**Titel:** Path spaces in cubical complexes modelling concurrent computations

**Purpose:** Lisbeth Fajstrup and I took to the Faculty of Mathematics, Informatics and Mechanics at Warsaw University in order to continue our collaboration with Dr. Krzysztof Ziemianski. The overall goal was to inform each other and to combine efforts concerning various equivalent ways to approach spaces of directed paths in cubical complexes as they occur in models for concurrent computation.

**Work:**  We worked together during four days in workshop style, alternating between short informal talks and the discussion and comparison of ideas, examples and possible approaches.

**Results:** Ziemianski presented a new way of describing the space of paths in a cubical complex giving rise to a (combinatorially structured) cellular complex making use of an approximation by so-called tame paths. Raussen described path spaces in a Euclidean cubical complex (as those arising from so-called PV-programs) by way of configuration spaces contained in a product of simplices. Together with Meshulam (Technion) he had applied well-known techniques in combinatorial algebraic topology (Ziegler-Zivalievic) to recover previous computations with Ziemianski in a simpler way, using Alexander duality. In the discussions, it turned out that it might be difficult to use duality techniques in the general case; it still looks promising for paths in spaces with “thin” obstructions (given by linear equations, not inequalities).
For applications in Computer Science, it would be interesting to state easily verifiable criteria that ensure that path spaces within geometric models for “Higher Dimensional Automata” are pathwise-connected, so that the result of a concurrent computation is completely independent from the interleaving order in which various processors proceed. Some results (mainly by Fajstrup, building on previous work by Raussen) were discussed and stated; it is not yet clear whether they are as general as possible; in particular concerning higher connectivity. Such results are also important for the investigation of serializability of Higher Dimensional Automata, in particular those arising when one PV-program is run concurrently with itself.
Ziemianski obtained last year the result that every simplicial complex is homotopy equivalent to a path space associated to some PV-space. He initiated a discussion about how to characterize simplicial complexes associated to PV-spaces with restricted capacities. This question seems to be difficult to answer in general; only some very preliminary results were obtained.

**Collaboration:** Collaboration with Ziemianski has turned out to be fruitful in the past; we are determined to continue it in the future. We plan to meet again along with workshops in Copenhagen (November 2014) and Aalborg (April 2015).

**Publications:** Plans for publications were discussed at the end of our workshop; they are at various stages so far. We have in progress drafts for publications about new alternative models for spaces of directed paths in (special) cubical complexes (cellular and configuration spaces) and also about criteria for the (higher) connectivity of path spaces in particular configurations.