity assumption is made and we include the case of the so-called hammock potential, when the random surface is uniformly chosen from the set of all surfaces satisfying a Lipschitz constraint. Our main result is that these surfaces delocalize, having fluctuations whose variance is at least of order log n, where n is the side length of the torus. We also show that the expected maximum of such surfaces is of order at least log n. The main tool in our analysis is an adaptation to the lattice setting of an algorithm of Richthammer, who developed a variant of a Mermin-Wagner type argument applicable to hard-core constraints. We rely also on the reflection positivity of the random surface model. The result answers a question mentioned by Brascamp, Lieb and Lebowitz on the hammock potential and a question of Velenik. Joint work with Piotr Miłoś.

Jérémie Bouttier (Saclay): *Introduction.*
Counting domino or rhombus tilings of a planar domain is a classical combinatorial problem which received historical contributions from MacMahon, Kasteleyn, Temperley and Fisher, etc. It is intimately connected with statistical physics (it models the adsorption of diatomic molecules on a surface) and probability theory (how does a large uniform random tiling look like ?). In this introductive talk, I will review some basic facts about tilings and related objects, paving the way to the more advanced talks of the day.

Filippo Colomo (Firenze): *Arctic curves of the six-vertex model.*

The six-vertex model with domain wall boundary conditions can be regarded as an `interacting' generalization of the famous `free-fermion' problem of domino tilings of the Aztec Diamond. In a suitable scaling limit, it is known to exhibit spatial phase separations, with the emergence of various regions of order and disorder, sharply separated by a smooth curve, called Arctic curve. Here we review the state of the art on the model, in particular: the determination of the explicit analytic form of the Arctic curve for arbitrary weights; the generalization of the treatment to domains of generic shape; the third-order phase transition induced by suitable modification of the aspect ratio of the domain.

Sylvie Corteel (Paris): *Dimers on Rail Yard Graphs.*

We introduce a general model of dimer coverings of certain plane bipartite graphs, which we call Rail Yard Graphs (RYG). Using a transfer matrix approach and the celebrated boson-fermion correspondence, the model can be reformulated as a Schur process (i.e. a random sequence of integer partitions). We obtain explicit expressions for the partition

function and for the inverse Kasteleyn matrix, which yields all dimer correlation functions. Plane partitions, domino tilings of the Aztec diamond and pyramid partitions arise as particular cases of our model. This is joint work with Cédric Boutillier (Paris 6), Jérémie Bouttier (CEA), Guillaume Chapuy (CNRS) and Sanjay Ramassamy (Brown U.).

Thomas Fernique (Villetaneuse): *From random to quasiperiodic tilings.*

We shall discuss questions that arise when modelling so-called quasicrystals by tilings, in particular rhombus tilings of the plane. Those can indeed be easily seen as surfaces in a higher dimensional space. Tilings modelling quasicrystals obtained by quenching then correspond to maximal entropy random surfaces, while the more recent and nicer annealed quasicrystals correspond to irrational planes. This rises various theoretical questions (ranging from Markov chain mixing through calculability and combinatorics), most of which are open.

Patrik Ferrari (Bonn): *From a 2+1 dimensional particle system to random tilings and random matrices.*

We will describe a 2+1 dimensional system of interacting particle system system that includes simultaneously a 1+1 dimensional particle system (the totally asymmetric simple exclusion process) and a random tiling model. In a special case in a discrete time setting, the latter is equivalent to the Aztec diamond. Further, under diffusion scaling limit one recovers the GUE minor measure of random matrices. This talk is based on the construction made with Alexei Borodin in arXiv:0804.3035 .

Alessandro Giuliani (Roma): *Height fluctuations in interacting dimers.*

Perfect matchings of Z^2 (also known as non-interacting dimers on the square lattice) are an exactly solvable 2D statistical mechanics model. It is known that the associated height function behaves at large distances like a massless gaussian field, with the variance of height gradients growing logarithmically with the distance. As soon as dimers mutually interact, via e.g. a local energy function favoring the alignment among neighboring dimers, the model is not solvable anymore and the dimer-dimer correlation functions decay polynomially at infinity with a non-universal (interaction-dependent) critical exponent. We prove that, nevertheless, the height fluctuations remain gaussian even in the presence of interactions, in the sense that all their moments converge to the gaussian ones at large distances. The proof is based on a combination of multiscale methods with the path-independence properties of the height function. Joint work with V. Mastropietro and F. Toninelli.

Leonid Petrov (Charlottesville): *Dynamics of random surfaces and interacting particle systems via spectral properties.*

Many interesting ensembles of random surfaces are "integrable", i.e., their distributions and asymptotics can be studied by algebraic methods. Often this integrability is triggered by remarkable algebraic identities, such as the Cauchy identity for summation of Schur (or, more generally, Macdonald) symmetric polynomials. With the help of these identities, integrable stochastic Markov dynamics on random surfaces (and also related interacting particle systems) can be modeled through multiplication operators in a "spectral" space. This approach also leads to discovery of new integrable Markov dynamics.

Full list of speakers and participants

BAUDEL Manon	MAPMO, Université d'Orléans	Orléans (France)
BERGLUND Nils	MAPMO, Université d'Orléans	Orléans (France)
BERTAIL Patrice	MODAL'X Université Paris 10	Nanterre (France)
BODINEAU Thierry	CMAP École Polytechnique	Palaiseau (France)
BOUTTIER Jérémie	CEA IphT	Gif sur Yvette (France)
BRUNET Eric	LPS - ENS	Paris (France)
CAMIA Federico	Mathematics, VU University	Amsterdam (The Netherlands)
CANTINI Luigi	LPTM Université de Cergy-Pontoise	Cergy-Pontoise (France)
CEPAS Olivier	Institut Néel	Grenoble (France)
CHEN Xinxin	Institut Camille Jordan	Villeurbanne (France)
CLERC Jean-Pierre	Polytech Marseille	Marseille (France)
COLOMO Filippo	INFN & Universita` di Firenze	Sesto Fiorentino (Italy)
COMETS Francis	Mathematics, Université Paris Diderot	Paris (France)
CONZE Jean-Pierre	IRMAR, Université de Rennes	Rennes (France)
CORTEEL Sylvie	LIAFA, Universite Paris Diderot	Paris (France)
DE BUYER Paul	Modal'X, Université Paris Ouest	Nanterre (France)
De TILIERE Béatrice	LPMA, Université Pierre et Marie Curie	Paris (France)
DUNLOP François	LPTM Université de Cergy-Pontoise	Cergy-Pontoise (France)
ERIGNOUX Clément	CMAP, Ecole Polytechnique	Palaiseau (France)
FERNANDEZ Bastien	Centre de Physique Théorique	Marseille (France)
FERNIQUE Thomas	LIPN & Univ. Paris 13	Villetaneuse (France)
FERRARI Patrik	Mathematics, Universität Bonn	Bonn (Deutschland)
FOURT Guy		Clermont-Ferrand (France)
GAO Hong	Physics, University of Potsdam	Potsdam (Deutschland)
GIACOMIN Giambattista	Mathematics, Université Paris Diderot	Paris (France)
GIULIANI Alessandro	Matematica e Fisica, Università Roma Tre	Rome (Italy)
GOBRON Thierry	LPTM Université de Cergy-Pontoise	Cergy-Pontoise (France)
GROSSKINSKY Stefan	Mathematics, University of Warwick	Coventry (U.-K.)
JONASSON Johan	Mathematics, Chalmers University	Göteborg (Sweden)
KLEIN Sarah	LPT, Université Paris-Sud	Orsay (France)
KOTECKY Roman	Charles University	Praha (Czech Republic)
LAQUERRIERE Benjamin	LMIA Université de La Rochelle	La Rochelle (France)
LASLIER Benoit	Statslab - Centre for Mathematical Science	Cambridge (U.-K.)

LECOMTE Vivien	LPMA Université Denis Diderot	Paris (France)
LIN Michael	Ben-Gurion University	Beer-Sheva (Israel)
LO Albert Y.	University of Science and Technology	Hong Kong
LOUIDOR Oren	Technion	Haifa (Israel)
LUCON Eric	MAP5 - Université Paris Descartes	Paris (France)
MACHICOANE Nathanaël	Laboratoire FAST	Orsay (France)
MARTINELLI Fabio	Mathematics and Physics, Università Roma Tre	Rome (Italy)
MIRACLE-SOLE Salvador	Centre de Physique Théorique	Marseille (France)
MUNGAN Muhittin	Bogazici University	Istanbul (Turkey)
NGUYEN Vu-Lan	LPMA Université Denis Diderot	Paris (France)
NOBUO Yoshida	Nagoya University	Nagoya (Japan)
OGUEY Christophe	LPTM Université de Cergy-Pontoise	Cergy-Pontoise (France)
PELED Ron	Mathematics, Tel Aviv University	Tel Aviv (Israel)
PERRET Anthony	LPTMS, Université Paris Sud	Orsay (France)
PETROV Leonid	Mathematics, University of Virginia	Charlottesville (USA)
POISAT Julien	CEREMADE, Université Paris-Dauphine	Paris (France)
ROSELLO Angelo	École normale supérieure de Cachan	Cachan (France)
RUIZ Jean	Centre de Physique Théorique	Marseille (France)
SAADA Ellen	MAP5, Université Paris Descartes	Paris (France)
SHLOSMAN Senya	Centre de Physique Théorique	Marseille (France)
SIRUGUE-COLLIN Madeleine	Centre de Physique Théorique	Marseille (France)
SPOHN Herbert	Zentrum Mathematics, Technische Universität	München (Deutschland)
SPORTIELLO Andrea	LIPN Université Paris-Nord	Villetaneuse (France)
TAMM Mikhail	Physics, Moscow State University	Moscow (Russia)
TARRES Pierre	CEREMADE, Université Paris-Dauphine	Paris (France)
THIERY Thimothée	LPT-ENS	Paris (France)
TONINELLI Cristina	LPMA, Université Denis Diderot	Paris (France)
TONINELLI Fabio	Institut Camille Jordan, Université Lyon 1	Villeurbanne (France)
TOUBOUL Jonathan	Collège de France	Paris (France)
TOURNIER Laurent	LAGA, Université Paris 13	Villetaneuse (France)
VROYLANDT Hadrien	ESPCI	Paris (France)
WON Yong Sul	Imperial College	London (U.-K.)
ZHANG Xiyun	University of Potsdam	Potsdam (Deutschland)