Knotted saddle periodic orbits in Beltrami flows – Scientific Report –

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1. *Purpose of the visit* The purpose of the visit was to investigate an algorithmic realization of the findnings of Enciso and Peralta-Salas [EPS]. They have shown that for any unbounded, locally finite link, there exists a smooth diffeomorphism close to the identity that transforms this link into a set of stream lines of a vector field that solves the steady, incompressible Euler equations in $\mathbb{R}^3$. These knots can be realized as saddle perodic orbits. They can be used as ground truth fields for the evaluation of methods that extract such structures, e.g., Kasten et al. [KRRS].

2. *Description of the work carried out during the visit*: The visit was granted for five days. I arrived at IST Austria at midday on Monday Feb 10 and departed in the late evening at Friday Feb 14.

   At the first day, I presented the findings of Enciso and Peralta-Salas and my first steps on the way to an implementation to Jan Reininghaus. We discussed the types of divergence-free vector fields in the context of the Helmholtz decomposition and we agreed that a numerial implementation using MATLAB should be done to solve the underlying eigenvalue problem. Therefore, we tested numerical algorithms at the second day that solve eigenvalue problems in one, two, and three dimensions for the Laplace operator, which is strongly connected to the curl operator. By discretizing the operator, the differential equations can be transferred to a matrix eigenvalue problem. We also discussed our problem with Chris Wojtan and Björn Hof. On Wednesday, we did a first implementation of the discretized curl operator.

   Having a solution to the eigenvalue problem, we saw that introducing the boundary conditions – predefined values that enforce the saddle periodic orbit to occur in the field - is not that simple. We therefore implemented an algorithm that searches for an optimal value for the eigenvalue of the discretized curl operator, while we also predefined values on the grid. On Thursday, we experimented with several optimization strategies to create a performant numerical algorithm. On Friday, we discussed our findings and searched for further optimization strategies and alternatives.

3. *Description of the main results obtained*: We were able to generate a field

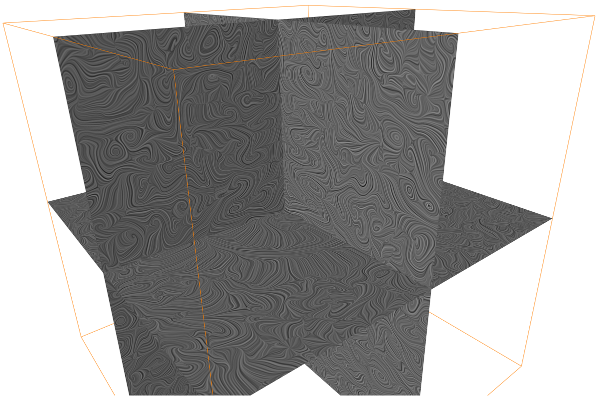

Figure 1: Simple saddle periodic orbit computed on a 32^3 grid that is nearly a solution to the Beltrami equation.

on a 32^3 grid that contains a ring as a saddle periodic orbit. A visualization of this field was done using Amira and can be seen in Fig. 1. One drawback of our current method is that it currently only works on a uniformly sampled grid. We have to investigate either different discretization strategies for the curl operator or multi grid methods to generate fields that contain more complicated structures.

4. *Future collaboration with host institution (if applicable)*: In the future, we plan to further investigate this problem. We plan to compute fields with either a higher resolution or a different grid type that contain knotted saddle periodic orbits.

5. *Projected publications/articles resulting or to result from your grant*: We plan to publish our findings and submit a paper to the TopoInVis workshop 2015. The submission will be done later this year the deadline should be in September 2014.

References

[EPS] A. Enciso and D. Peralta-Salas. Knots and links in steady solutions of the euler equations. *Annals of Mathematics*, 249:204–249.

[KRRS] J. Kasten, J. Reininghaus, W. Reich, and G. Scheuermann. Towards the extraction of saddle periodic orbits. submitted.